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Raman Light Scattering from Excitons and Magnons in Cobalt Fluoride

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We have observed one-particle, inelastic, polarized light scattering from low-lying excitons and magnons in single-crystal cobalt fluoride (CoF_2) at $2^{\circ}K$. Because of the orbital nature of these excitations, the cross sections $(10^{-33} - 10^{-32} \text{ cm}^2/\text{sr})$ are much larger than for one-magnon scattering in the other case, which has been measured (FeF₂, \sim 10⁻³⁵ cm²/sr). Our measured cross sections are in good agreement with a recent calculation by Ishikawa and Moriya.

The first observation' of one-magnon light scattering in magnetic insulators was in FeF_2 where the magnons are predominantly spinlike excitations and the scattering mechanism requires spin-orbit coupling in the virtual intermediate state in order to be finite for an electric dipole process.² Subsequent measurements have concentrated on two-magnon scattering.^{1,3} This is centrated on two-magnon scattering. 1,3 This is because for spinlike excitations it is a stronger process (proceeding via inter-ion exchange cou $pling^{1,4}$, and in low-anistropy materials such as manganese and nickel compounds, the one-magnon energy is only a few cm^{-1} , and hence difficult to measure. We wish to report here the observation of scattering by single magnons and low-lying excitons in CoF_2 . The latter arise from excitations within the low-lying ${}^{4}T$, levels of the cobalt ions, split by spin-orbit coupling, orthorhombic crystal field, and inter-ion exchange interactions. The scattering mechanism involves a direct coupling of the \widetilde{E} vector of the incident light to the orbital part of the excitations, and this gives rise to larger cross sections than for spin excitations. In fact, the exciton scattering is comparable in magnitude with the phonon scattering. Measured cross sections are in good agreement with those recently calculated by Ishikawa and Moriya. '

The symmetry group of CoF_2 below T_N is $D_{4h}^{14}(D_{2h}^{12})$ and branches of the excitation spectrum at $k = 0$ are labeled by unitary subgroup representations Γ_1^+ , Γ_2^+ , and Γ_3^+ + Γ_4^+ . In the absence of off-diagonal inter-ion exchange coupling, all branches show the sublattice degeneracy of 2. Without this restriction the $\Gamma_1^{ \bullet}, \, \Gamma_2^{ \bullet}$ pair may show a Davydov splitting while Γ_3^+ and Γ_4^+ are required to be degenerate by the antiunitary operators of $D_{4h}^{14}(D_{2h}^{12})$. The low-lying electronic levels of the Co^{2+} ion have been studied by Johnson, Dietz, and Guggenheim⁶ and Gladney.⁷ The dispersion of the low-lying excitations in CoF, has been measured by neutron scattering, δ and the ones we will be concerned with here are the magnons with an energy of 37 cm^{-1} at $\vec{k} = 0$, the Davydov-split Γ_1^{\bullet} , Γ_2^{\bullet} excitons at 169 cm⁻¹ and 204 cm⁻¹, and the Γ_3^{\bullet} + Γ_4^{\bullet} branch at 194 cm⁻¹. In addition there are four (Γ_1^+, Γ_2^+) , and four $(\Gamma_{1}^{\dagger}+\Gamma_{4}^{\dagger})$ branches around 800 cm⁻¹ which have not yet been observed by any technique. Light scattering is allowed by symmetry from all

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branches, but only Γ_2^+ , Γ_3^+ , and Γ_4^+ are magnetic-dipole infrared active. The nonvanishing elements of the scattering tensor for the corepresentations can be determined group-theoretically and are given by'

$$
D\Gamma_1^* = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & c \end{pmatrix}, \quad D\Gamma_2^* = \begin{pmatrix} 0 & d & 0 \\ d^* & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
$$

$$
D\Gamma_3^*, \Gamma_4^* = \begin{pmatrix} 0 & 0 & f \\ 0 & 0 & 0 \\ g & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -f^* \\ 0 & -g^* & 0 \end{pmatrix}.
$$
 (1)

The individual modes can be observed by a proper choice of scattering geometry. Note that the scattering tensor is not antisymmetric, as is the case for spin-like magnons, where $f = -g$ and $Im(f) = Im(g)$, nor symmetric as is the case for phonons, and we have observed this in the relative scattering cross sections. As Fleury and Loudon¹⁰ have pointed out, this lack of antisymmetry is expected for CoF_2 because of the orbital nature of the low-lying excitations, and is not observed in FeF₂.

A single crystal of CoF_2 , oriented with faces normal to the $[100]$, $[010]$, and $[001]$ directions, was immersed in liquid helium at 2'K. The beam from a 60-mW He-Ne laser was incident vertically and the 90' scattered light analyzed with a 1-m double monochromator and detected by conventional photon-counting techniques. The results obtained are shown in Figs. 1 and ² and

summarized in Table I. Scattering from the Γ_2^* exciton at 204 cm^{-1} was below the sensitivity limit of our system. The assignment of the electronic scattering is made on the basis of its polarization properties, energy, and qualitative temperature dependence. All of the lines broaden rapidly towards T_N (e.g., the Γ_1^{-+} width goes from 2 to \sim 40 cm⁻¹) and the scattering cross section essentially goes to zero at this temperature. Our polarization measurements of the 169-cm⁻¹ line show that this mode is the Γ_1^* and positively determines the sign of the Davydov splitting, i.e., Γ_1^{\dagger} is lower than Γ_{2}^{\dagger} . The absorption coefficient of CoF₂ at 6328 Å is 1 cm⁻¹ for E ||c and
7 cm⁻¹ for $E \perp c$ ¹¹. Thus the effective scattering 7 cm⁻¹ for $E + c$.¹¹ Thus the effective scattering volume is different for these two directions of excitation. The laser beam was kept close to the surface of the crystal to minimize reabsorption of the scattered light. Relative cross sections were determined by comparison with the E_g phonon at 256 cm⁻¹ which has a symmetric scattering tensor. An approximate absolute cross section was obtained by measuring the 992-cm ' line of benzene and using Skinner and Nilsen's 12 value for its absolute cross section. In table I we also show results for the strongest phonon scattering, viz., that from the $A_{1 \, \mathrm g}(\Gamma_1^{\mathrel{\;\,\circ\,}})$ and scattering, viz., that from the $A_{1g}(\Gamma_1^{\dagger})$ and
 $E_g(\Gamma_3^{\dagger} + \Gamma_4^{\dagger})$ modes.¹³ Macfarlane and Ushioda⁻ have already reported room-temperature measurements of all of the $\vec{k} = 0$ phonons. Both the $A_{1g}(\Gamma_1^{\dagger})$ and $E_{g}(\Gamma_3^{\dagger}+\Gamma_4^{\dagger})$ modes couple appreciably to electronic excitations at low temperature. This was first shown for $E_g(\Gamma_s^{\dagger} + \Gamma_4^{\dagger})$ by Allen

FIG. 1. Polarized spectra of one-magnon light scattering in CoF_2 at 2°K. The scale is linear in the wavelength shift from the He-Ne laser line at 6828 A.

and Guggenheim¹⁵ who measured a temperature-dependent absorption cross section (which, in contrast to light scattering, is zero for the pure phonon mode), and a linewidth minimum around T_N . We have confirmed these linewidth measurements and also find a very similar behavior for the $A_{1g}(\Gamma_1^{\dagger})$ mode. Further details will be presented in the near future.

The differential extinction coefficient for incident and scattered polarizations μ and ν (defined as the fraction of light scattered per centimeter of path per steradian) can be written¹⁶

$$
\left(\frac{dh}{d\Omega}\right)_{\mu\nu} = \int_0^\infty d\omega \, \frac{\omega_0 \omega^3}{2\pi c^4} \int_{-\infty}^\infty dt \, e^{i(\omega - \omega_0)t} \langle \alpha_{\mu\nu}(0) \alpha_{\nu\mu}(t) \rangle,\tag{2}
$$

where ω_0 is the incident frequency and ω the scattered frequency. To obtain the cross section we must divide by 2.8×10^{22} , the number of cobalt ions per cubic centimeter. The matrix elements of the polarizability operator for the Stokes scattering from excitation $|n\rangle$ are

$$
\langle 0 | \alpha_{\mu\nu} | n \rangle = \sum_{u} \left[\frac{\langle 0 | P_{\mu} | u \rangle \langle u | P_{\nu} | n \rangle}{E_{u} - E_{0} - h \omega} + \frac{\langle 0 | P_{\nu} | u \rangle \langle u | P_{\mu} | n \rangle}{E_{u} - E_{0} + h \omega} \right].
$$
\n(3)

FIG. 2. Polarized spectra of single-exciton light scattering in CoF_2 at 2°K. Also shown are the strongest phonon lines $\bar{A}_{1\text{g}}(\Gamma_1^{\text{+}})$ and $E_{\text{g}}(\Gamma_3^{\text{+}}+\Gamma_4^{\text{+}})$. The relative intensities of the peaks have not been corrected for the polarization response of the spectrometer, but this correction has been made in Table I. Note that the gain used for the $A_{1g}(\Gamma_1^{\ +})$ line in (b) is $\frac{1}{b}$ that in all the other spectra. As in Fig. 1, the energy scale is linear in the wavelength shift from 6328 Å. An approximate cross-section calibration was made relative to the 992 cm^{-1} line of benzene (see Table I).

Ishikawa and Moriya⁵ have recently calculated the these matrix elements of the orbital-dependent polarizability for CoF₂ under the assumption that the odd-parity states E_u are $3d^64p$ and $3d^64f$ cobalt levels at $\sim 10^5$ cm⁻¹. There are lower lying charge-transfer states around 5×10^4 cm⁻¹,¹¹ but this assumption should give a good order of magnitude for the absolute extinction coefficient (or equivalently the cross section). The basis states for $|0\rangle$ and $|n\rangle$ were obtained from an effective Hamiltonian calculation within the ${}^{4}T_1$ term. The dipole operator couples the orbital parts of the $3d^{7}$ ⁴T₁ states 10) and \ket{n} directly to the odd-parity states $|u\rangle$ without the need for spin-orbit coupling (although, of course, spin-orbit coupling helps determine the linear combinations of ${}^{4}T_{1}$ states in $|0\rangle$ and $|n\rangle$). This is in contrast to the mechanism for scattering from spinlike excitations such as in FeF_2 .

The results of this theory are compared with the experimental values in Table I. We have used Ishikawa and Moriya's⁵ results and evaluated the cross sections for the He-Ne laser frequency, including the frequency dependence of the polarizability. The agreement is very satisfactory, particularly with regard to the relative cross sections, and supports their model. Further improvement could be obtained by carrying out the sum in Eq. (3) over the odd-parity states E_u experimentally observed in $\widehat{\text{CoF}_2}$,¹⁷ and by using better values for $\langle 0|P_{\mu,\nu}|u\rangle$ which in the present case have all been taken as 2.5 D. The main anomaly is in the ratio of the XX to YY intensity for the Γ_1^* exciton which is observed to be very low.

We have not yet observed the two-particle scattering (two-magnon, two-exciton or exciton-magnon) and can set an upper limit for $dh/d\Omega$ of

Table I. Summary of light scattering from magnetic excitations in CoF_2 . Unless otherwise stated, measurements are at 2'K.

Excitation Symmetry at k=0	Energy $(cm-1)$	Width ^a (m^{-1})	Scattering Geometry $\mu\nu$	$dh/d\Omega$ - Expt. ^b $(10^{-9}$ cm $^{-1}/$ sr $)$	$dh/d\Omega$ - Theory ^C $(10^{-9}$ cm ⁻¹ /sr)
Magnon $\Gamma_3^+ + \Gamma_4^+$ 37.5±.5		≤ 2.0	ΥZ ZX	.13 .27	\cdot 5 1.1
Excitons r_1^+	169.0	≤2.0	XХ ZZ	\cdot	$^{2.8}_{4.7}$
r_2^+	204 ^d		YΧ	\leq . 1	0.3
r_3^+ + r_4^+	194	≤3.0	YZ ZX	.45 .60	1.4 2.5
Phonons $(A_{1g})r_1^+$	373	4	XΧ ZZ	$\frac{2}{5}$	
$A_{1g}^{\ e}$	366	10	XX ZZ	$\frac{1}{2}$	
$(E_q) r_3^+ + r_4^+$	256	5	YZ $= 7Y$	3	
E_g^e	245	10	YZ $= 7Y$	1.5	

^a Spectral slit width = 2 cm^{-1} .

^bAbsolute values $\pm 50\%$; relative values $\pm 20\%$. For comparison, the 992-cm⁻¹ line of benzene has $dh/d\Omega = 25 \times 10^{-9}$ cm⁻¹/sr.

~Derived from Ishikawa and Moriya, Bef. 5.

d This energy obtained from neutron scattering, Ref. 5.

 $^{\rm e}$ At 290 $^{\rm e}$ K. See also Ref. 14.

 \sim 10^{-10} cm⁻¹/sr for these processes. This is again in contrast to scattering from spinlike excitations, where the two-magnon scattering is stronger than the one-magnon.¹

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