

the more intense γ rays. In order to establish the correct mass assignment, Ho and Tb foils were bombarded with different beam energies. Comparison of these samples, the time dependence of the intensities of the observed γ rays, and the growth of the known daughter products² allowed a very reliable mass assignment.

Preliminary coincidence experiments (4096 \times 4096 channels) favor the existence of energy levels at 49.5 ($\frac{5}{2}^-$), 141.8 ($\frac{7}{2}^-$), 258.9 ($\frac{9}{2}^-$), 494.1, 506.4, and 1008 keV. Further coincidence studies are in progress in order to establish a more complete level scheme of Hf¹⁷¹.

In addition to Ta¹⁷¹, Ta¹⁷⁰ was observed in the same experiment, produced via the reactions Ho¹⁶⁵(C¹², 7n)Ta¹⁷⁰ and Tb¹⁵⁹(O¹⁶, 5n)Ta¹⁷⁰. The 2⁺-0⁺ and 4⁺-2⁺ ground rotational band transi-

tions in Hf¹⁷⁰ were known from earlier in-beam conversion electron studies.³ Here, the energies of these transitions were determined as 100.9 \pm 0.1 and 221.3 \pm 0.2 keV, respectively. The observation of these γ rays decaying with 7 \pm 2 min half-life adds strong weight to the assignment of the activity to Ta¹⁷⁰.

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π -Carbon Elastic Scattering Near the 33 Resonance

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Elastic π^- -C scattering is calculated with the Kisslinger optical model from 120 to 280 MeV. With parameters obtained from π^- -N phase shifts, general qualitative agreement is obtained with the recent Binon *et al.* data. Modifying the parameters further improves the fit.

We have calculated π^- -carbon elastic-scattering differential cross sections at energies near the 33 π -nucleon resonance using the Kisslinger optical model.^{1,2} Our results for optical parameters derived from two-body phase shifts agree well qualitatively with the recent experiments

of Binon *et al.*³ at seven energies from 120 to 280 MeV. This contrasts to the large differences between theory and experiment recently found in pion double-charge-exchange reactions,⁴ in the production of C¹¹ by π^\pm bombardment⁵ of C¹², etc.

Table I. Kisslinger model parameters. Units are F³. "Free parameters" are those derived directly from π -N phase shifts, while "Fermi averaged parameters" are corrected for nucleon motion. "Best fit" curves in Figs. 1 and 2 are computed with Fermi averaged b_0 values.

Energy (MeV)	Free parameters				Fermi averaged parameters				Fitted parameters	
	Re b ₀	Im b ₀	Re b ₁	Im b ₁	Re b ₀	Im b ₀	Re b ₁	Im b ₁	Re b ₁	Im b ₁
120	-0.95	0.40	8.13	5.59	-0.87	0.42	7.34	7.15	9.24	3.14
150	-0.81	0.36	5.63	8.52	-0.73	0.38	3.84	8.27	-0.09	13.75
180	-0.72	0.33	1.71	8.81	-0.61	0.35	0.64	7.48	0.87	9.48
200	-0.67	0.31	-1.21	7.45	-0.54	0.33	-0.83	6.40	-0.39	8.61
230	-0.60	0.30	-2.46	4.99	-0.44	0.32	-1.98	4.66	-0.39	7.10
260	-0.51	0.28	-2.67	3.22	-0.34	0.31	-2.27	3.24	-1.55	5.33
280	-0.48	0.27	-2.62	2.42	-0.27	0.31	-2.18	2.53	-1.69	4.48

The optical potential in momentum space is approximately given by⁶

$$\langle \vec{p}' | V | \vec{p} \rangle = A \langle \vec{p}' | t | \vec{p} \rangle \rho(\vec{p}' - \vec{p}), \quad (1)$$

where t is the π - N amplitude, and ρ is the Fourier transform of the nuclear density. Both s and p waves contribute even well below and above the resonance at $T_r \approx 190$ MeV, so that

$$\langle \vec{p}' | t | \vec{p} \rangle \approx a_0 + a_1 \vec{p} \cdot \vec{p}'. \quad (2)$$

Here a_0 and a_1 are nearly constant for $T \ll T_r$, but a_1 varies rapidly near T_r . With (2), in coordinate space we have

$$U \equiv 2EV(r) = -Ab_0 p_0^2 \rho(r) - Ab_1 \vec{p} \cdot \rho(r) \vec{p}. \quad (3)$$

Here for $l=0$, 1

$$b_l = \frac{4\pi}{p_0^3} \left(\frac{\mu^2 + M^2 + 2EM}{M^2} \right) [k f(0)_{c.m.}]_l, \quad (4)$$

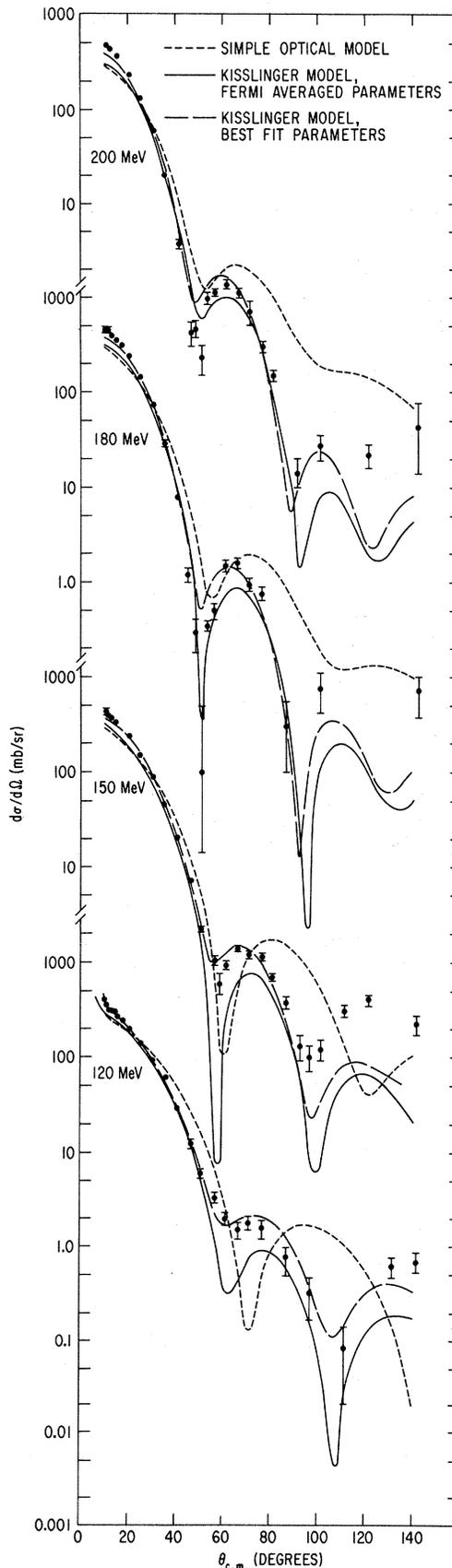
where μ , p_0 , and E are the pion mass, lab momentum, and total lab energy, k is the c.m. momentum, and M is the nucleon mass. [The π - N amplitude $f(0)$ is averaged over π - p and π - n for a zero isospin nucleus such as carbon.]

Earlier¹ we applied this model to the limited data then available for carbon and a few other nuclei at energies around 80 MeV. We numerically integrated⁷ a Klein-Gordon equation with a quadratic Coulomb but not a V^2 term included. Good agreement with experiment was found with b 's from free π - N phase shifts and with densities given by electron scattering data. At lower energies similar results were obtained, except that because of an accidental cancellation in the s -wave phase shifts ($\delta_1 + 2\delta_3 \approx 0$), $Re b_0$ had to be adjusted considerably; this can be understood in terms of higher-order terms in the optical potential.⁸

Near T_r , the rapid variation of a_1 with kinetic energy suggests an effective position dependence of b_1 , since the pion kinetic energy will vary with position. Indeed, the use of the impulse approximation (free amplitudes) for the individual scatterings in the multiple scattering series approximately summed by (1) is only marginally valid for so large an amplitude.⁹

Nevertheless, we decided to compare the Kisslinger model predictions with the new π^- -C

FIG. 1. Elastic π^- -C scattering from 120 to 200 MeV. Kisslinger and simple optical models with parameters from free π - N phase shifts corrected for Fermi motion, and Kisslinger model with b_1 adjusted for best fit. (See Table I for parameters used).



data³ to see how near the resonance it would actually work before attempting any more elaborate description. We took

$$\rho(r) = \rho_0 [1 + (Z-2)r^2/3a^2] \exp(-r^2/a^2),$$

$$a = 1.5 \text{ F} \quad (5)$$

in agreement with electron scattering data¹⁰ and our earlier π -C fits. For comparison, we also calculated with the usual simple (velocity independent) optical potential, $V \sim f(0)\rho$, which is obtained here by replacing (2) with

$$\langle \vec{p}' | t | \vec{p} \rangle \approx \langle \vec{p}' | t | \vec{p} \rangle = a_0 + a_1 p_0^2 \quad (6)$$

or by $b_0 \rightarrow b_0 + b_1$, $b_1 \rightarrow 0$ in (3).

The results given by both models are shown in Figs. 1 and 2 for b 's obtained by averaging those obtained from π -N phase shifts over the Fermi motion of the nucleons (Table I).¹¹ The Kisslinger model successfully predicts the positions of all the observed minima, i.e., the first minimum at all energies and the second minima for $T \leq 200$ MeV. It also suggests the observed deepening of the first minimum near the resonance. Typically, the calculated points are within a factor of 2 or less of the measured values, as the latter vary with angle by four orders of magnitude. By contrast, the simple optical model gives a much poorer fit at each energy, and fails to give the right qualitative behavior beyond the first minimum.

Very good fits to the data can be obtained with the Kisslinger model if b_1 is varied moderately (Figs. 1 and 2 and Table I).¹² However, the points beyond the second minimum could not be fit by varying the b 's. Limited studies of the effects of density variations using Saxon-Woods shapes also failed to improve the large-angle results.

Corrections to the Kisslinger model which will modify the large momentum-transfer scattering are under study. These may help to explain the unexpected success of the model as well as improving the fit at large angles. We also plan to investigate inelastic scattering.

In conclusion, the Kisslinger model is generally successful in predicting the features of π^- -C elastic scattering throughout the 33-resonance region. Wave functions obtained from it should be fairly reliable for use in calculating more complex processes.

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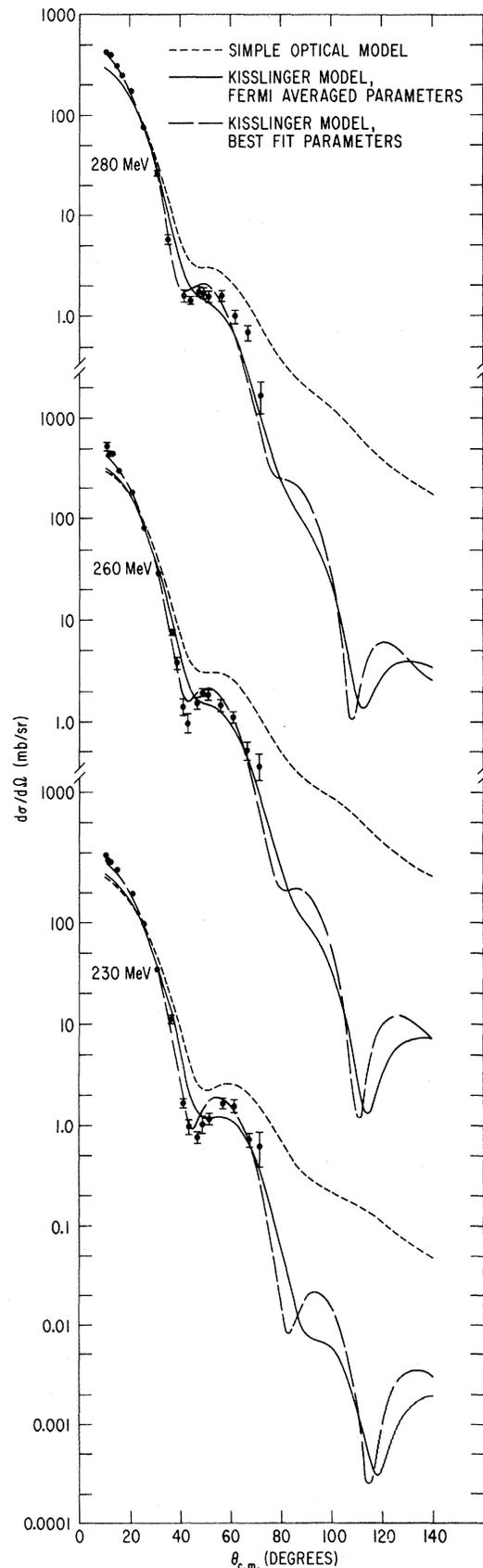


FIG. 2. Elastic π^- -C scattering from 230 to 280 MeV as in Fig. 1.

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Anomalies in the Cross Sections of (d, n) Reactions Leading to Isobaric Analog States*

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The (d, n) reactions on the nuclei ^{92}Mo , ^{94}Mo , ^{96}Mo , and ^{96}Zr leading to the isobaric analog states in the residual nuclei have been studied. The observed (d, n)-reaction cross sections show strong deviations from the distorted-wave Born-approximation calculations based upon the known neutron spectroscopic factors of the corresponding parent analog states.

In this communication we present evidence that the (d, n) reactions on several target nuclei in the $N=50$ region leading to the isobaric analog states of the target-plus-neutron system display anomalous behavior. The anomalies consist of large deviations of the observed (d, n)-reaction cross sections from the values predicted by distorted-wave Born-approximation (DWBA) calculations. We find that the (d, n) cross sections for the reaction leading to $d_{5/2}$ and $d_{3/2}$ isobaric analog states are strongly enhanced, whereas those leading to $s_{1/2}$ isobaric analog states tend to be smaller than the DWBA values. For almost all of the nuclei studied, the isobaric analog of the ground state of the target-plus-neutron system is populated far more strongly than any other isobaric state. Recently, McGrath et al.¹ have reported the measurement of ($^3\text{He}, d$) cross sections on $^{90,92,94}\text{Zr}$ and $^{92,94,96,98}\text{Mo}$ targets to unbound isobaric analog states. All expected tran-

sitions are observed in their work except those to $3s_{1/2}$ analog states which are inhibited. Preliminary attempts to understand this anomaly in terms of the distorted-wave Born approximation have been reported to be unsuccessful.

The data in the present work were obtained during a systematic study of the (d, n) reaction as a tool for obtaining information on the single-proton character of states in the $N \cong 50$ region.² The experiments were performed using the Oak Ridge National Laboratory tandem Van de Graaff accelerator in conjunction with the pulsed and bunched-ion source. The experimental equipment and procedures have been described elsewhere.³ The deuteron-beam energy was 12 MeV, and the neutron flight path from the target to the detector was 12 m. Rolled foils of ^{92}Mo ($910 \mu\text{g}/\text{cm}^2$), ^{94}Mo ($1 \text{ mg}/\text{cm}^2$), ^{96}Mo ($960 \mu\text{g}/\text{cm}^2$), and ^{96}Zr ($450 \mu\text{g}/\text{cm}^2$) were used as targets. Figure 1 shows four (d, n) neutron time-of-flight spectra