Table I. Comparison of experimental and theoretical spectroscopic-factor products. The experimental cross section and DWBA theory have been normalized in the energy range specified.

| Reaction | Coulomb barrier (MeV) | Lab energy (MeV) | $S_1S_2$ (Expt) | $S_1S_2$ (Theor) |
|---|---|---|---|---|
| $^{9}Be(^{16}O, ^{17}O)^{8}Be$ | 18.55 | 7-13 | 0.62 | 0.46 |
| $^{13}C(^{16}O, ^{17}O)^{12}C$ | 21.10 | 12-16 | 0.50 | 0.49 |

The individual spectroscopic factors cannot be determined independently from these results. However, taking the values of the spectroscopic factors for the $^{9}$Be and $^{13}$C ground states from the theory,[3] values of 0.82 and 1.07 are obtained for the $^{17}$O first excited state. These are in reasonable agreement with that calculated by Brown.[6]

The results presented above show that the analysis of one-neutron transfer reactions, sufficiently below the Coulomb barrier so that nuclear effects can be discounted, can produce realistic energy-independent spectroscopic factors for states in light nuclei. Furthermore, with the sensitivity of the present measurement technique, cross sections of a few microbarns can be measured, and therefore many more transfer reactions to excited states can be studied than was previously possible.

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# Magnetic-Dipole Transitions in the $(\pi d_{3/2})(\nu f_{7/2})$ Quartet*

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The mean lives of the second and third excited states in $^{40}$K are $(0.65\pm0.15)\times10^{-12}$ and $(1.6\pm0.3)\times10^{-12}$ sec, respectively. The relative speeds of the three $M1$ transitions between the four lowest states are incompatible with these states being pure $(\pi d_{3/2})^{-1}\times(\nu f_{7/2})$—even if effective moments are used; neither small components of the most likely shell-model impurities nor any possible $E2$ admixture improves the fit. A similar situation appears to hold in $^{38}$Cl.

The $1d_{3/2}$ shell closes with the addition of the twentieth nucleon, with the next nucleons going into the $1f_{7/2}$ orbit. In this picture $^{40}$Ca is a doubly closed-shell nucleus and $^{40}$K is described by a $d_{3/2}$ proton hole and an $f_{7/2}$ excess neutron. Support for this model comes from the facts that the low-lying levels of $^{40}$K have the right spins (Fig. 1) and that the energies of the states can be fitted by the model.[1] Even more impressive is the high accuracy with which the low-lying $^{38}$Cl spectrum, taken to be dominated by the $(\pi d_{3/2})(\nu f_{7/2})$ configuration, can be predicted from the $^{40}$K spectrum.[2] Another test is the electromagnetic properties of the states of the multiplet.

Is is immediately apparent that this simplest picture cannot be completely correct since it predicts the ground-state magnetic moment of $^{40}$K to be $-1.46\mu_N$ while the observed value is $-1.29\mu_N$. Similar discrepancies[3] exist for the "single-neutron" nucleus $^{41}$Ca ($-1.91\mu_N$ vs $-1.60\mu_N$) and the "proton-hole" nucleus $^{39}$K ($0.13\mu_N$ vs $0.39\mu_N$). A more consistent picture emerges if the notion of effective moments is introduced, in which case the measured moments of $^{39}$K and $^{41}$Ca can be used to predict the value $-1.25\mu_N$ for $^{40}$K. Effective moments can also be used to calculate $M1$ transition rates.

A straightforward calculation of the speed of an $M1$ transition in the multiplet yields

$$1/\tau = (1.673\times10^{12})E_\gamma^{\,3}(2J_f+1)\Big[(-1)^{J_i}\sqrt{2}\,W(J_i J_f \tfrac{7}{2}\tfrac{7}{2};1\tfrac{3}{2})\langle(\nu f_{7/2})\|\vec{\mu}\|(\nu f_{7/2})\rangle + (-1)^{J_f}\,W(J_i J_f \tfrac{3}{2}\tfrac{3}{2};1\tfrac{7}{2})\langle(\pi d_{3/2})\|\vec{\mu}\|(\pi d_{3/2})\rangle\Big]^2, \tag{1}$$

in which $E_\gamma$ is in MeV and $\tau$ is in seconds. Since $|J_i - J_f| = 1$, the relationship

$$\frac{W(J_1J_2J_aJ_a;1J_b)}{W(J_1J_2J_bJ_b;1J_a)} = (-1)^{2(J_a-J_b)}\left(\frac{J_b(J_b+1)(2J_b+1)}{J_a(J_a+1)(2J_a+1)}\right)^{1/2} \tag{2}$$

holds, and hence Eq. (1) becomes[4]

$$1/\tau = (3.345\times10^{12})E_\gamma^{\,3}(2J_f+1)W^2(J_iJ_f\tfrac{7}{2}\tfrac{7}{2};1\tfrac{3}{2})\big[\langle(\nu f_{7/2})\|\vec{\mu}\|(\nu f_{7/2})\rangle - (21/5)^{1/2}\langle(\pi d_{3/2})\|\vec{\mu}\|(\pi d_{3/2})\rangle\big]^2. \tag{3}$$

Thus all of the $M1$ transitions are proportional to the square of the same linear combination of reduced single-particle matrix elements, and the ratios of the different transition speeds depend only on the $\gamma$-ray energies and on the spins of the initial and final states. Equation (3) can be rewritten to define the quantity

$$|M_{df}|^2 \equiv \big[\langle(\nu f_{7/2})\|\vec{\mu}\|(\nu f_{7/2})\rangle - (21/5)^{1/2}\langle(\pi d_{3/2})\|\vec{\mu}\|(\pi d_{3/2})\rangle\big]^2 = \frac{2.99\times10^{-13}}{\tau E_\gamma^{\,3}(2J_f+1)W^2(J_iJ_f\tfrac{7}{2}\tfrac{7}{2};1\tfrac{3}{2})}, \tag{4}$$

which should be the same for all $M1$ transitions within the multiplet. In order to test the validity of Eq. (4), the lifetimes of the second and third excited states of $^{40}$K have been measured.

The attenuated-Doppler-shift method was used to measure the lifetimes. The states were populated by the reaction $^{39}$K($d,p$), with the recoil direction defined by coincidence between the $\gamma$ and the outgoing proton. Two proton detectors were used simultaneously—one at or near 0° and therefore in coincidence with $^{40}$K recoils having almost zero velocity, and the other placed so that the recoils were moving nearly along the axis of

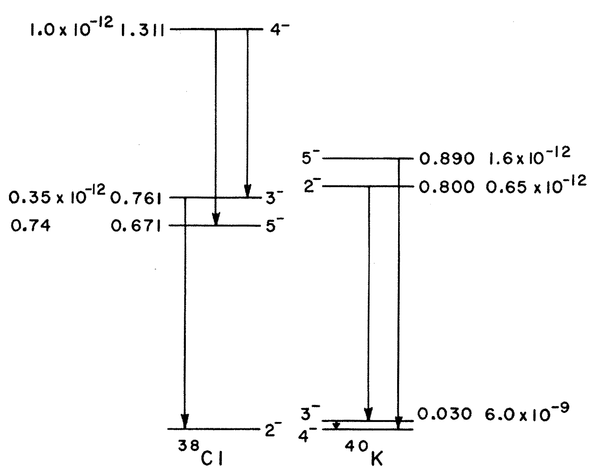


FIG. 1. Energy-level diagram of the first four states in $^{38}$Cl and in $^{40}$K. Also shown are the lifetimes of the states and the various $M1$ transitions.

the $\gamma$ detector. For both states the unattenuated shift was about 4.5 keV. Data were taken both with KI targets whose thickness was large compared with the range of the recoils, and with thin gold-backed targets. False shifts were checked for by substituting targets for which long-lived states are populated. When it became apparent that the results were at variance with theory, the experiment was repeated with a number of additional checks—including measurements of states so short-lived that they showed the full Doppler shift. The usual slowing-down formulas[5] were used to compute the shift as a function of lifetime. A summary of the results for the second and third excited states of $^{40}$K is given in Table I.

The lifetime of the first excited state has been measured by Lynch and Holland,[6] by Hafemeister and Shera,[7] and by Boulter, Prestwich, and Arad.[8] The three measurements are consistent, and we adopt the value $(6.0\pm0.4)\times10^{-9}$ sec. From the speed of each of the three $M1$ transitions, $|M_{df}|^2$ can be determined from Eq. (4); and, if the model is correct, all three transitions should lead to the same value. However, it can be seen from Table II that the three transition rates are not consistent with a single $|M_{df}|^2$. The 30-keV transition is much too low in energy to contain a significant $E2$ component; and any $E2$ admixtures to the 890-keV transition would only

Table I. Mean lives of states that are considered to be members of a $(\pi d_{3/2}) \times (\nu f_{7/2})$ quartet. The quantity $F$ is the ratio of the measured shift to the unattenuated shift.

| Nucleus | $E_x$ (MeV) | $E_\gamma$ (MeV) | Stopping material | F | Measured $\tau$ (psec) | Adopted $\tau$ (psec) |
|---|---|---|---|---|---|---|
| $^{40}$K | 0.800 | 0.770 | KI | 0.36 ± 0.04[a] | 0.70 ± 0.15 | 0.65 ± 0.15 |
| $^{40}$K | 0.800 | 0.770 | Au | 0.15 ± 0.06[b] | 0.60 ± 0.25 | |
| $^{40}$K | 0.890 | 0.890 | KI | 0.18 ± 0.03[a] | 1.6 ± 0.3 | 1.6 ± 0.3 |
| $^{40}$K | 0.890 | 0.890 | Au | 0.00 ± 0.06[b] | > 1.4 | |
| $^{38}$Cl | 0.761 | 0.761 | $PbCl_2$ | 0.40 ± 0.10 | $0.35^{+0.20}_{-0.15}$ | $0.35^{+0.20}_{-0.15}$ |
| $^{38}$Cl | 1.311 | 0.640 | $PbCl_2$ | 0.18 ± 0.06 | 1.0 ± 0.3 | 1.0 ± 0.3 |
| $^{38}$Cl | 1.311 | 0.550 | $PbCl_2$ | 0.23 ± 0.10 | 0.8 ± 0.4 | |

[a]Average of four separate measurements.
[b]Average of two separate measurements.

make the situation worse since even if this is taken to be a pure $M1$ transition, it is still too slow (by about a factor of 2) relative to the 30-keV transition.

The fact that the ratios of the three transitions are not consistent with a single $|M_{df}|^2$ indicates that significant proportions of other configurations are present, although the effect of these should have been allowed for to some degree by the use of effective moments. Furthermore, the strongest other components are expected to be $(\pi s_{1/2})^{-1}(\nu f_{7/2})$ and $(\pi d_{3/2})^{-1}(\nu p_{3/2})$, neither of which will contribute an $M1$ amplitude for decay to a $(\pi d_{3/2})^{-1}(\nu f_{7/2})$ configuration.

After the present experiments were completed, we became aware of some recent results of Bass and Wechsung,[9] who have used the attenuated-Doppler-shift technique to measure the lifetimes of a number of states in $^{40}$K. These workers obtain results in good agreement with those reported here, namely $(0.50 \pm 0.15) \times 10^{-12}$ sec for the second excited state and $(1.5 \pm 0.4) \times 10^{-12}$ sec for the third excited state. Bass and Wechsung consider a number of $^{40}$K states built of $(\pi d_{3/2})^{-1} \times (\nu f_{7/2})$, $(\pi s_{1/2})^{-1}(\nu f_{7/2})$, and $(\pi d_{3/2})^{-1}(\nu p_{3/2})$ components; the first four states are taken to be almost ($\geq 97\%$) pure $(d_{3/2})^{-1}f_{7/2}$. Using effective moments and charges, they find that transition rates can be fitted within about a factor of 2 —which they consider to be very good agreement. In contrast, we find that in comparing transitions within this multiplet, which Bass and Wechsung do not do in detail, a factor-of-2 disagreement is very serious since it cannot be remedied

Table II. The quantity $|M_{df}|^2$ defined by Eq. (4) for the $M1$ transitions in the $(\pi d_{3/2})(\nu f_{7/2})$ multiplet. If the configurations were pure, $|M_{df}|^2$ would be a constant in each nucleus.

| Transition | $^{40}$K | $^{38}$Cl |
|---|---|---|
| $3^- \leftrightarrow 2^-$ | $6.1 \pm 1.5$ | $15^{+10}_{-5}$ |
| $4^- \leftrightarrow 3^-$ | $8.7 \pm 0.8$ | $2.7^{+1.6}_{-1.0}$ |
| $5^- \leftrightarrow 4^-$ | $3.2 \pm 0.6$ | $8.9 \pm 2.5$ |
| Calc. using free moments | 6.24 | 6.24 |
| Calc. using effective moments | 8.09[a] | 13.09[b] |

[a]Obtained using $\mu(^{41}Ca)$ and $\mu(^{39}K)$. [b]Obtained using $\mu(^{41}Ca)$ and $\mu(^{37}Cl)$.

by varying the choice of effective moments. Rather, major alterations in the wave function are necessary if theory is to be brought into agreement with experiment.

As noted above, a major success of the $(d_{3/2})^{-1} \times f_{7/2}$ model for the first four $^{40}$K states is the fact that the spectroscopy of the first four $^{38}$Cl levels can be derived from the spectroscopy of the first four $^{40}$K levels. In this picture $|M_{df}|^2$ should also be a constant for $M1$ transitions among the first four levels in $^{38}$Cl, and we also measured the lifetimes of the third and fourth excited states in $^{38}$Cl (Fig. 1), again using the attenuated-Doppler-shift technique. The results are included in Table I. Since the fourth excited state can decay by an $M1$ transition to either the second or the third excited state, the branching ratio as well as the lifetime of the state is needed in order to determine the partial widths. In fact, the branching ratio is in itself a test of the model: The prediction is that the ratio of the speed of the $1.311 \rightarrow 0.671$ transition to the speed of the $1.311 \rightarrow 0.761$ transition should be 1.21. However, the experimental branching ratio was 4.4 in one spectrum and 3.4 in the other. While some proton-gamma angular correlations may be present, these are expected to be small, and we take the branching ratio to be $3.9 \pm 1.0$. Table II contains the values of $|M_{df}|^2$ extracted from the three $^{38}$Cl transitions; and it is apparent that here too if only the $(\pi d_{3/2})(\nu f_{7/2})$ configuration is considered, the relative transition rates are not adequately explained. It should be noted that again small admixtures of what are expected to be the strongest impurities—namely the $(\pi s_{1/2})^{-1} \times (\nu f_{7/2})$ and the $(\pi d_{3/2})(\nu p_{3/2})$ configurations—will have little effect on the $M1$ transition rates.

In summary, then, it must be said that the $(\pi d_{3/2})(\nu f_{7/2})$ model, which is so successful in predicting many of the properties of the low-lying $^{40}$K and $^{38}$Cl states, fails badly when it comes to predicting relative $M1$ transition rates within the multiplet—even when effective moments are used. In order to explain the observed rates, either the $(\pi s_{1/2})(\nu f_{7/2}) + (\pi d_{3/2})(\nu p_{3/2})$ components must be large (~30%) or admixtures of higher configurations not adequately taken into account by the use of effective moments must be present.

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