charge distribution.

In summary, we feel that our data strongly demonstrate the presence of $E2$ atomic transitions in ^{208}Pb. The importance of taking such transitions into consideration for interpretation of muonic x-ray data in terms of nuclear charge distributions cannot be overstressed. These transition energies can nearly coincide with those of inner $E1$ transitions and thus may confuse the interpretations.

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Current-Induced Ion Heating in a Toroidal Plasma*

S. Q. Mah, H. M. Skarsgard, and A. R. Strilchuk
Physics Department, University of Saskatchewan, Saskatoon, Canada
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Time-resolved measurements of plasma temperatures, current, and density have been made in a turbulent heating experiment. Rapid ion heating ceases at the instant when the mean electron velocity falls below the critical value for generation of ion-acoustic waves.

Turbulent plasma heating in high-voltage discharges has been observed previously in both linear and toroidal geometries. In some experiments[1,2] only the electron or the diamagnetic temperatures were measured while in others[3-7] energetic ions were detected but the time variation of the ion temperature was not determined. This information is important if a satisfactory explanation of the ion heating is to be found since the plasma parameters on which the heating process depends generally change rapidly during the discharge. Observations of the frequency range of electrostatic fluctuations indicate that ion-acoustic waves may be responsible in some cases[6] for ion heating. However, nonlinear processes[8] can influence the frequency spectrum and conclusions drawn from linear theory may not be reliable. In this paper we report the time dependence, in a turbulent heating experiment, of the ion temperature as well as the electron temperature, plasma density, and discharge current. These measurements make possible a detailed comparison between the conditions for which rapid ion heating is observed and those for which ion-acoustic wave production can occur.

The experimental system (Fig. 1) includes a toroidal, glass-walled chamber wound with eighteen equally spaced coils (not shown) to provide

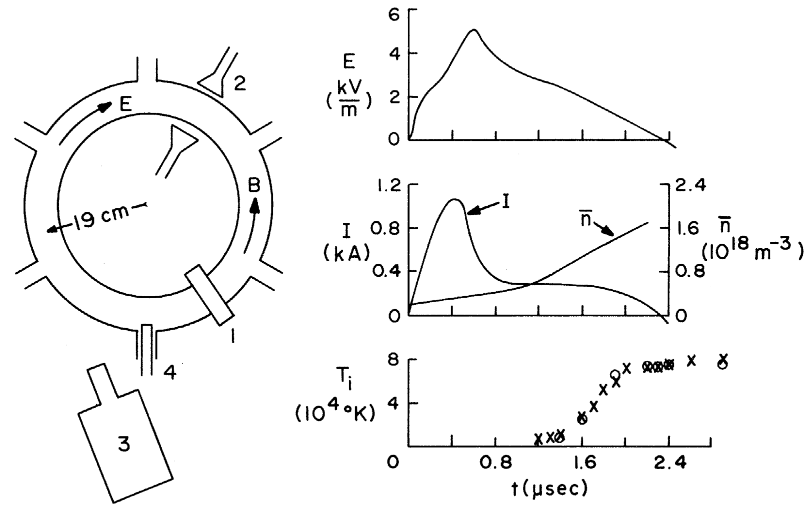

FIG. 1. Experimental system. 1, Rogofsky coil; 2, microwave interferometer; 3, Fabry-Perot interferometer; 4, orbit analyzer probe. Measurements shown are for argon gas at $p = 0.3$ mTorr, $B = 1500$ G, and a capacitor voltage $V_c = 8.5$ kV. Ion temperature measurements were made in the tangential (crosses) and radial (circles) directions relative to the discharge current.

a magnetic field B of up to 2 kG. Although pulsed (~3 msec duration) this field is effectively constant for the duration (~2 μsec) of the turbulent heating experiment. After the magnetic field has been turned on a plasma is formed by striking an rf discharge in argon at a pressure of 0.3 mTorr. Simultaneous with the termination of the rf field a strong electric field E is applied by discharging two 5-μF capacitor banks in series through an air-cored winding symmetrically placed about the plane of the torus.[9] The field E is determined by using a pickup coil and correcting for the induced field due to the plasma self-inductance. The plasma current I is measured with a Rogofsky coil (current transformer) and the density with a 2-cm microwave interferometer. By taking account of the shape of the density profile—found from measurements with movable Langmuir probes to have a peak about 1 cm outside the minor axis of the chamber—we obtain the average electron density $\bar{n}$ over the current ring of radius 2.5 cm.

The ion temperature is found by measuring the Doppler broadening of the 4806-Å emission from Ar II with a Fabry-Perot interferometer. Piezoelectric scanning is used to obtain the line profile in several successive discharges; a monitor of the total line intensity by means of a grating spectrometer provides normalization corrections for small variations in light intensity from one discharge to the next. Pressure broadening effects are negligible for our conditions and self-absorption is unimportant. The plasma is optically thin; low light intensity, particularly at early times, is the main difficulty in the experiment. Identical measurements in both the radial and tangential directions, relative to the discharge current, confirm that the broadening is due to thermal rather than macroscopic motion.[10] The electron temperature at time $t=0$ (Fig. 1) is determined from measurements of the plasma diamagnetism[11] and at early times in the discharge (up to 0.2 μsec) by means of a charge-selective orbit analyzer probe.[12] This technique has not worked at later times in the discharge, because of small signal-to-noise ratios. It has been possible, however, to determine the electron temperature during the latter part of the discharge from the observed rate of increase of plasma density and known values for the ionization cross section.[13] Thus $\bar{n}^{-1}d\bar{n}/dt = n_g\langle\sigma v\rangle$, where n_g is the concentration of neutral gas atoms and $\langle\sigma v\rangle$ is the product of electron velocity and total ionization cross section (defined as the cross section for production of a single charge regardless of the multiplicity of charge on the ions produced), averaged over a Maxwellian electron energy distribution. (For our conditions the electron drift velocity is small compared with the thermal velocity.) This method is useful in partially ionized plasmas if the electron temperature is not too large. It is assumed that plasma losses are negligible in comparison with the ionization increase. This assumption seems to be justified at electron temperatures below about 500 V ($=\kappa T_e/e$) considering known loss processes such as drift across the inhomogeneous toroidal magnetic field[11] and anomalous (Bohm) diffusion. It can be checked, however, in the following way. For argon $\langle\sigma v\rangle$ has a maximum at an electron temperature of 300 V. Now it so happens that we also observe a prominent peak in the ionization rate $\bar{n}^{-1}d\bar{n}/dt$ (at ~1.3 μsec in Fig. 1). If we consider this peak to correspond to an electron temperature of 300 V (for which $\langle\sigma v\rangle$ is known) we can calculate the ionization rate independently. The result agrees closely with the measured value. This agreement would not be observed if plasma losses were significant.

The discharge behavior is most conveniently discussed in terms of two separate time intervals. In the interval up to about 1 μsec there is initially a short period of free electron acceleration[4] followed by less than normal acceleration as the current builds up to a peak of about 1 kA. During this interval rapid electron heating takes place, accounting for most of the power going into the plasma. Recent work, still incomplete and to be reported separately, indicates that this behavior can be understood in terms of high-frequency instabilities—of the order of the electron plasma frequency ω_{pe}—if the influence of trapped electrons is taken into account. In this paper we consider the behavior during the second time interval (after ~1 μsec) which begins with the onset of rapid ion heating. The current is limited by anomalous resistance (about two orders of magnitude higher than classical) and changes only slowly during this time, while the electron temperature is decreasing rapidly.

The initial rate of ion heating is ~10^3 times larger than collisional energy transfer from the electrons and represents (at larger applied electric fields) up to 5% of the power fed into the discharge. The most striking feature of the measured ion temperature is its sudden leveling off (at ~1.9 μsec in Fig. 1). It was suspected initially that the confinement limit for the magnetic field employed was reached at that instant and in fact the ion Larmor radius r_{ci} at the final ion temperature is comparable with r_p, the plasma radius ($r_{ci} \approx r_p/2$). However the ions only move ~3 mm (1/30 of a Larmor orbit) during the entire period of rapid heating so the confinement limit is not relevant here. Neither can ion cyclotron oscillations be responsible for the heating. Ion-acoustic waves ($f \leq f_{pi} \simeq 30$ MHz) have a frequency, however, which is more consistent with the time scale of the observed ion heating. Moreover, ion-acoustic waves should grow for the plasma conditions found during the time that rapid heating is observed and, even more significant, they are not expected to be generated for the plasma conditions observed after the cessation of rapid heating. This can be seen by comparing the experimental electron-drift velocity $u = I/Ae\bar{n}$, where A is the area of cross section of the current ring, and the critical velocity[14] for growth of ion waves u_c. For a given ion mass the latter depends only on the electron and ion temperatures which are known experimentally. It should be noted that we do not at all times have $T_e \gg T_i$ and the critical velocity is not given simply by the "ion-sound velocity" $(\kappa T_e/M)^{1/2}$. Ion Landau damping becomes important at later times and the critical velocity depends sensitively on T_i as well as T_e.

Figure 2 shows experimental values of T_e, T_i, and u as well as the critical velocity u_c, for three different capacitor voltages. In each case the electron-drift velocity u is well above the critical value during the period of rapid heating but falls below it near the time at which rapid heating ceases. This is most clearly evident in the discharges at higher voltage [Figs. 2(b) and 2(c)] which show a more sudden leveling off of the ion temperature. We consider these observations as strong evidence that ion-acoustic waves are responsible for the ion heating. It follows that rapid ion heating should continue for a longer time if $u > u_c$ is maintained, for example, by programming the electric field E so as to keep the electron temperature higher and thereby reduce u_c.

If we assume that all the wave energy created goes into heating the ions we expect[15] $d(\frac{3}{2}\bar{n}\kappa T_i)/dt = 2\gamma W_F$, where γ is the growth rate and W_F is the energy density of the fluctuations. By calculating the growth rate of ion-acoustic waves the measured rate of ion heating can then be used to estimate the fluctuation energy. For example, at $t = 1.5$ μsec in Fig. 2(b) we find from linear theory[16] $\gamma = 0.9\times10^7$ sec^{-1} (for $k = \lambda_e^{-1}$ where λ_e is the electron Debye length) and $W_F/W_T = 4\times10^{-3}$ with $W_T = \frac{3}{2}\bar{n}\kappa(T_e + T_i)$.

The final ion temperature is independent of both the capacitor voltage (see Fig. 2) and the plasma density (for a factor of 2 increase over the value at which the measurements in Fig. 2 were taken). However it does depend on the magnetic field; we find that $(T_i)_{\text{final}}$ increases by about a factor of 2 as B is changed from 750 to 2000 G.

The plasma resistance $R = EL/I$ ($L = 2\pi\times0.19$ m)

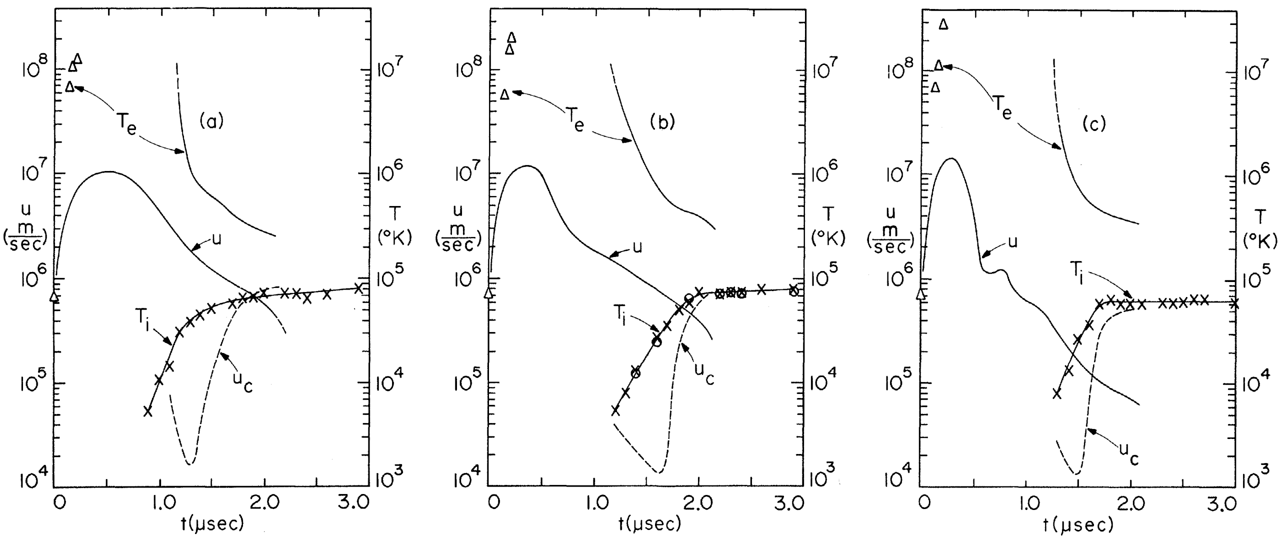

FIG. 2. Measured ion (T_i) and electron (T_e) temperatures, mean electron velocity (u), and the critical velocity for generation of ion waves (u_c), for capacitor voltages (a) 6.5, (b) 8.5, and (c) 12 kV. In each case $B = 1500$ G and $p = 0.3$ mTorr of argon.

in general decreases slightly ($\lesssim$ a factor of 2) during the interval from 1 to 2 μsec. It also depends on the capacitor voltage used; resistances of approximately 5, 10, and 50 Ω are observed for capacitor voltages of 6.5, 8.5, and 12 kV, respectively. The classical resistance, determined by electron-atom collisions for our conditions, is nearly the same for each case and varies from 0.2 to 0.06 Ω during the time interval of interest. Although a sign of plasma turbulence, the anomalous resistance observed cannot be entirely explained by existing nonlinear theories of ion waves. Sagdeev[15,8] predicts an effective collision frequency $\nu_S \approx 10^{-2}\omega_{pe}T_e u/T_i\bar{v}_e$ with $\bar{v}_e = (\kappa T_e/m)^{1/2}$ and, for our geometry, a resistance $R_S \approx 10^{12}T_e u/\omega_{pe}T_i\bar{v}_e$ (Ω). From measured values for T_e, u, ω_{pe}, and T_i we find that R_S decreases markedly (by more than a factor 10^2) during the time interval from 1 to 2 μsec, in contrast to the observed resistance. Recently Dupree[17] has calculated the resistivity for a plasma with current-driven ion-acoustic wave instability, by considering the dynamical friction force between ballistic clumps of plasma which are formed through resonant scattering of particles by the waves. The effective collision frequency $\nu_D \approx (\bar{k}\lambda_e/40\pi)\omega_{pe}$ leads to the resistance $R_D \approx 0.5 \times 10^{12}\bar{k}\lambda_e/\omega_{pe}$ (Ω), where $\bar{k}$ is an appropriately averaged wave vector parallel to u. Choosing $\bar{k} = \lambda_e^{-1}$ we find fair agreement between R_D and experiment both as to absolute magnitude and the variation with time during the discharge, although the observed dependence on capacitor voltage is not explained by this theory. In particular, with $V_c = 8.5$ kV [Figs. 1 and 2(b)] the observed and calculated resistances agree (within about 20%) as ω_{pe} increases with time; for $V_c = 6.5$ and 12 kV, however, the calculated resistance R_D lies too high (~2 times) and too low (~5 times), respectively.

The observed resistance is still anomalously large during the interval that $\bar{v}_i < u < u_c$ ($\bar{v}_i$ is the ion thermal velocity), when ion-acoustic waves responsible for ion heating are no longer generated but lower frequency drift waves are expected to grow.[18] Surprisingly, the resistance calculated from Dupree's theory also shows fair agreement with experiment in this interval. Moreover, ν_D is similar to the collision frequency $\nu_B = (2\pi)^{-1}(m/M)^{1/3}\omega_{pe}$ found in plasmas[19,6] in which high-frequency instabilities are observed in addition to lower frequency (possibly ion-acoustic wave) fluctuations. Thus it appears that Dupree's resistivity (with $\bar{k} \sim \lambda_e^{-1}$), although derived for a plasma with ion-acoustic wave fluctuations, is at least a crude approximation to that actually observed for a wider range of conditions in turbulent plasmas.

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