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Redetermination of the Magnetic Moment of the Proton in Nuclear Magnetons*

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A redetermination of the magnetic moment of the proton in nuclear magnetons has been performed using an omegatron. Special care was taken to make the phase of the rf electric field uniform inside the omegatron, and an almost rectangular line shape consistent with simple theory was obtained. Measurements were made both before and after gold plating the omegatron to check on the effects of nonuniform surface potentials. The value obtained uncorrected for diamagnetism was 2.792 783 ± 0.000 016 (±6 ppm).

The magnetic moment of the proton in nuclear magnetons can be obtained by measuring the cyclotron frequency f_c and spin resonance frequency f_s in the same magnetic field. Simple theory gives $f_s/f_c = \mu_p/\mu_N$, where μ_N is the nuclear magneton and μ_p is the proton moment. Standard techniques were used to measure f_s with an accuracy of 5×10^{-7}.

The rectangular charge-collecting omegatron shown in Fig. 1 was used to measure f_c. It is similar in design to that of Sommer, Thomas, and Hipple (STH).[1] A beam of 70-eV electrons, which is highly collimated by the magnetic field, produces ions by collisions with hydrogen gas. The desired ions spiral out toward the collector due to the force applied by a vertical rf electric field whose frequency is approximately the cyclotron-resonance frequency. Those ions hitting the ion collector form a current. The information of interest comes from the variation of this ion current with the frequency of the rf electric field.

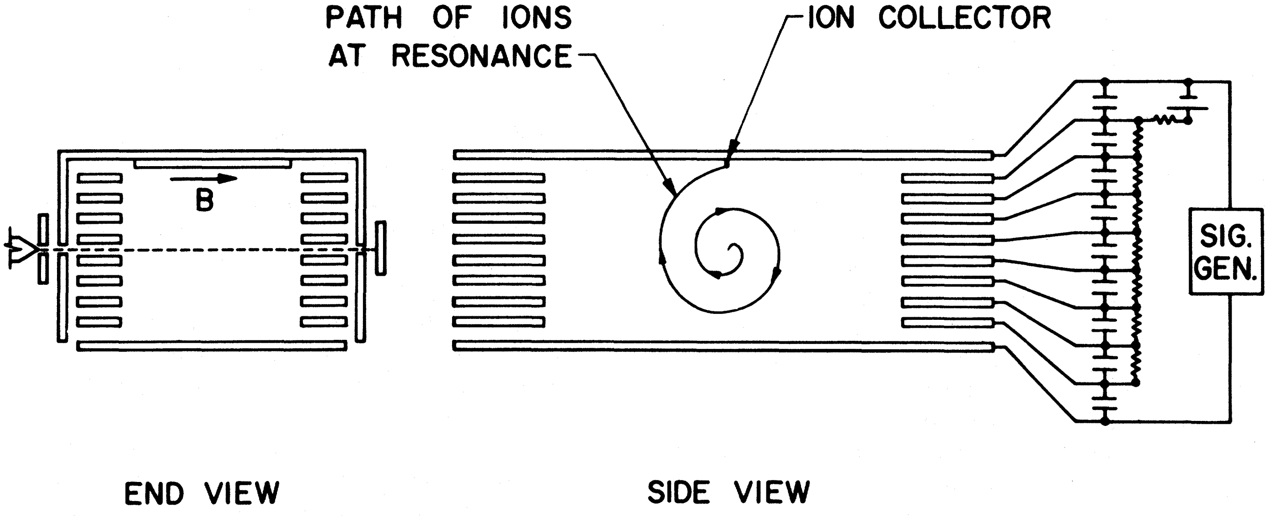

FIG. 1. Schematic representation of the omegatron. The size is 1.2 times actual with the plate thicknesses being 3 to 5 times actual.

The rf field is produced by applying an rf voltage between the bottom plate and the rf grounded top plate. The rf field uniformity is achieved by using eight rectangular window-frame shaped guard rings. The volume inside the end plates and guard rings is 20×20×40 mm. A capacitive rf potential divider with short leads connecting to the plates and guard rings is used. Each element is 820 pF. Special attention is given to reducing to a minimum any phase shift between the stack of guard rings and the end plates.

For an ion starting from rest in a homogeneous magnetic field B the radius of the particle's orbit as a function of time and in the absence of an electrostatic field, in approximation, is (in mks units)

$$r = \frac{E_{\mathrm{rf}}}{B} \frac{\sin[\frac{1}{2}(\omega_{\mathrm{rf}} - \omega_c)t]}{\omega_{\mathrm{rf}} - \omega_c},$$

where ω_{rf} is the angular frequency of the rf electric field E_{rf} and ω_c is the cyclotron angular frequency. Thus, in the absence of collisions and drift, all ions would reach the collector at $r = R$ for $|\omega_{\mathrm{rf}} - \omega_c| < E_{\mathrm{rf}}/(BR)$ and no ions would be collected for $|\omega_{\mathrm{rf}} - \omega_c| > E_{\mathrm{rf}}/(BR)$. These conditions correspond to a rectangular-shaped line with infinitely steep sides. The actual shape of the sides of the line is controlled by at least six considerations: the strength of the rf electric field, the initial velocities of ions at production, the effects of collisions, the magnetic-field homogeneity, the variation of potential over surfaces, and the rf phase uniformity in the omegatron. A high rf electric field produces a line which is less susceptible to the influence of surface potential variations, drift, and collisions. No attempt was made to measure lines of low E_{rf}, and only lines of almost rectangular shape with steep sides were used.

Line shapes such as the one shown in Fig. 2 were obtained. They are similar to those predicted by simple theory. These almost rectangular shapes have not been used in previous μ_p/μ_n experiments. Typical asymmetries (difference in center frequency between 15 and 85% of peak) for proton lines were five parts in 10^7 and those for other ions used were less than two parts in 10^6. The linewidth at 90% of maximum was typically 80% of the linewidth at 5% of maximum for lines 100-ppm wide. This improved shape is believed to result from a combination of increased trapping time due to the higher magnetic field and also the reduction of the phase difference of the rf electric field between the guard rings and the end plates which resulted from having the complete voltage divider only 40 mm from the guard rings. Strongly asymmetric lines were obtained when a phase difference was introduced between the guard rings and the end plates.

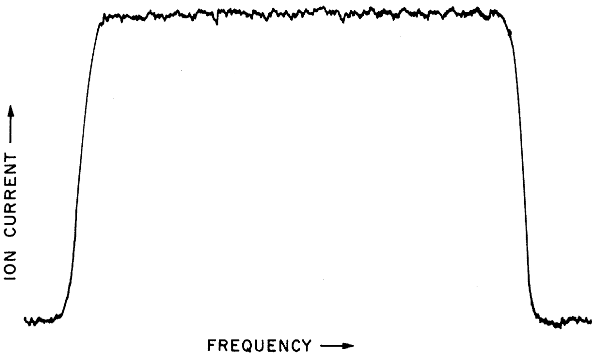

FIG. 2. The proton line shape shown above is 110-ppm wide and the error associated with asymmetry and random error is less than 1 ppm.

Frequency shifts as well as asymmetries can occur if the phase of the rf field seen by the ions near their starting points is different from that of the averaged field over their orbits just before they are collected. These can result from two different types of rf field inhomogeneities. First, they can arise from a variation of the phase of the rf electric field across the volume of the omegatron. Second, shifts can result from a change in average direction of the rf electric field with orbit size or orbit drift. These effects can be most easily described by decomposing the electric field into its two counter-rotating components. Only one of these components is effective in producing ion acceleration. This component may be represented as $(E_0/2)\exp[j(\omega t + \varphi)]$, where E_0 is the amplitude of the rf electric field, ω is the angular frequency of the rf source, and φ is either the phase of the field or the angle between the direction of the component of the rf electric field which is perpendicular to the magnetic-field direction and the x axis. φ is a constant over the volume only in a uniform rf field. The average frequency of this field as seen by the ion should be $\omega + d\varphi/T$, where T is the rotation period and $d\varphi$ is the difference in the average phase or direction of the field between one orbit and the next. In the present experiment the effect of deviations from rf uniformity is less than ±0.5 ppm.

The electric field required to trap the particles in the magnetic-field direction causes the principal shift of a single ion-resonance curve center. This shift is corrected for in a similar way to that used by STH; however, here only ions of low relative concentration were cycled in order that the shift due to change of space charge with change of resonant ion could be made negligible. Measurements on at least two ion species are required to correct for the shift due to a combination of space charge and the electrostatic trapping field. One can extrapolate to the electric-field-free result by going to zero mass while assuming the electric field to be independent of the ion observed. Simple theory of the electrostatic shift for fields of hyperbolic symmetry gives $f_{\mathrm{obs}} = f_c[1 - E(r)M/(B^2er)]$, where B is the magnetic-field intensity, $E(r)$ is the radial electric field at a distance r from the center of the electron beam averaged over one orbit, and M is the mass of the ion. This relation holds to better than 0.1 ppm for the parameters which characterize this experiment in the absence of drift. Since $E(r)/r$ is not constant with radius in a rectangular omegatron, we can more correctly write $f_c = f_c(1 - KM)$, where K is a complete path average of $E(r)/(B^2er)$. Figure 3 shows nine sets of ion resonances taken with the gold-plated omegatron.

After extensive shimming, the total magnetic-field inhomogeneity was reduced to ±6 ppm. The corresponding estimated uncertainty in μ_p/μ_n is ±4 ppm. This uncertainty estimate assumes that little averaging takes place and that the different ions may average in slightly different ways.

Great effort was made to make other systematic errors as small as possible. All materials used were tested for magnetic contamination and gave an effect of less than ±1.0 ppm. Electric-field penetration from the electron filament and electron-collector holes is estimated at ±1.5 ppm. The filament current caused a shift of 0.4 ppm with an uncertainty of ±0.1 ppm. Uncertainty in the relativistic mass corrections is set at ±1 ppm. Uncertainties in the ion masses used contribute less than ±0.1 ppm, line asymmetries ±1 ppm. Possible changes of rf voltage amplitude across the line contribute ±0.5 ppm, and frequency measurement ±0.1 ppm.

Wide ranges of variables are included in the final results. The experimental conditions included variations of 0.2 to 4.3 μA in the electron emission, 0.1 to 3.5 V of the trapping potential, $(1.4 \text{ to } 16)\times10^{-7}$ Torr on the total gas pressure, and E_{rf} values giving linewidths for H^+ of 35 to 160 ppm. The hydrogen and deuterium gases were leaked separately into the vacuum system through a NiPd leak, with the H_2 concentration varying from 20 to 80%. All results fell within ±3 ppm. The final data consist of five runs in which each of three ions were measured and nineteen runs in which two ions were measured.

The principal criterion used for acceptable data was the steepness of the sides of the lines. This was of particular importance in the early runs on the omegatron with patch surface potentials from oxide layers. There, no reproducible zero-mass extrapolation was obtained with small values of E_{rf}. However, with larger E_{rf} the lines became increasingly rectangular and quite symmetric, and the zero-mass extrapolation was consistent from run to run.

The spin-resonance frequency of the proton was

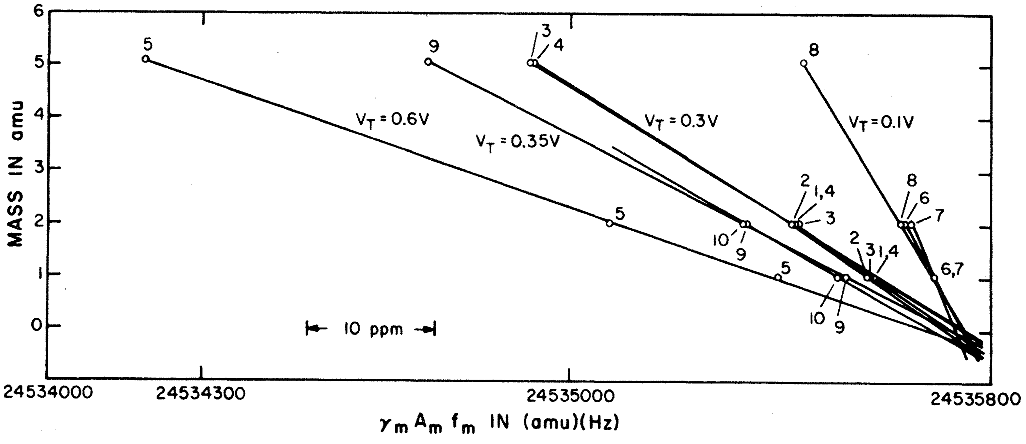

FIG. 3. Extrapolation to zero mass for the data taken with the omegatron gold plated. Mass-one ions were H^+, mass-two D^+, and mass-five ions were HD_2^+. The intercept represents the value of the proton-cyclotron frequency at zero electrostatic field times the proton rest mass. γ_m is the effective relativistic mass increase in the ion m assuming little drift in the orbit. V_T is the trapping potential in volts. A_m and f_m are the mass and cyclotron-resonance frequency of the ion m.

obtained by measuring the spin-resonance frequency of the deuteron in spherical sample of undoped heavy water and using a conversion ratio.

The plates and guard rings of the omegatron were made from oxygen-free high-conductivity copper. For the majority of the measurements they were uncoated, and patches on them which appeared to be oxidized probably gave substantial surface potential differences. A result obtained with the omegatron uncoated[2] included a ±9 ppm uncertainty to allow for possible electric-field gradient effects. These are frequency shifts due to static electric fields which are not corrected for in zero-mass extrapolation if ions of different mass see different average values of $E(r)/r$. The omegatron was then gold plated, and the result obtained from number of additional runs was only 3 ppm higher. This is taken as strong evidence that the uncertainty of the uncoated omegatron set of measurements due to electric field gradient effects is about 3 ppm. The uncertainty of the final result is taken as ±3 ppm since the coated omegatron could have some remnant electric-field gradients smaller than the uncoated omegatron. The values before and after gold plating were averaged, since the larger number of runs under a variety of operating conditions before plating was felt to balance the advantage of reduced surface potentials.

The final result obtained for μ_p'/μ_N in the form uncorrected for the diamagnetism of the nuclear resonance sample used is $2.792\,783 \pm 0.000\,016$, or ±6 ppm. This error is intended to be a 70% confidence interval and represents the square root of the sum of the squares of the systematic and random errors. The result includes a 0.4 ppm correction for filament current shift of the magnetic field and an effective relativistic correction for the ion masses of 3.3 ppm for H^+ and 0.8 ppm for D^+.

The result given is in agreement within the quoted errors with all direct determinations of μ_p'/μ_N except those of STH and Marion and Winkler. The result closest to the result of the present work is that of Mamyrin and Frantsuzov, which is $2.792\,794 \pm 0.000\,017$.[3] It should be noted that the indirect value of μ_p'/μ_N determined by use of the faraday as obtained by the National Bureau of Standards in 1960 is lower by 25 ppm than the value reported here, with the standard error in the difference being 13 ppm.

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[2] D. O. Fystrom, thesis, University of Colorado, Boulder, 1969 (unpublished).

[3] B. A. Manyrin and A. A. Frantsuzov, in *Proceedings of the Third International Conference on Atomic Masses and Related Constants, Winnipeg, Canada, 1967*, edited by R. C. Barber (Univ. of Manitoba, Winnipeg, Canada, 1968), p. 427, Dokl. Akad. Nauk SSSR 159, 777 (1964) [Sov. Phys. Dokl. 9, 1082 (1965)].