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Dimensions of Currents and Current Commutators*

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We study the scale dimensions of current components and Schwinger terms. We state and prove necessary and sufficient conditions for the spatial and temporal components of hadronic currents to have the same dimension. The incompatibility of well-defined c-number Schwinger terms with universal dimensionality of current components is demonstrated. Experimental implications are discussed.

The discovery of scaling in deep inelastic electron-proton scattering has engendered renewed interest in the notion that strong interactions become scale invariant at small distances.[1] Recent work of Wilson[2] suggests that it may be possible to implement this idea within the framework of quantum field theory provided one adheres to a meaningful definition of the scale dimensionality d of field operators. For a spin-zero field $\varphi(x)$, an operational definition for d is the following: If the renormalized propagator $\Delta_F'(q^2)$ behaves as $(-q^2)^{d-2}$ as $q^2 \to -\infty$, the scale dimension of the field is d. (Very similar definitions can be given for fields with other spins.)

It is likely that, in general, d is anomalous, i.e., $\neq 1$.[3] Furthermore, this asymptotic behavior of the propagator will follow from considerations of broken scale invariance[4] if d arises by commuting at equal times $\varphi(x)$ with D, the generator of infinitesimal scale transformations:

$$i[D(x_0), \varphi(x)] = x^\mu \partial_\mu \varphi(x) + d\varphi(x). \quad (1)$$

In this note we study the scale dimensions of current components and Schwinger terms. We state and prove necessary and sufficient conditions for the temporal and spatial components of currents to have the same dimension. We further demonstrate the incompatibility of well-defined

c-number Schwinger terms, when currents have universal scale dimensions. Some experimental consequences of our considerations are discussed.

If scale transformations are to leave unaltered Gell-Mann's SU(3)⊗SU(3) charge algebra, it is clear that the local charge densities J_0^α must have the scale dimension 3, subject to the assumption that they do have a well-defined dimension. If one wishes to consider the electromagnetic current density without reference to the non-Abelian current-algebra relations, than the same conclusion may be arrived at if there exists a charge operator of well-defined scale dimensionality. These are, of course, well-known results; what does not seem to be so well known[5] is an answer to the following question: What is the dimensionality of the spatial components of $J_\mu^\alpha(x)$?

The importance of the above question is the following: Lepton-induced reactions are our most reliable probe into hadron structure; if the hadronic currents which couple to the leptons have a dimensionality which depends on the Minkowski index μ, one cannot talk of asymptotic scale invariance in these reactions without running afoul of the principle of relativity!

We denote by $\theta_{\mu\nu}$ the "new improved energy-momentum tensor" appropriate to considerations of scale transformations.[6] Its trace is denoted by $\theta[\equiv g^{\mu\nu}\theta_{\mu\nu}]$. In terms of this tensor the scale current D^μ and the dimension operator D are given by

$$D^\mu=\theta^{\mu\nu}x_\nu;\quad D=\int D_0 d^3x;\quad \partial_\mu D^\mu=\theta. \tag{2}$$

Note that the genesis of the problem at hand lies in the failure of the dimension operator to commute symmetrically with the generators of the Poincaré group, P^α and $M^{\mu\nu}$. Explicitly, one has[7]

$$i[D(0),P_0]=P_0-\int d^3x\,\theta(0,\vec{x}), \tag{3}$$

$$i[D(0),M_{0i}]=\int d^3x\,x_i\theta(0,\vec{x}). \tag{4}$$

We now state our results:

(a) From Lorentz covariance, one easily shows that the necessary and sufficient condition for the scale dimension J_μ^α to be independent of μ is that the following commutator be zero:

$$i[J_0^\alpha(0),\int d^3x\,x_i\theta(0,\vec{x})]=0. \tag{5}$$

(b) With a conventional assumption (to be stated precisely below) concerning the dilation and SU(3)⊗SU(3) symmetry-breaking interactions, Eq. (5) is shown to be equivalent to

$$[Q^\alpha(0),\theta(0,\vec{x})]=\int d^3y[J_0^\alpha(0,\vec{x}),\theta(0,\vec{y})] \tag{6a}$$

$$=0 \text{ if } \dot{Q}^\alpha=0. \tag{6b}$$

Even a stronger statement may be shown to be equivalent to (5): The commutator of θ with the charge density must be free of all gradient terms. A sufficient condition for the validity of (6a) and (6b) is

$$[J_0^\alpha(0,\vec{x}),\int d^3y\,\mathcal{H}_I(0,\vec{y})]=0, \tag{7}$$

where $\mathcal{H}_I$ is the entire symmetry [dilation and SU(3)⊗SU(3)] breaking piece in the Hamiltonian density.

(c) When the currents have universal scale dimensionality in all components, the Schwinger term in the commutator between the temporal and spatial densities must have scale dimensionality 2. This precludes the Schwinger term from being a well-defined c-number. If it is accepted that it is a c-number, then it must be a divergent quantity.

To prove statement (a), we perform an infinitesimal boost, generated by M_{0i}, on each side of the following commutator (which we assume to hold):

$$i[D(0),J_0^\alpha(0)]=3J_0^\alpha(0). \tag{8}$$

Using Eq. (4) (and assuming the validity of the Jacobi identity!) we obtain

$$i[D(0),J_i^\alpha(0)]=3J_i^\alpha(0)+i\int d^3x\,x_i[\theta(0,\vec{x}),J_0^\alpha(0)]. \tag{9}$$

Hence statement (a).

To prove statement (b) we assume that $\mathcal{H}_I$ is not so singular as to negate the divergence conditions[8,9]

$$\partial^\lambda J_\lambda^\alpha(0)=-i[Q^\alpha(0),\mathcal{H}_I(0)] \tag{10}$$

$$\partial^\lambda D_\lambda(0)=-i[D(0),\mathcal{H}_I(0)]+4\mathcal{H}_I(0). \tag{11}$$

By performing an infinitesimal SU(3)⊗SU(3) transformation, generated by Q^α, on each side of Eq. (11), we obtain

$$[Q^\alpha(0),\theta(0)]=[D(0),\partial^\lambda J_\lambda^\alpha(0)]+i4\partial^\lambda J_\lambda^\alpha(0)=\partial^\lambda\{[D(0),J_\lambda^\alpha(0)]+i3J_\lambda^\alpha(0)\}-g^{\lambda 0}[\dot{D}(0),J_\lambda^\alpha(0)]$$
$$=i(3-d_s)\partial^iJ_i^\alpha(0)+[J_0^\alpha(0),\int d^3y\,\theta(0,\vec{y})]. \tag{12}$$

Here d_s indicates the dimension of the spatial components of $J_\mu{}^\alpha$. Equations (6a) and (6b) follow from Eq. (12). Applying an infinitesimal scale transformation to Eq. (7), one verifies immediately that Eq. (7) implies (6a) and (6b).

Finally, a local form of Eq. (12) may be derived by applying an infinitesimal scale transformation to the commutator,

$$i[\theta_{00}(0,\vec{y}), J_0{}^\alpha(0)] = \Phi^\alpha(0)\delta(\vec{y}) + J_i{}^\alpha(0,\vec{y})\partial^i\delta(\vec{y}), \quad \Phi^\alpha(x) = \partial^\lambda J_\lambda{}^\alpha(x). \tag{13}$$

One obtains, after some tedious but straightforward algebra,

$$[J_0{}^\alpha(0,\vec{x}), \theta(0)] = [Q^\alpha(0), \theta(0)]\delta(\vec{x}) - i(\tag{14}$$ sionless systems of charged parti-

Hence the absence of all gradient ternlar systems should be some kind of

To prove statement (c), we perform an infinitesimal scale transformation on the commutator

$$[J_0{}^\alpha(0,\vec{x}), J_i{}^\beta(0)] = if^{\alpha\beta\gamma}J_i{}^\gamma(0)\delta(\vec{x}) + S_{ij}{}^{\alpha\beta}(0)\partial^j\delta(\vec{x}). \tag{15}$$

We obtain

$$(3 + d_s + x_k\partial^k)[J_0{}^\alpha(0,\vec{x}), J_i{}^\beta(0)] = d_s if^{\alpha\beta\gamma}J_i{}^\gamma(0)\delta(\vec{x}) + i[D, S_{ij}{}^{\alpha\beta}(0)]\partial^j\delta(\vec{x}), \tag{16}$$

whence

$$i[D, S_{ij}{}^{\alpha\beta}(0)] = (d_s - 1)S_{ij}{}^{\alpha\beta}(0). \tag{17}$$

Statement (c) is thereby established. It is easy to see that if there are terms on the right-hand side of (15) involving n derivatives of the δ function, then the scale dimension of the coefficient of such a term is $d_s - n$.

When $d_s = 3$, Eq. (17) is incompatible with a well-defined c-number for $S_{ij}{}^{\alpha\beta}$, as in the algebra of fields. In models where the Schwinger term may be evaluated canonically (scalar electrodynamics, σ model), (17) is verified canonically. When the Schwinger term is a noncanonical c-number, as appears to be the case in spinor electrodynamics, a contradiction is avoided since that object comes out to be infinite. Note that a c-number term in (15) proportional to the third derivative of the δ function is consistent with $d_s = 3$.

Examples. – (i) Conventional current algebra with underlying quark structure[10]: Here the currents are bilinear in Fermi fields and

$$\mathcal{H}_I = \epsilon_0\bar{\psi}\psi + \epsilon_8\bar{\psi}\lambda_8\psi + \mathcal{H}_I{}', \tag{18}$$

where $\mathcal{H}_I{}'$ is an SU(3)⊗SU(3) singlet. We do not expect any gradient terms in commuting $J_0{}^\alpha$ with the first two terms on the right-hand side[11] of Eq. (18) – at least for conserved currents; with a well-chosen[12] $\mathcal{H}_I{}'$ we can guarantee that $d_s = 3$.

(ii) Field algebra[13] based on Yang-Mills theory with mass term: In this scheme the currents are taken to be proportional to canonical vector and axial-vector fields. We find that the meson mass terms explicitly break Eqs. (5) and (6). Using either Eq. (9) or Eq. (12) (and commuting naively!), one finds in fact that $d_s = 1$. This is consistent with statement (c), since in this model the Schwinger term is a finite c-number. This value for d_s is, of course, the naive dimension of the canonical vector field; it may be changed by the interaction. It would be a remarkable dynamical accident, however, if it achieves precisely the value 3. Furthermore, note that if d_s does migrate away from 1, the Schwinger term in the algebra of fields can no longer be a finite c-number. (The time component of the vector field is not a canonical variable, but a dependent one. That is why it can carry dimension 3.)

Remarks. – (i) From a physical viewpoint, the following is a consequence of this note: If high-energy lepton-induced reactions such as $e^- + p \to e^- +$ hadrons, $\nu_\mu + p \to \mu^- +$ hadrons, $e^+ + e^- \to$ hadrons, etc. really exhibit features that can be traced back to asymptotic scale invariance, field algebra cannot provide us with a suitable framework for describing these processes in any simple way. It is interesting that measurements of the "transverse-to-longitudinal ratio" in the first-mentioned reaction give results in disagreement with field-algebra expectations.[14]

(ii) If we accept that the Schwinger term in nature (rather than in models) is a c-number,[15] we must conclude that it is a divergent quantity if asymptotic scale invariance holds. It therefore follows that the total electroannihilation cross section $\sigma(q^2)$ for leptons into hadrons with final mass q^2 must decrease more slowly[16] than $1/q^4$. If the decrease is $1/q^2$, as has been frequently suggested, this provides a quadratically divergent Schwinger term, entirely consistent with the present point of view. Note that this quadratic divergence necessarily implies the existence of a finite, third-derivative gradient term in the $[j^0, j^i]$ commutator. Such a term will be given by the asymptotic form of $q^2\sigma(q^2)$. These statements can be easily derived from the spectral representation for the vacuum polarization tensor.

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[4]By this we mean that the (off-shell) matrix elements of θ (defined below) are damped at high frequencies. We shall have no occasion to consider exact scale invariance, which for us implies that $\theta = 0$ as an operator condition.

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[9]Note that (11) implies $i[D(0), \theta^{00}(0)] = 4\theta^{00}(0) - \theta(0)$, where $\theta^{00} = \bar{\theta}^{00} + \mathcal{H}_I$ and $i[D(0), \bar{\theta}^{00}(0)] = 4\bar{\theta}^{00}(0)$. We may consider the validity of this relation, which is slightly more general than (11), as the fundamental assumption from which it is easy to show that our results follow.

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