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Coupled-Channel Born-Approximation Calculation of Two-Nucleon Transfer Reactions in Deformed Nuclei

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Two-neutron transfer reactions in strongly deformed nuclei are discussed in the framework of the coupled-channel Born approximation. In particular, the reaction ^{176}Yb$(p,t)^{174}$Yb is analyzed.

The amplitude of the coupled-channel Born approximation (CCBA) for a transfer reaction $A(a,b)B$ with $a=b+x$, may schematically be written as[1]

$$T=\langle\Psi_b^{(-)}(s_A+x,x_b,r_b)\,|\,V(x,x_b)\,|\,\Psi_a^{(+)}(x_A,x_a,r_a)\rangle, \tag{1}$$

where the wave functions $\Psi^{(\pm)}$ are solutions of appropriate coupled-channel equations[2] that describe the inelastic scattering on an essentially equal footing with the elastic scattering. Thus (1) is a natural extension of the distorted-wave Born-approximation (DWBA) amplitude in which the $\Psi^{(\pm)}$ describe only elastic scattering.

Equation (1) has been applied exactly only in very few cases so far.[3] Ascuitto and Glendenning[4] developed the so-called "source term" method which uses a different approach from the usual CCBA but is believed to yield an amplitude that agrees with (1). They performed calculations[5] for (p,t) processes between moderately collective nuclei (Ni) and found that CCBA predicts, for example, twice as large a cross section to the collective 2^+ state as the normal DWBA calculation does. In the present article we apply (1) to a (p,t) process between two strongly deformed nuclei, and compare the results with those of corresponding DWBA calculations.[6]

In CCBA, the functions $\Psi^{(\pm)}$ in (1) may very explicitly be written as

$$\Psi_{m_a'M_{nA}';\,m_aM_{nA}}=\frac{4\pi}{k_ar_a}\sum_i i^{l_a'}(\hat{l}_a'/\hat{l}_a)\chi_{l_a'j_a'n_A';\,l_aj_an_A}^{J}(r_a)(l_am_{la}s_am_a|j_am_{ja})(j_am_{jA}I_{na}M_{nA}|JM)$$
$$\times(l_a'm_{la}'s_am_a'|j_a'm_{ja}')(j_a'm_{ja}'I_{nA}'M_{nA}'|JM)Y_{l_am_{la}}^{*}(\Omega_{\vec{k}_a})$$
$$\times Y_{l_a'm_{la}'}(\Omega_{\vec{r}_a})\phi_{I_{nA}M_{nA}}(x_A)\varphi_{s_am_a}(x_a), \tag{2a}$$

$$\Psi_{m_b'M_{nB}';\,m_bM_{nB}}^{(-)*}=(-)^{I_{nB}-I_{nB}'+m_b'-m_b+M_{nB}'}\Psi_{-m_b'-M_{nB}';\,-m_b-M_{nB}}^{(+)}. \tag{2b}$$

Since we use a notation very similar to that used in Ref. 2 and by Satchler[7], the meaning of (2) is clear. We just note that the function $\chi_{l_a'j_a'n_A';\,l_aj_an_A}^{J}(r_a)$ describes the radial part of the relative motion between A and a with angular momenta (l_a',j_a'), in the channel in which A lies in its n_A'th state, when the only incoming wave present is the one having angular momenta (l_a,j_a) in the channel in which A lies in its n_Ath state. The superscript J is the total angular momentum of this set of coupled partial waves, while $\phi(x_A)$ and $\varphi(x_a)$ are internal wave functions of A and a, respectively.

Inserting (2) into (1) and performing algebra which extends that of Ref. 7, the amplitude (1) under

the zero-range approximation is given as[8]

$$T_{M_{nB}m_b;\,M_{nA}m_a}(\theta)=\sum\hat{j}\frac{4\pi}{k_a k_b}\frac{B}{A}(-)^{(|m|-m)/2}\left[\frac{(l_b-|m|)!}{(l_b+|m|)!}\right]^{1/2}P_{l_b m}(\theta)(l_a 0 s m_a|j_a m_a)(j_a m_a I_{nA}M_{nA}|JM)$$
$$\times(l_b-m s_b m_b|j_b-m_{jb})(j_b m_{jb}I_{nB}-M_{nB}|J-M)\int\chi_{l_b' j_b' n_B';\,l_b j_b n_B}{}^{J}(A/B'a)$$
$$\times F_{lsj}(I_{nA}'I_{nB}';r_a)\chi_{l_a' j_a' n_A';\,l_a j_a n_A}{}^{J}(r_a)dr_a i^{l-(l_a'+l_b')}(-)^{j_b-j_a'+I_{nB}-I_{nA}'}$$
$$\times(\hat{l}_a'\hat{l}_b')^2\hat{j}_a'\hat{j}_b'\hat{s}\hat{I}_{nB}'W(j_a'j_b'I_{nA}'I_{nB}';jJ)(l_a'0l_b'0|l0)\begin{Bmatrix}l_a' & s_a & j_a'\\ l_b' & s_b & j_b'\\ l & s & j\end{Bmatrix}. \qquad (3)$$

This expression is essentially the same as given by Penny and Satchler[1] but is much simplier. See Ref. 8 for this simplification.

If the transferred neutrons are coupled to spin zero, the radial form factor will be written as[9]

$$F_{l0l}(I_{nA}'I_{nB}';R)=i^l V(-)^{l+I_{nA}'-I_{nB}'}\sum_{j_1\le j_2}\langle\phi_{I_{nB}'}|(1+\delta_{j_1 j_2})^{-1/2}\{[b_{j_1 j_2}]_l\phi_{I_{nA}'}\}I_{nB}\rangle\sum_{1,2}[K_{1/2}(\sigma_1)K_{1/2}(\sigma_2)]_0$$
$$\times\int d\Omega_{\vec{R}}d\vec{r}\,Y_{l0}(\Omega_{\vec{R}})\Theta(\vec{r},\vec{R})\{a[\Phi_{j_1}(\vec{n}_1)\Phi_{j_2}(\vec{n}_2)]_l\}, \qquad (4)$$

where $\vec{r}$ and $\vec{R}$ are, respectively, the relative and center-of-mass coordinates of the two transferred neutrons and b_j is the annihilation operator of a shell-model particle carrying angular momentum j. The last factor in the brackets denotes a normalized, antisymmetrized two-neutron shell-model wave function. The single-particle neutron states are calculated in Saxon-Woods wells using the separation-energy procedure. In deriving (3) and (4), the interaction $V(x, x_b)$ in (1) has been assumed to be independent of all the nucleon spins and of the relative motion of the two neutrons, and is such that the proton interacts with the center of mass of the two neutrons with zero range. This last assumption allows us to make the coordinate R play the role of r_a in (3). The rest of the notation in (4) is the same as in Ref. 9.

The numerical calculations of the amplitude (3) and the corresponding cross sections were performed by using a computer program called MARS.[8] This program uses as a subroutine the coupled-channel program JUPITOR-1[2,10] to obtain the functions $\chi_{l'j'n';\,ljn}{}^{J}(r)$ and then reads in the form factor (4). In evaluating the integral in (4) the method and the computer program TWOPAR by Bayman and Kallio[9] were used. The spectroscopic amplitude, i.e., the first bracketed factor in the summand in (4), was calculated by using the Nilsson and BCS models as in Ref. 6. All the numerical calculations of the present work are done for the analysis of the ^{176}Yb$(p, t)^{174}$Yb processes[11] populating members of the ground-state rotational band of ^{174}Yb. The spectroscopic amplitudes were evaluated by using nuclear-structure information extracted from the data given by Burkes et al. and Chasman[12] and by Duchworth, Baker, and van Rookhuysen[13] and Stilson and Grodzins.[14]

Before comparing the predictions of CCBA with experiment, it is enlightening to give comparisons between the predictions of CCBA and DWBA. We thus made a CCBA calculation with no coupling in the incident channel and a 0^+-2^+ coupling in the exit channel, and a DWBA calculation using the same optical parameters,[15] except that the CCBA had the deformation $\beta_2=0.3$ as an extra parameter. Although in this case the angular distributions were not very different, the cross sections to the 0^+ and 2^+ states were larger in CCBA than in DWBA, by a factor of 2 and 5, respectively. In order to understand this difference, we first note that in DWBA only the diagonal radial wave functions, i.e., only the functions $\chi_{l'j'n';\,ljn}{}^{J}(r)$ with $(l'j'n')=(ljn)$ appear, while nondiagonal radial wave functions can also appear in CCBA. We shall call an overlap integral in (3), involving only diagonal wave functions, a diagonal integral, and call the rest nondiagonal integrals. We found that the diagonal integrals in CCBA are only very slightly different from their corresponding values in DWBA. In spite of the known fact that the nondiagonal wave functions are usually much smaller than the diagonal ones, the nondiagonal integrals often

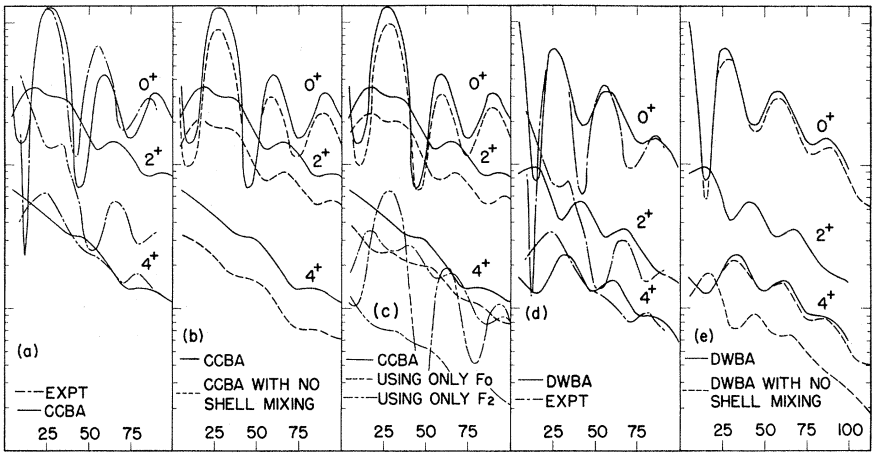

FIG. 1. CCBA results are given in (a), (b), and (c), and DWBA results are given in (d) and (e). Among the same type of curves, the 0^+, 2^+, and 4^+ cross sections appear in decreasing magnitudes, except that the 2^+ cross section with no shell mixing in (e) is smaller than the corresponding 4^+ cross section. The same (arbitrary) unit has been used for both CCBA and DWBA calculations. The experimental cross sections are normalized so that the first 0^+ maximum is reproduced by both CCBA and DWBA.

have magnitudes comparable with those of the diagonal integrals. This is due to the fact that the contribution from the nuclear interior is essentially zero in most diagonal integrals, because of the rapid oscillation of the form factor, as well as that of the distorted waves. Such a cancelation is found to be weaker in nondiagonal integrals.

Since the rapid oscillation of the form factor is a feature common to any multinucleon transfer reactions, it may turn out that CCBA differs more from DWBA for such reactions than for one-nucleon transfer reactions (even for transitions for which the DWBA process is not forbidden). A more systematic survey on this point, taking into account also the effect of other mechanisms like momentum mismatch, will be of great interest.

In order to analyze the data of Ref. 11, the CCBA calculations were performed considering a 0^+-2^+ coupling in the incident channel, and 0^+-2^+-4^+ coupling in the exit channel. A fairly extensive, though not exhaustive, search has been made of the optical-model parameters aiming at fits to existing scattering as well as (p, t) data. The best set of parameters used in obtaining CCBA curves in Fig. 1 is presented in Table I. It explains well the elastic and inelastic proton scattering data[11] from ^{176}Yb and the elastic triton scattering data[17] from ^{182}W. The DWBA parameters, presented in Table I and used to obtain corresponding curves in Fig. 1, also gave fits to elastic scattering data in both channels.

The use of different parameters for CCBA and DWBA is justified since in both approaches the basic idea is to see whether the optical parameters that fit the scattering also fit the reaction data. Only internal consistency is required.

It is seen in Figs. 1(a) and 1(d), that the ratios between the 0^+, 2^+, and 4^+ cross sections [to be called $\sigma(0^+)$, $\sigma(2^+)$, and $\sigma(4^+)$ henceforth] are very much the same for DWBA and CCBA. Moreover, these ratios, as well as the angular distributions, agree with experiment to the same degree of satisfaction. Thus, so far as the fitting of the relative cross-section data is concerned, DWBA and CCBA do not differ very much from each other. However, the absolute value of the CCBA cross section is larger than that of the DWBA.

An important difference between DWBA and CCBA can be seen by comparing Fig. 1(b) with Fig. 1(e). As was noted in Ref. 6, and is seen in Fig. 1(e), it is necessary to mix the major-oscillator shells in the Nilsson model if the DWBA ratio $\sigma(2^+)/\sigma(0^+)$ is to reproduce the experimental value. We thus performed CCBA calculations using two sets of form factors, one with the major-shell mixing, and the other without it. The results are given in Fig. 1(b). As is seen, the shell mixing in CCBA is much less significant than in DWBA [Fig. 1(e)]. Since the shell mixing affects[6] rather largely the form factor F_2, but very little F_0 and F_4, the relative insensitivity to the shell mixing of the CCBA $\sigma(2^+)$ indicates that F_2 is not contributing to

Table I. Optical parameters (following the notation of Ref. 10).

	V	R_{v0}	A_v	W	R_{w0}	A_w	W_D	R_D	A_D	W_0	W_2	W_4	β_2[a]	β_4[a]	
(a)	55.600	1.250	0.720				17.800	1.250	0.470	0.8	0.9	1	0.230	−0.045	proton CCBA
(b)	54.633	1.222	0.670				18.060	1.040	0.900	0.8	0.9	1	0	0	proton DWBA
(c)	181.220	1.240	0.600	13.440	1.718	0.870				0.8	1	1	0.230	−0.045	triton CCBA
(d)	162.700	1.240	0.705	32.800	1.218	1.035				0.8	1	1	0	0	triton DWBA

[a]See Hendrie *et al.*, Ref. 16.

$\sigma(2^+)$ significantly. Further support for this conclusion is furnished by the three sets of CCBA displayed in Fig. 1(c), which were calculated by including (1) all the form factors F_0, F_2, and F_4; (2) only F_0; and (3) only F_2. As is seen, $\sigma(2^+)$ for case (2) is about 7 times as large as that for case (3), indicating that in the full form-factor calculation, the two-step processes, rather than the one-step process, account for the major part of $\sigma(2^+)$. However, the fact that $\sigma(2^+)$ of case (3) in Fig. 1(c) is smaller than $\sigma(2^+)$ of Fig. 1(d) indicates that the two-step process can sometimes work destructively. A similar difference between DWBA and CCBA may be observed in predicting the influence of the Y_4 deformation in populating the 4^+ state.[6]

The form factor used in most of the above calculations was obtained by assuming the quadrupole-deformation parameter β_2 to be[14] 0.3, and by including all the levels belonging to the $N=4$, 5, and 6 oscillator shells in the BCS calculation. We also constructed form factors by reducing the value of β_2 to[16] 0.23 and/or including only 22 Nilsson orbits in the BCS calculation. Apart from some changes in absolute magnitudes, the resulting cross sections were quite insensitive to those variations.

In conclusion, we have found that DWBA and CCBA work equally well insofar as the fitting of the relative cross-section data is concerned. However, it was also found that sometimes quite different information can be extracted if data fitting is done with DWBA rahter than CCBA. Thus DWBA would have to be used with caution particularly when multinucleon-transfer reaction data is analyzed, though clearly more experience with CCBA calculations has to be accumulated in order to known under what circumstances the CCBA is indispensable. In parallel with such a survey, it may also be important to make similar investigations in other features of the calculation, i.e., those of the form factors (including the zero-range and the separation-energy approximations), spectroscopic amplitudes, etc.

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Effect of the Earth's Revolution Around the Sun on the Proposed Gyroscope Test of the Lense-Thirring Effect

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We analyze the proposed Stanford experiment (precession of the spin of a gyroscope in an Earth satellite) to test the Lense-Thirring effect. We show that the sun also makes a contribution to the precession which must be included, particularly if one wishes to distinguish between the Einstein and Brans-Dicke theories.

Modern technology is making possible new tests of Einstein's general theory of relativity. One of these is Schiff's[1,2] proposed gyroscope experiment. Everitt and Fairbank[3] and Fairbank[4] expect to carry out this experiment in the near future by launching a satellite containing two pairs of superconducting gyroscopes into a polar orbit around the Earth; the spin of one pair (gyro No. 1) will be parallel to the Earth's axis and the spin of the other pair (gyro No. 2) will be perpendicular to the plane of the orbit.[2-4] Not only is this test capable of distinguishing[5] between the gravitational theories of Einstein and of Brans and Dicke[6] (BD), but it is the only experiment which is sensitive to the off-diagonal terms in the metric tensor. The latter terms result from the Earth's rotation and were calculated by Lense and Thirring[7] soon after Einstein's work.

The angular velocity of precession of the spin axis $\vec{S}$ of a gyroscope in Einstein theory, $\vec{\Omega}_E$ say, may be written as[1,2,5]

$$\vec{\Omega}_E = \vec{\Omega}_T + \vec{\Omega}_{DS} + \vec{\Omega}_{LT} + \vec{\Omega}_Q, \tag{1}$$

where $\vec{\Omega}_T$, $\vec{\Omega}_{DS}$, $\vec{\Omega}_{LT}$, and $\vec{\Omega}_Q$ are the so-called Thomas, de Sitter, Lense-Thirring, and quadrupole-moment[8,9] contributions, respectively. From henceforth, we will regard the $\vec{\Omega}$'s as being averaged over a period of the motion. It is possible to have $\vec{\Omega}_T$ essentially zero[1] by putting the gyroscope in a satellite. The importance of selecting a polar orbit results from the fact that $\vec{\Omega}_{DS}$ and Ω_{LT} are at right angles[1-4] for such an orbit. For definiteness, consider the Earth's angular velocity to be in the z direction and the polar orbit to be in the xz plane so that the orbital angular momentum of the satellite points in the y direction. Then $\vec{\Omega}_{DS}$ lies along y and $\vec{\Omega}_{LT}$