Distorted-Wave Analysis of Charge Asymmetry Effects in the Reaction $d(\alpha, t)^3$He†

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The fore-aft asymmetry of the angular distribution for the reaction $d(\alpha,t)^3$He at 82 MeV is compared with the result of a simple distorted-wave Born-approximation calculation. We find that if Coulomb effects are included, the general features of the asymmetry can be reproduced.

For isospin-conserving forces the Barshay-Temmer theorem[1] predicts that the angular distribution of the products of the reaction

$$\alpha + d \rightarrow t + {}^3\mathrm{He} \qquad (1)$$

will be symmetric about 90° in the c.m. system. The asymmetries that Gross et al.[2] and Wagner, Foster, and Greeneberg[3] have recently observed in the angular distribution of this reaction are surprisingly large when compared with previous results[4,5] of the reactions ^{10}B(α, ^{7}Li)^{7}Be and ^{12}C(^{14}N, ^{13}C)^{13}N. It is not easy to distinguish unambiguously between the effects of isospin violations[6,7] in the Coulomb and nuclear forces in reactions of the type (1). Before the observed asymmetries can be used to decide for or against the existence of isospin-nonconserving nuclear forces,[8] it is necessary to estimate the effect of the Coulomb forces on the angular distribution of process (1).

To this end we construct the simplest possible model of Reaction (1) and treat this model in distorted-wave Born approximation (DWBA). We suppose the system to be made up of four inert particles: a neutron n, a proton p, and two deuterons d_1 and d_2. The α particle is composed of n, p, and d particles. The Hamiltonian of the model has the form

$$H = K + V_{np} + V_{nd_1} + V_{nd_2} + V_{pd_1} + V_{pd_2} + V_{d_1d_2}, \qquad (2)$$

where K is the total kinetic energy.

Since d_1 and d_2 are identical, it is necessary to symmetrize the (d, t) transition amplitude with respect to all their coordinates. Thus we have

$$T_{dt} = T_{d_1t_1} + T_{d_1t_2}. \qquad (3)$$

Here $T_{d_1t_1}$ is the amplitude for the reaction in which d_1 is incident on α, and a triton containing d_1 is emitted in a certain direction. In the same way, $T_{d_1t_2}$ is the amplitude for the reaction in which d_1 is incident on α, and a triton containing d_2 is emitted in the same direction as before (see Fig. 1). We use the DWBA to evaluate $T_{d_1t_1}$ and $T_{d_1t_2}$:

$$T_{d_1t_1}{}^{\mathrm{DWBA}} = \langle \chi_{d_1}{}^- | V_{d_1n} | \chi_{t_1}{}^+ \rangle, \quad T_{d_1t_2}{}^{\mathrm{DWBA}} = \langle \chi_{d_1}{}^- | V_{d_1p} | \chi_{t_2}{}^+ \rangle,$$

where

$$\chi_{d_1}{}^- = \chi_{d\alpha}{}^-(\vec{r}_{d_1} - \vec{r}_\alpha)\varphi_\alpha(d_2np), \quad \chi_{t_1}{}^+ = \chi_{t^3\mathrm{He}}{}^+(\vec{r}_{33})\varphi_t(d_1n)\varphi_{^3\mathrm{He}}(d_2p),$$

with $\vec{r}_{33} = \vec{r}_t - \vec{r}_{^3\mathrm{He}}$, and $\chi_{t_2}{}^+ = P_{d_1d_2}\chi_{t_1}{}^+$, where $P_{d_1d_2}$ is the operator that exchanges all coordinates of d_1 and d_2. Here $\chi_{d\alpha}{}^-$ and $\chi_{t^3\mathrm{He}}{}^+$ are the distorted waves corresponding to suitable optical potentials. The functions φ_α, φ_t, and $\varphi_{^3\mathrm{He}}$ describe the internal motion of the α particle, triton, and ^{3}He, respectively.

These approximations are represented by the diagrams of Fig. 1. We note that $P_{d_1d_2}\chi_{t_1}{}^+ = \chi_{t^3\mathrm{He}}{}^+(-\vec{r}_{33}) \times \varphi_t(d_2n)\varphi_{^3\mathrm{He}}(d_1p)$, provided that the mass difference between t and ^{3}He can be neglected. The usual zero-range approximations are

$$V_{d_1n}\varphi_t(d_1n) \approx D_{0n}\delta(\vec{r}_{d_1} - \vec{r}_n), \quad V_{d_1p}\varphi_{^3\mathrm{He}}(d_1p) \approx D_{0p}\delta(\vec{r}_{d_1} - \vec{r}_p).$$

We use these approximations, and later make corrections for finite-range effects. The form factor F and spectroscopic factor S are defined by

$$S_n{}^{1/2}F_n(\vec{r}_n) = \int d\vec{r}_{d_1}\int d\vec{r}_p\,\varphi_\alpha(d_1np)\varphi_{^3\mathrm{He}}(d_1p), \quad \int d\vec{r}_n |F_n|^2 = 1, \qquad (4)$$

with similar equations for p. Because the α particle has isospin 0, the dominant part of φ_α must be symmetric under combined exchange of the spin and space coordinates of n and p. If F_n and F_p are conventionally taken to have the same sign, it then follows that $S_n{}^{1/2}$ and $S_p{}^{1/2}$ must have the same sign.

Table I. Optical-model parameters in MeV and F used in potential of the form (Ref. 13), $V(r) = V_c - V_0(1+e^{x})^{-1} - iW_0(1+e^{x'})^{-1}$.

Channel	V_0	r_0	a	W_0	r_{0w}	a_w	r_c	Ref.
^{3}He$+t$	169	1.14	0.675	32.1	1.82	0.566	1.4	13
$\alpha+d$	86.9	1.008	0.70	3.73	1.750	1.33	1.36	14

The DWBA expression for T_{dt} of relation (3) finally reduces to

$$T_{dt}^{\mathrm{DWBA}} = S_n^{1/2} D_{0n} \int d\vec{r}\, \chi_{d\alpha}^{-}(\tfrac{3}{4}\vec{r}) F_n(\vec{r}) \chi_{t^3\mathrm{He}}^{+}(\vec{r}) + S_p^{1/2} D_{0p} \int d\vec{r}\, \chi_{d\alpha}^{-}(\tfrac{3}{4}\vec{r}) F_p(\vec{r}) \chi_{t^3\mathrm{He}}^{+}(-\vec{r}). \quad (5)$$

This expression is evaluated by expanding the distorted waves in partial waves.[9]

Tamura's[10] code DWMAIN was modified to calculate the linear combination of overlap integrals. The form factors F_n and F_p were calculated by the well-depth method, assuming pure $l=0$ angular momentum transfer. As suggested by Thompson and Hering[11] we used potentials of the Hulthén form with a short-range parameter[11] $\beta = 1.33$ F^{-1}. The Schrödinger equation was solved for this potential and the strength was adjusted to reproduce the observed value of the separation energy ϵ. In the calculation of F_p the Coulomb potential due to a uniformly charged sphere of radius 2.4 F was included. The difference in F_n and F_p was quite small, their overlap differing from unity by less than 10^{-5}. However, the difference rises to ~7% at 10 F.

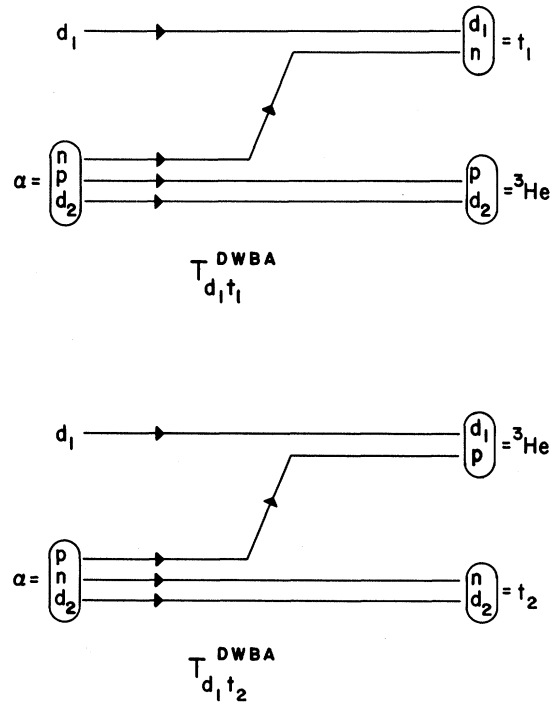

FIG. 1. Diagrammatic representation of the model employed to calculate process (1).

Thompson and Hering[11] have calculated D_{0n} and D_{0p}, assuming that the deuteron and nucleon are bound by a Hulthén potential. They obtain $D_{0n} = 161$ MeV F$^{3/2}$ and $D_{0p} = 153$ MeV F$^{3/2}$. These values neglect the first-order effect of the Coulomb potential on the relative motion of d and p. Our calculations by numerical integration of the Schrödinger equation gave $D_{0n} = 159$ MeV F$^{3/2}$ and $D_{0p} = 145$ MeV F$^{3/2}$, showing an appreciable increase in the Coulomb effect. Therefore we have varied the ratio $(S_n^{1/2} D_{0n})/(S_p^{1/2} D_{0p})$ between 1.00 and 1.05.

Since the range of the V_{dn} and V_{dp} is not small compared with the size of the α particle, finite-range effects may be important. We corrected for such effects by Smith's[12] method for Hulthén interactions. This involves the insertion of damping factors $\Lambda_n(\vec{r})$ and $\Lambda_p(\vec{r})$ in the integrals (5). We calculated Λ_n and Λ_p up to terms of order $1/\beta^2$ in Smith's[12] Eq. (8). The factors Λ_n and Λ_p differ because of mass differences and Coulomb effects. However, the asymmetry in the Λ's is much smaller than that in the form factors.

The distorted waves were calculated from Woods-Saxon optical potentials. The parameters are listed in Table I. The ^{3}He$+t$ potential is poorly known. We therefore used parameters[13] that are successful in describing scattering of ^{3}He from various p-shell nuclei. Parameters for $\alpha+d$ scattering are known[14] at c.m. energies between 2 and 16.5 MeV. Although the c.m. energy is 27.3 MeV, here we have taken those parameters.

The upper part of Fig. 2 shows the results of a DWBA calculation. No internal cutoff was used in calculating the radial integrals. The positions and relative heights of the maxima are well reproduced. This feature seems to be rather stable against changes in the details of the calculation. We investigated the effects of including a reasonable $d+\alpha$ spin-orbit potential,[14] vary-

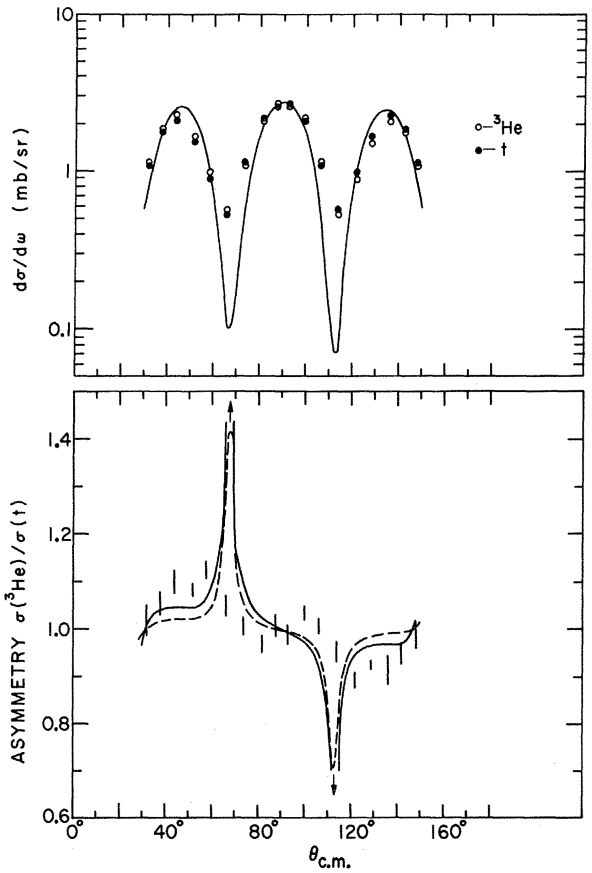

FIG. 2. Upper part: angular distribution for the reaction $d(\alpha, {}^3\mathrm{He})t$ at $E_\alpha = 82$ MeV together with a DWBA calculation with $(S_n{}^{1/2}D_{0n})/(S_p{}^{1/2}D_{0p}) = 1.03$; lower part: experimental asymmetry ratio $\sigma({}^3\mathrm{He})/\sigma(t)$ together with theoretical predictions for $(S_n{}^{1/2}D_{0n})/S_p{}^{1/2}D_{0p}) = 1.03$ (solid curve) and 1.05 (dashed curve). The experimental points in this figure are taken from Fig. 1 of Ref. 2. The curve in the upper part corresponds to the detection of ^{3}He.

ing the absorptive potential by a factor of 2, varying all $^3\mathrm{He}+t$ potential parameters by ±10% around the values given in Table I, and omitting the finite-range corrections and using Woods-Saxon potentials to generate the form factors. Four other $^3\mathrm{He}+t$ potentials from the literature were also used. In all these cases the basic structure of the angular distributions persisted, although changes in the $^3\mathrm{He}+t$ potentials produced some shift in the maxima near 45° and 135°. We were even able to reproduce the angular distribution using plane waves for $\chi_{da}{}^-$ and $\chi_{t^3\mathrm{He}}{}^+$ by introducing an inner cutoff at about 3.5 F. The stability of the result gives us some confidence that the crudity of our calculation will not invalidate it.

In the present extreme cluster model, in which the deuterons are regarded as elementary, the spectroscopic factors S_n and S_p may reach maximum values of 1. We obtain a ratio of experiment to theory of ~0.8, a satisfactory result in view of the notorious sensitivity of spectroscopic factors to the distorting potentials.

The bottom part of Fig. 2 shows the experimental and theoretical ratios of the ^{3}He and t yields. The theoretical asymmetry is always of of the correct sign. Away from the minima in the cross section the asymmetry is of the right order of magnitude but generally somewhat too small. Close to the minima, however, the asymmetry becomes far too large. Since the minima result from cancelation of the d_1t_1 and d_1t_2 amplitudes, such "spikes" are to be expected. The numerical inaccuracies of the calculation are too small to affect these spikes materially. Averaging over the experimental angular resolution is not sufficient to wash out the spikes. However, any effect that reduces the depths of the minima is likely also to reduce the asymmetry. For example, the neglected compound nucleus and $l = 2$ transfer mechanisms may dominate at the minima. We have also neglected exchange effects arising from the identity of n (p) with the neutron (proton) within the deuteron, as well as all processes in which more than one particle is transferred. Addition of an arbitrary isotropic cross section of 0.4 mb reduces the spikes by a factor of ~4.

Most of the asymmetry comes from the Coulomb effect on the ratio $(S_n{}^{1/2}D_{0n})/(S_p{}^{1/2}D_{0p})$. In fact, if we put this ratio equal to 1, the asymmetry is only ~1% except at the spikes, where it rises to ~4%. This shows that the reaction is not dominated by the distant part of the form factors where F_n and F_p differ considerably. It is not surprising that the Coulomb potential should have a large effect on the D_0's. By a simple transformation one can write[11] $D_0 = \epsilon \int d\vec{r}\, \varphi$. The perturbation of the separation energy ϵ is a diagonal element of the Coulomb potential, and this is large.

Our simple model can explain the major part of the observed asymmetry, but it is not yet adequate to probe the isospin-nonconserving part of the nuclear forces by first accurately removing the Coulomb effects. Even within the framework of DWBA the model obviously can be improved in several respects, and the success of the present version seems to justify the effort involved. True finite-range calculations are needed for realism in discussing such light nuclei.[15] It may be necessary to antisymmetrize be-

tween the target and projectile nucleons. The use of more realistic models of the three- and four-particle bound state would permit a better treatment of the Coulomb perturbation and enable the form factors to be calculated without resorting to the well-depth method. Better knowledge of the distorting potentials would be of great value. Since the fore-aft asymmetry depends chiefly on the $T=1$ part of $\chi_{t^3\mathrm{He}}$ the isospin dependence of the $^3\mathrm{He}+t$ potential is important. Polarization measurements would be required in order to gain information on the isospin dependence.

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[1]S. Barshay and G. M. Temmer, Phys. Rev. Lett. 12, 728 (1964).

[2]E. E. Gross, E. Newman, W. J. Roberts, R. W. Rutkowski, and A. Zucker, Phys. Rev. Lett. 24, 473 (1970).

[3]G. J. Wagner, C. Foster, and B. Greenberg, private communication.

[4]H. T. Fortune, A. Richter, and B. Zeidman, Phys. Lett. 30B, 175 (1969).

[5]W. von Oertzen, J. C. Jaqmart, M. Liu, F. Poughgeon, J. C. Roynette, and M. Riou, Phys. Lett. 28B, 482 (1969).

[6]D. Robson and A. Richter, to be published.

[7]W. von Oertzen, Z. Phys. 228, 182 (1969).

[8]E. M. Henley, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), Chap. II.

[9]G. R. Satchler, Nucl. Phys. 55, 1 (1964).

[10]T. Tamura, private communication.

[11]W. J. Thompson and W. R. Hering, Phys. Rev. Lett. 24, 272 (1970).

[12]W. R. Smith, Nucl. Phys. A94, 550 (1967).

[13]J. C. Hiebert, E. Newman, and R. H. Bassel, Phys. Rev. 154, 898 (1967).

[14]B. H. Choi, W. J. Thompson, and J. Y. Park, Bull. Amer. Phys. Soc. 15, 652 (1970).

[15]Such calculations are in progress at Florida State University (D. Robson, private communication).

Lower Bounds on the Threshold Charge-Exchange Amplitude from Di-Pion Mass Distributions in Final States*

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It is argued that the differential rates of production of neutral di-pions in a reaction of the type $X\to Y+(\pi\pi)$ have the form $d\sigma_{+-}/k^2dk=A+Bk+O(k^2)$, $d\sigma_{00}/k^2dk=C-Bk+O(k^2)$ for small values of k, the pion relative momentum in the di-pion rest frame. Determination of the coefficients A, B, and C would yield the bound $|\alpha_2-\alpha_0|\geq 3|B|/2(AC)^{1/2}$, where a_I is the $\pi\pi$ scattering length for isospin I.

Weinberg's current-algebra calculation[1] of the pion-pion scattering lengths has renewed interest in the experimental determination of the low-energy $\pi\pi$ phase shifts. At the present time this information is obtained mainly by extracting the $\pi\pi$ effects from the process $\pi+N\to\pi+\pi+N$ according to the Chew-Low[2] proposal, namely, by analytic extrapolation in the nucleon momentum transfer Δ^2 to the neighborhood of the unphysical value $\Delta^2=-\mu^2$, where μ is the pion mass. To obtain the $\pi\pi$ phase shifts from such an analysis, detailed knowledge of the $\pi\pi$ angular distribution as a function of Δ^2 is required.

In view of the inconclusiveness of these efforts so far, in particular with respect to the isospin-0 scattering length,[3] it may be of interest to know that some information on the threshold charge-exchange $\pi\pi$ amplitude may be directly obtainable in a model-independent way from the mass distribution of neutral low-mass di-pion systems produced in the final state of any process of the type $X\to Y+(\pi\pi)$, where X and Y are systems of one or more particles.

To be specific, let us consider the reaction

$$a+b\to c+(\pi\pi)_I,$$