a broad range of the phonon spectrum and are relatively insensitive to the details of the dispersion relation. For liquid He^4 the difficulty in deriving the correct form of the phonon dispersion relation from C_V is increased by the dominance of $C_{V,r}$ at higher temperatures. The significance of the γ values in Table I is that they give the correct signs of the dominant terms in Eq. (1) and represent the best simple approximation to the dispersion relation for phonons in the range $0.2 \lesssim \epsilon/k_B \lesssim 1$ K. The data for $V=27.58$ cm^3/mole, for example, could also be approximated, but not as well, by $\gamma=0$ and $\delta=-4.5 \times 10^{76}$ g^{-4} cm^{-4} sec^4 ($C_V=C_{V,r}+AT^3+CT^7$); other more complicated dispersion curves might fit the data slightly better than the $\gamma=-4.1 \times 10^{37}$ g^{-2} cm^{-2} sec^2, $\delta=0$ curve.

The inelastic scattering of neutrons from liquid He^4 at the saturated vapor pressure has been used by Woods and Cowley[12] to show that the dispersion is negative for $\epsilon/k_B > 8$ K. Their data are not inconsistent with the C_V data because their longest wavelength data correspond to $\epsilon/k_B \approx 4$ K, and $\gamma=-4.1\times 10^{37}$ g^{-2} cm^{-2} sec^2 falls within their assigned error limits up to approximately 5.5 K. In fact, an extrapolation of their data from above $\epsilon/k_B = 8$ K, where their accuracy is higher, could be taken as suggesting a region of positive dispersion at low energy, in agreement with the C_V data.

There is apparently no theoretical basis for negative γ values. Eckstein and Varga[13] have derived γ values of the order of magnitude of 10^{37} g^{-2} cm^{-2} sec^2 from the hydrodynamic Hamiltonian, but they are positive.

The strong volume dependence of γ suggests that measurement of the attenuation of sound as a function of pressure might be of interest. The discrepancy between the observed attenuation and the theoretical maximum that is found near zero pressure may disappear at higher pressures where γ is positive.

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Experimental Observation of Nonlinear Landau Damping of Plasma Waves in a Magnetic Field*

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Experiments on nonlinear Landau damping and growth of longitudinal plasma waves that propagate nearly perpendicular to a magnetic field have been carried out. The measured value of the nonlinear wave-particle coupling coefficient is in good agreement with theory.

Nonlinear Landau damping (or growth) is one of the fundamental mechanisms in nonlinear plasma theory. It is expected to play an important role in the development of plasma turbulence, explosive instabilities, and plasma heating. Although in recent years much effort has been given to developing the theory of this process,[1-3] experimental results supporting such theories have been lacking. In this Letter we report (a) the experimental observation of nonlinear Landau damping (growth) of longitudinal plasma waves that propagate nearly perpendicularly to an external dc magnetic field, and (b) the quantitative measurement of the matrix elements (coupling coefficients) for nonlinear Landau damping in a magnetic field. The measurements are in good agreement with theory.

Physically, nonlinear Landau damping can be viewed as the scattering of two (or more) waves nonresonantly; the virtual beat waves produced by such scattering can then interact with particles by Landau damping. For a plasma in a magnetic field, the resonant condition for a simple wave-wave-particle process is given by

$$\omega_{\vec{k}''}+\omega_{\vec{k}'}-(k_{\parallel}''+k_{\parallel}')v_{\parallel}=l\omega_c, \tag{1}$$

where $\omega_{\vec{k}''}$ and $\omega_{\vec{k}'}$ are the wave frequencies, $k_{\parallel}''$ and $k_{\parallel}'$ are the components of the wave vectors parallel to the magnetic field, $v_{\parallel}$ is the particle velocity, ω_c is the cyclotron frequency, and l is an integer. Thus, this process is fundamentally different from the well-known resonant mode-mode coupling, in which case the beat wave itself is also a resonant mode (i.e., $\omega_{\vec{k}'}+\omega_{\vec{k}''}=\omega_{\vec{k}}$).[1-3] We note that if all values of $v_{\parallel}$ are in resonance, then Eq. (1) reduces to[3]

$$\omega_{\vec{k}''}+\omega_{\vec{k}'}=l\omega_c. \tag{1a}$$

In our experiments we observed the following: (a) Nonresonant decay of an externally launched, finite-amplitude cyclotron harmonic wave (or Bernstein mode[4]), $\omega_{k''}$, into a small-amplitude one, $\omega_{k'}$, such that $|(\omega_{k''}-\omega_{k'}-l\omega_c)/l\omega_c|<10^{-2}$. (b) Amplification of a small test wave with increasing amplitudes of the pump wave. In these experiments the small-amplitude test wave, with frequency $\omega_{k'}$, was launched externally by a second probe. (c) Damping of the finite-amplitude pump wave when the perturbed waves grew to sufficiently large amplitudes. We note that in the present experiments the conditions were adjusted so that no rf signals were observed at the difference (or sum) frequencies, $\omega=\omega_{k''}+\omega_{k'}$. Thus, resonant mode-mode coupling or parametric theory cannot explain the present results. In particular, we have previously reported observation of the decay of cyclotron harmonic waves by the process of resonant mode-mode coupling.[5,6] The decay of the finite-amplitude wave occurred either into one weakly damped and one strongly damped wave,[5] or two weakly damped propagating waves.[6] In the previous cases the difference frequency $(\omega_{k''}-\omega_{k'})$ was always well separated from an exact harmonic of the cyclotron frequency, and it was associated with waves. The perturbed-wave amplitudes were governed by a second-order wave kinetic equation. In the present case, as we shall see, the amplification rates of the perturbed wave are governed by a third-order kinetic equation.

Using a dressed-particle model, Rosenbluth et al. have calculated the matrix elements for nonlinear Landau damping of flute modes ($k_{\parallel}=0$) in a plasma in a magnetic field.[3] Recently we have obtained the matrix elements by a third-order perturbation theory directly from the Vlasov equation, including finite $k_{\parallel}'$, $k_{\parallel}''$. In the limit $k_{\parallel}'$, $k_{\parallel}''\to 0$, the results of our calculation substantiate the validity of the dressed-particle model of Rosenbluth et al. Since our measurements were carried out in space under steady-state conditions, we have derived the spatial analog of the Rosenbluth et al. wave kinetic equation.[7] In the limit $k_{\parallel}'\ll k_{\perp}'$, $k_{\parallel}''\ll k_{\perp}''$, $|E_{k''}|\gg|E_{k'}|$, a narrow frequency spectrum, and Eq. (1a), we obtained the following equations:

$$\frac{\partial E_{\omega',k'}}{\partial x}+\alpha E_{\omega',k'}=\frac{CE_{\omega',k'}E_{\omega'',k''}^2}{\partial D/\partial k'}, \tag{2}$$

$$\frac{\partial E_{\omega'',k''}}{\partial x}+\alpha_0 E_{\omega'',k''}=0. \tag{3}$$

Here, $C=\eta\tilde{c}$, α and α_0 are the linear spatial damping rates, D is the plasma dispersion function, and

$$\eta=\frac{1}{4}\left(\frac{\omega_p}{\omega_c}\right)^4\frac{\omega_c}{v_{\mathrm{th}}k_{\parallel}}(\pi/2)^{1/2}\frac{l}{n_0\kappa T_e},$$

where $k_{\parallel}=k_{\parallel}''-k_{\parallel}'$ and ω_p, v_{th}, n_0, and T_e are the plasma frequency, thermal velocity, number density, and electron temperature, respectively. The quantity $(\tilde{c}/\omega_c^4)^{1/2}$ is given by Eq. (IX-9) of Rosenbluth et al.[3] Note that we have assumed that the nonlinear terms have no effect on the amplitude of the finite-amplitude wave. This assumption will, of course, break down when the perturbed wave $\omega'(k')$ grows to values sufficiently large to attain a significant fraction of the energy of the finite-amplitude wave $\omega''(k'')$.

Equations (2) and (3) can be combined, integrated, and rewritten in a form that is useful for comparison with experimental measurements:

$$\ln[E_{\omega',k'}]=[CE_0^2\exp(-2\alpha_0x)]\left[\frac{\partial D}{\partial k'}(-2\alpha_0)\right]^{-1} -\alpha x+\mathrm{const}, \tag{4}$$

where $E_0=E_{\omega'',k''}(x=0)$. Thus, if we take the difference of Eq. (4) for two different values of $E_{k'', \omega''}$ at one position in space, we obtain

$$\tilde{c} = \ln\left[\frac{E_{\omega', k'}(2)}{E_{\omega', k'}(1)}\right]\frac{\partial D}{\partial k'}(-2\alpha_0) \times\{\eta[E_{\omega'', k''}{}^2(2)-E_{\omega'', k''}{}^2(1)]\}^{-1}. \quad (5)$$

In the present experiments all the quantities on the right-hand side of Eq. (5) were measured, and thus the coupling coefficient was determined.

The experiments were carried out in a helium plasma produced by a hot-cathode discharge at one end of a linear machine, with the magnetic field in the experimental region uniform to better than 0.1%. The plasma parameters were as follows: electron temperature $T_e = 4.0$ eV, electron density $n_0 = 4\times10^{10}$ cm^{-3}, and collision frequency $\nu_{e0} = 5\times10^6$ sec^{-1}, giving a mean free path $\lambda_{e0} = 40$ cm. The plasma column was approximately 12 cm in diameter, and was substantially uniform 3 cm radially. Three radially movable coaxial probes, 90° apart, provided with either T-shaped tips or double grids (with the plane of the T and the grids parallel to the magnetic field) were employed to launch or detect waves. In these experiments we used a calibrated interferometer to study the wave dynamics.

In Fig. 1(a) we exhibit typical decay spectra which were obtained from the receiving probe. The probe was located a few centimeters in the radial direction from the transmitting probe used to launch the finite-amplitude wave. In Fig. 1(b) we show the location of the frequencies $\omega_{k'}$, $\omega_{k''}$ and the wave numbers $k_\perp'$, $k_\perp''$ (as measured by the interferometer) on the dispersion diagram of cyclotron harmonic waves.[8] Also shown are the difference frequencies of each doublet, and the difference wave numbers (which are associated with momentum taken up by the magnetic field[3]). Similar data have been obtained for various values of the density, magnetic field, and frequency. We see that the difference frequencies are the cyclotron frequency or its second harmonic. We also note the lack of rf signals on the spectrum at the difference frequency $\omega_{k''} - \omega_{k'}$. In particular, in these data $\omega_{k''}$, ω_c, n_0, and pump-wave amplitude E_0 have been carefully adjusted to ensure that no other competing processes, such as resonant scattering, occurred simultaneously with the nonresonant decay.

By varying the transmitter power, we could study the variation of the amplitude of the perturbed wave $E_{\omega', k'}$ (which grew from background noise) as a function of E_0. Utilizing techniques

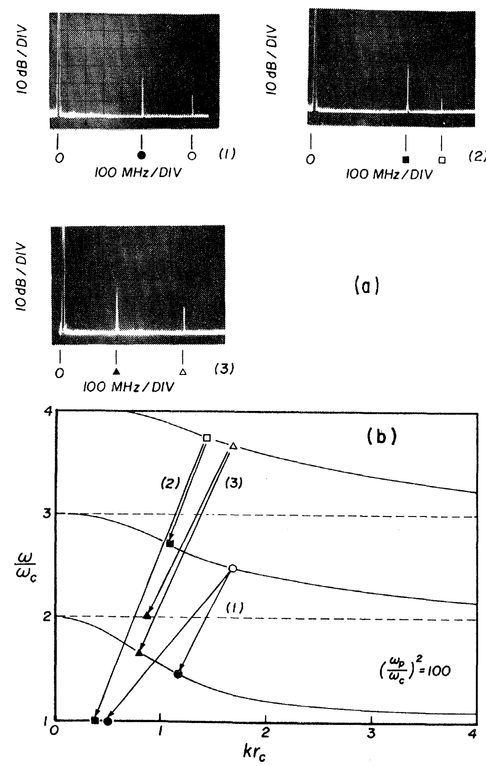

FIG. 1. (a) Spectra for three distinct cases of nonresonant decay. The finite-amplitude wave signal (hollowed symbols) is greatly attenuated by filtering. (1) $f_c = 290$ MHz, (2) $f_c = 187$ MHz, (3) $f_c = 190$ MHz. (b) Location of spectra (1), (2), and (3) on the dispersion curve of cyclotron harmonic waves.

discussed in Ref. 6, we have been able to measure the pump-wave electric field, and thus determine the coupling coefficient (i.e., matrix element) for nonlinear Landau damping. Alternatively, we have reduced the pump-wave amplitudes to values considerably below those used in the decay (i.e., $e\varphi/kT_e < 10^{-2}$) and observed amplification of a small-amplitude test wave $E_{\omega', k'}$ launched by a second transmitting probe, as shown in Fig. 2. The frequencies were selected to correspond exactly to those observed in the decay, and $E_{k', \omega'}$ was adjusted to be 40-50 dB lower than $E_{k'', \omega''}$, the pump wave. Again, no rf signals were observed at the difference (or sum) frequencies. A calibrated interferometer was used to measure $E_{k', \omega'}$ as a function of $E_{k'', \omega''}$ a few centimeters radially from the second transmitting probe. Using the data of Fig. 2, we show in Fig. 3 the plot of $\log(E_{k'}{}^2)$ and $E_{k''}{}^2$ as a function of the transmitter power. We note that the third-order theory is followed through four decades! In the same figure we see the damping of the finite-amplitude wave (dashed curve) for

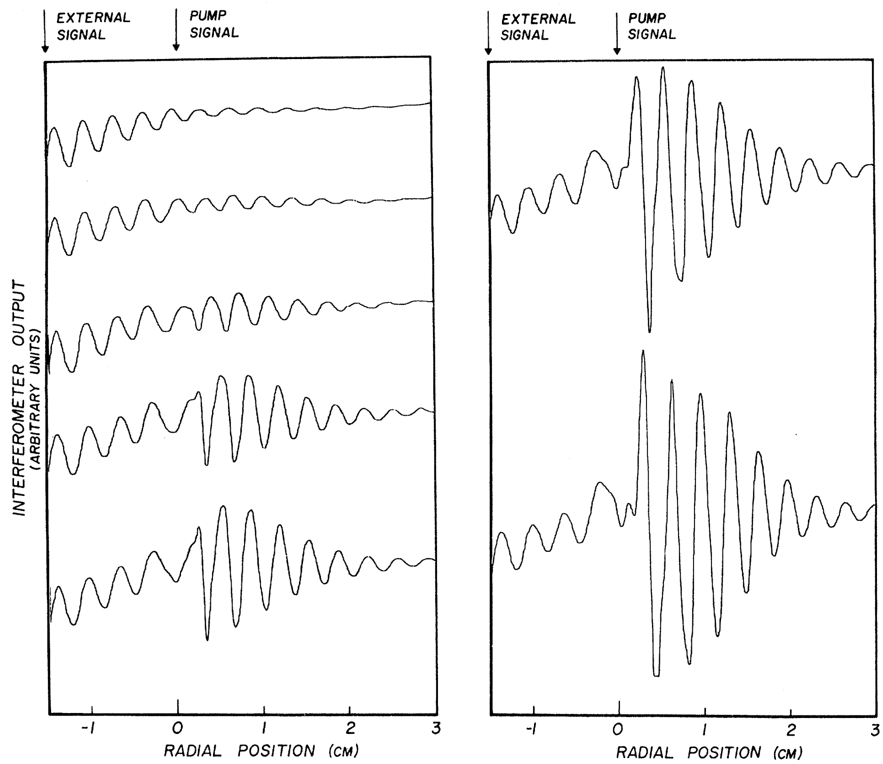

FIG. 2. Interferometer traces showing the amplification of the externally excited test wave $\omega'(k')$ for case (2) of Fig. 1. The arrows indicate the location of the probes for exciting the test wave and the pump wave.

large values of $E_{k'}^2$. The experimental value of $\tilde{c}$ was obtained by combining the amplification rate with the measured values of η, α_0, E_0, and $\partial D/\partial k_\perp$ according to Eq. (5). For the three cases shown in Fig. 1, the measured values of $\tilde{c}$ are 6.4×10^{-2}, 9.4×10^{-2}, and 4.3×10^{-2}. The corresponding values from the theory are 4.7×10^{-2}, 5.2×10^{-2}, and 4.8×10^{-2}. This is considered to be good agreement. We remark that the coupling coefficients obtained in the nonlinear decay experiments were essentially the same as those given here.

The main sources of error were the uncertainties in the measurements of n_0, E_0, and $k_\parallel$. The absolute value of n_0 was determined (better than within a factor of 2) by four different techniques: Langmuir probe (using the theory of Laframboise), cutoff of transmission between two T probes at the upper hybrid frequency, standard microwave interferometry, and an observation of a beam-plasma interaction cutoff frequency. In order to measure E_0, one must first obtain the probe coupling coefficients. The techniques of this measurement have been discussed recently,[6] and its accuracy is estimated to be within a factor of 2. The importance of $k_\parallel$ in the propagation of cyclotron harmonic waves (launched by a T probe) has been demonstrated recently in connection with echo studies.[9] In the present work the axial wave amplitudes $E_{k'}$, $E_{k''}$ were mapped

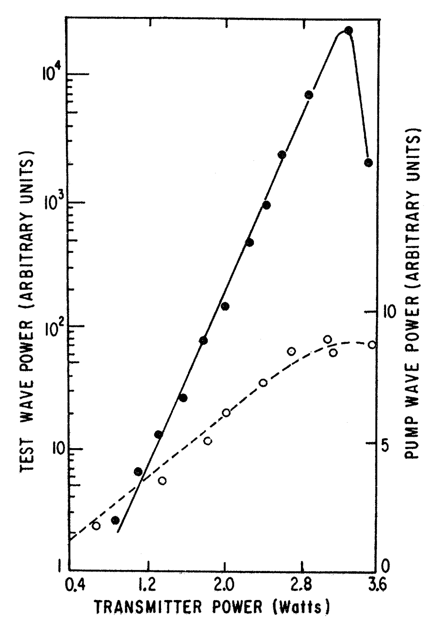

FIG. 3. $E_{\omega',k'}^2$ (solid curve) and $E_{\omega'',k''}^2$ (dashed curve) versus transmitter power at 1 cm radially from the transmitter probe.

out separately, by means of an axially and radially movable probe. We found, by interferometry, that the wave signals were primarily concentrated within the axial length of the T probes or grids. When the detecting probe was outside of this region, the signal dropped by 60 dB. Our measurements indicated that in the axial direction the waves were essentially standing waves, with the dominant Fourier components having twice the probe lengths; i.e., $k_{\parallel}' \simeq \pi/l'$, etc. Thus, by approximating the standing waves in the axial direction with $\cos(k_{\parallel}'z)$, $\cos(k_{\parallel}''z)$, we find from theory that for $k_{\parallel} \ll k_{\perp}$ the dominant contributions to the matrix elements are given by $k_{\parallel} = k_{\parallel}' + k_{\parallel}''$. By varying the lengths of the receiving and transmitting probes in the nonlinear Landau damping experiments, we have verified that this model of wave excitation gave good agreement with theory.

In summary, we believe that we have observed experimentally nonlinear Landau damping of plasma waves in the presence of an external magnetic field. The qualitative features of the experimental results agreed well with the predictions of the nonlinear Landau damping theory. In particular, the observed decay of a finite-amplitude wave into a single perturbed wave cannot be explained by resonant mode-mode coupling or parametric theory. The amplitude variation of the perturbed wave as a function of the pump wave followed the predictions of the third-order theory, through at least four decades. The measured nonlinear wave-particle coupling coefficient was found to agree with theory within experimental error. A detailed account of this work will be published elsewhere.

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[8]The dispersion curves shown in Fig. 1(b) are those with $k_{\parallel}=0$. Nevertheless, for the experimentally measured values of $k_{\parallel}/k_{\perp}(k_{\parallel} \ll k_{\perp})$, it can be shown that the *real* parts of $k_{\perp}$ and ω are closely approximated by the curves used.

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Electron Cyclotron Drift Instability*

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Electron cyclotron waves are shown to grow unstably in the presence of small relative drifts of a warm ion distribution perpendicular to the B field. Numerical simulations show that this instability causes anomalous diffusion of plasma across the magnetic field and heating of the electron thermal spread to values much greater than the relative ion-electron drift velocity.

Most methods for heating and confining high-temperature plasma involve a flow of electric currents normal to magnetic field lines in the plasma. This Letter reports the theoretical properties of a high-frequency electrostatic plasma instability which is driven by the relative drift of ions and electrons in such a normal current flow and which is expected to occur and cause anomalous resistance to the current in a wide range of applications. The calculations presented here are done for an infinite, uniform, collisionless plasma with a fixed magnetic field B_z. The electrons have no drift and the ions have a drift v_d in the x direction. The ion and electron temperatures are $T_i = Mv_i^2/2$ and $T_e = mv_e^2/2$.

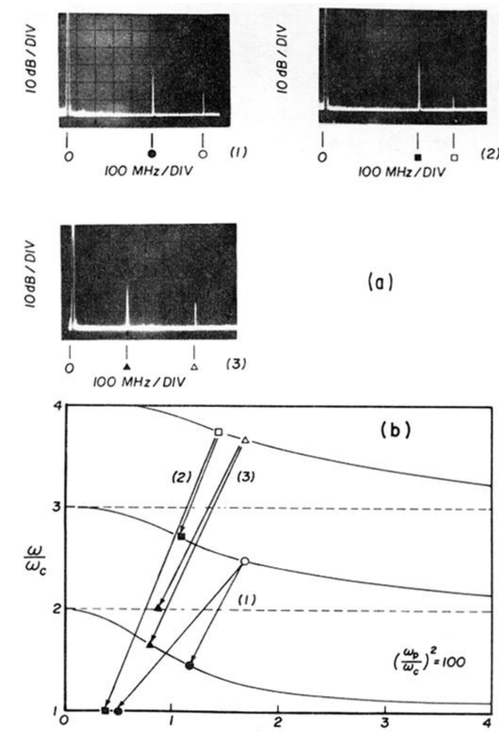

FIG. 1. (a) Spectra for three distinct cases of nonresonant decay. The finite-amplitude wave signal (hollowed symbols) is greatly attenuated by filtering. (1) $f_c = 290$ MHz, (2) $f_c = 187$ MHz, (3) $f_c = 190$ MHz. (b) Location of spectra (1), (2), and (3) on the dispersion curve of cyclotron harmonic waves.