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[13]For similar reasons the existence of the transport coefficients in the Burnett hydrodynamical equations is not clear in three dimensions.

Calorimetric Evidence for Positive Phonon Dispersion in Liquid Helium-4†

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The constant-volume heat capacity of liquid He^4 has been measured from 0.3 K to above the λ point. Below 0.9 K the phonon heat capacity corresponds to a dispersion relation $\epsilon = cp(1-\gamma p^2)$ with γ varying from -4.1×10^{37} g^{-2} cm^{-2} sec^2 at the saturated vapor pressure to 19.6×10^{37} g^{-2} cm^{-2} sec^2 near the freezing pressure.

During the past few years, extensive studies[1-6] of the propagation of sound in liquid He^4 have revealed inadequacies in the theoretical expressions for the velocity and attenuation. In a recent Letter, Maris and Massey[7] have pointed out that the discrepancies between theory and experiment might be resolved if the coefficient γ in the equation representing the phonon region of the energy-momentum relation,

$$\epsilon = cp(1-\gamma p^2-\delta p^4\cdots), \tag{1}$$

were negative. For example, negative values would account for the observed attenuation exceeding the theoretical maximum. However, γ has generally been assumed to be positive, and no direct experimental evidence for negative values has been published previously. In this Letter we report measurements of the constant-volume heat capacity C_V that provide support for the suggestion that γ is negative. The measurements show that for pressures near the saturation vapor pressure and for low phonon energies the dispersion is positive [dominated by terms in Eq. (1) with negative coefficients] and can be approximated by $\gamma = -4.1\times10^{37}$ g^{-2} cm^{-2} sec^2.

During the measurements the capillary used to fill the constant-volume cell was closed by a valve at the entrance to the cell, and evacuated.[8] This eliminated the various problems associated with a connecting capillary filled with He^4. A germanium thermometer that had been calibrated against the susceptibility of a single crystal of cerium magnesium nitrate was used. The heat capacity of copper has been measured on the same temperature scale and the results were in good agreement with accepted values,[9] demonstrating the reliability of the calibration. Systematic errors in C_V are believed to be less than 1%, and to vary only slowly with temperature.

The low-temperature expression for the phonon heat capacity corresponding to Eq. (1) is

$$C_{V,ph} = \frac{\pi V(2\pi k_B)^4}{15h^3c^3}\left[T^3 + \frac{25\gamma}{7}\left(\frac{2\pi k_B}{c}\right)^2 T^5 + \frac{3\gamma^2+\delta}{7}\left(\frac{2\pi k_B}{c}\right)^4 T^7\cdots\right] \equiv AT^3+BT^5+CT^7\cdots, \tag{2}$$

where k_B is the Boltzmann constant and V is the molar volume. For temperatures of approximately 1 K or less the roton contribution to the heat capacity should be adequately approximated by[10]

$$C_{V,r} = R\left[\left(\frac{\Delta}{k_BT}\right)^{3/2} + \left(\frac{\Delta}{k_BT}\right)^{1/2} + \frac{3}{4}\left(\frac{\Delta}{k_BT}\right)^{-1/2}\right]\exp(-\Delta/k_BT), \tag{3}$$

where $R \equiv 2p_0^2\mu^{1/2}k_BV/(2\pi)^{3/2}\hbar^3$. In the derivation of Eq. (3) the energy-momentum relation for rotons is approximated by $\epsilon = \Delta + (p-p_0)^2/2\mu$.

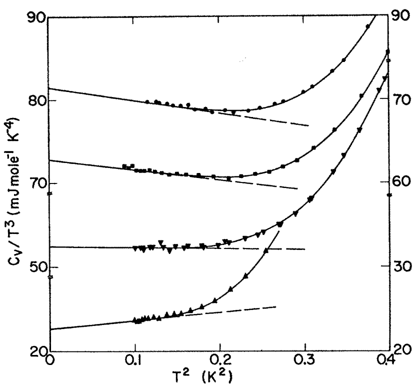

FIG. 1. The constant-volume heat capacity C_V of liquid He^4, plotted as C_V/T^3 vs T^2. The solid curves are least squares fits by $C_V = C_{V,r} + AT^3 + BT^5$, and the dashed lines represent $AT^3 + BT^5$. Circles, 27.58 cm^3/mole; squares, 27.11 cm^3/mole; inverted triangles, 26.23 cm^3/mole; triangles, 23.79 cm^3/mole.

Figure 1 shows C_V/T^3 vs T^2 for liquid He^4 at four molar volumes. (The data labelled $V = 27.58$ cm^3/mole were actually taken at the saturation vapor pressure but the correction to constant volume is negligible at these temperatures.) The minimum in C_V/T^3 for the higher molar volumes shows that the contribution from higher-order terms in Eq. (1) is negative below about 0.5 K. The data were fitted by a least-squares procedure with the three expressions $C_V = C_{V,r} + AT^3$, $C_V = C_{V,r} + AT^3 + BT^5$, and $C_V = C_{V,r} + AT^3 + CT^7$, in which A, B, C, R, and Δ were taken as adjustable parameters. For each expression the rms deviation was obtained as a function of a variable high-temperature cutoff temperature T_u by fitting only the data for $T \leq T_u$. The best fits were obtained for $C_V = C_{V,r} + AT^3 + BT^5$. With that expression the rms deviations and derived parameters were almost independent of T_u for $T_u \lesssim 0.9$ K. The deviations appear to be random and their rms values are between 0.2 and 0.4%. The expressions obtained for C_V are shown by the solid curves in Fig. 1, and the corresponding values of the parameters are given in Table I. For those parameters that can be compared with values derived from more direct measurements, the agreement is good. For $V = 27.58$ cm^3/mole, inelastic scattering of neutrons[11] gives $\Delta/k_B = 8.65$ K and the value derived from C_V is 8.62 K. For the same molar volume, Whitney and Chase[4] have measured the temperature dependence of the sound velocity c. From their data and absolute measurements above 1 K, they have derived 0-K values of c between 2.38 and 2.39 $\times 10^4$ cm/sec. The heat-capacity measurements give 2.40×10^4 cm/sec. The discrepancy is somewhat larger than we would expect, but it could be caused by the simplification of Eq. (2) to two terms. The average value of the Grüneisen constant $[\equiv -(V/c)(\partial c/\partial V)]$ obtained from C_V is 2.6, and direct measurements[6] of the pressure dependence of c give values of 2.2 to 2.8, depending on pressure.

The data could also be fitted quite well with $C_V = C_{V,r} + AT^3 + CT^7$, but the rms deviations were consistently higher. As suggested by inspection of Fig. 1, the expression $C_V = C_{V,r} + AT^3$ does not fit the data well. The rms deviations were large and decreased steadily with decreasing T_u. The values of R and Δ were strongly dependent on T_u and were obviously incorrect when T_u was made low enough to give resonable rms deviations. For example, for $V = 27.58$ cm^3/mole and $T_u = 0.95$ K, the rms deviations from $C_V = C_{V,r} + AT^3 + BT^5$ were 0.2%, but from $C_V = C_{V,r} + AT^3$ they were 0.8% and the deviations were not random. With T_u decreased to 0.6 K the rms deviation from $C_V = C_{V,r} + AT^3$ was still 0.53%, but Δ/k_B had increased to 12.5 K.

The heat capacities below 0.9 K are satisfactorily represented by the first two terms of Eq. (1) with the γ values in Table I. [The corresponding γ^2 contributions to the T^7 term in Eq. (2) are negligible at these temperatures.] If the first two terms of Eq. (1) correctly express the form of the dispersion relation for phonons with $\epsilon/k_B \lesssim 1$ K, the uncertainty in the γ values is approximately $\pm 10\% \pm 5 \times 10^{36}$ g^{-2} cm^{-2} sec^2. However, C_V measurements sample

Table I. Parameters obtained by fitting the C_V data with $C_V = C_{V,r} + AT^3 + BT^5$.

V $\left(\frac{\text{cm}^3}{\text{mole}}\right)$	A $\left(\frac{\text{mJ}}{\text{mole K}^4}\right)$	B $\left(\frac{\text{mJ}}{\text{mole K}^6}\right)$	R $\left(\frac{\text{mJ}}{\text{mole K}}\right)$	Δ/k_B (K)	c $\left(\frac{\text{cm}}{\text{sec}}\right)$	γ $\left(\frac{\text{sec}^2}{\text{g}^2\text{cm}^2}\right)$
27.58	81.57	−15.6	6.63×10^4	8.62	2.397×10^4	-4.1×10^{37}
27.11	72.88	−12.3	6.75×10^4	8.61	2.475×10^4	-3.8×10^{37}
26.23	52.41	−1.0	6.38×10^4	8.43	2.732×10^4	-0.5×10^{37}
23.79	22.72	9.8	5.47×10^4	7.67	3.495×10^4	19.6×10^{37}

a broad range of the phonon spectrum and are relatively insensitive to the details of the dispersion relation. For liquid He^4 the difficulty in deriving the correct form of the phonon dispersion relation from C_V is increased by the dominance of $C_{V,r}$ at higher temperatures. The significance of the γ values in Table I is that they give the correct signs of the dominant terms in Eq. (1) and represent the best simple approximation to the dispersion relation for phonons in the range $0.2 \lesssim \epsilon/k_B \lesssim 1$ K. The data for $V=27.58$ cm^3/mole, for example, could also be approximated, but not as well, by $\gamma=0$ and $\delta=-4.5 \times 10^{76}$ g^{-4} cm^{-4} sec^4 ($C_V=C_{V,r}+AT^3+CT^7$); other more complicated dispersion curves might fit the data slightly better than the $\gamma=-4.1 \times 10^{37}$ g^{-2} cm^{-2} sec^2, $\delta=0$ curve.

The inelastic scattering of neutrons from liquid He^4 at the saturated vapor pressure has been used by Woods and Cowley[12] to show that the dispersion is negative for $\epsilon/k_B > 8$ K. Their data are not inconsistent with the C_V data because their longest wavelength data correspond to $\epsilon/k_B \approx 4$ K, and $\gamma=-4.1\times 10^{37}$ g^{-2} cm^{-2} sec^2 falls within their assigned error limits up to approximately 5.5 K. In fact, an extrapolation of their data from above $\epsilon/k_B = 8$ K, where their accuracy is higher, could be taken as suggesting a region of positive dispersion at low energy, in agreement with the C_V data.

There is apparently no theoretical basis for negative γ values. Eckstein and Varga[13] have derived γ values of the order of magnitude of 10^{37} g^{-2} cm^{-2} sec^2 from the hydrodynamic Hamiltonian, but they are positive.

The strong volume dependence of γ suggests that measurement of the attenuation of sound as a function of pressure might be of interest. The discrepancy between the observed attenuation and the theoretical maximum that is found near zero pressure may disappear at higher pressures where γ is positive.

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Experimental Observation of Nonlinear Landau Damping of Plasma Waves in a Magnetic Field*

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Experiments on nonlinear Landau damping and growth of longitudinal plasma waves that propagate nearly perpendicular to a magnetic field have been carried out. The measured value of the nonlinear wave-particle coupling coefficient is in good agreement with theory.

Nonlinear Landau damping (or growth) is one of the fundamental mechanisms in nonlinear plasma theory. It is expected to play an important role in the development of plasma turbulence, explosive instabilities, and plasma heating. Although in recent years much effort has been given to developing the theory of this process,[1-3] experimental results supporting such theories have been lacking. In this Letter we report (a) the experimental observation of nonlinear Landau