$2\pi^0$ Mass Spectrum and $\delta_0^{\,0}$ from $\pi^- + p \to \pi^0 + n$ at 10 GeV/c*

E. I. Shibata,† D. H. Frisch, and M. A. Wahlig‡
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
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An enhancement in the $2\pi^0$ mass spectrum in the region $M_{\pi\pi} < 1$ GeV has been observed in $\pi^- + p \to \pi^0 + \pi^0 + n$ at 10 GeV/c. This is interpreted as a broad isospin-0, s-wave resonance at 600 ± 100 MeV with width 400 ± 100 MeV. Because of limited mass resolution, we cannot rule out several narrower resonances with these quantum numbers.

In a previous paper,[1] we reported the observation of two enhancements in the $2\pi^0$ mass spectrum as seen in the reaction

$$\pi^- + p \to \pi^0 + \pi^0 + n. \qquad (1)$$

One of the enhancements is the f^0 meson, centered at 1275 ± 25 MeV; from the decay angular distribution for small-momentum-transfer events the f^0 spin was determined[1] to be 2. The second enhancement is broad, extending from threshold to ~1 GeV, and was not interpreted for two reasons: (a) Depending on then poorly known cross sections, possible backgrounds of up to 40% might have distorted the shape of our $2\pi^0$ mass spectrum significantly; (b) our di-pion mass resolution was (and still is) not good enough to isolate the narrow peaks which were claimed to be in this mass region.[2] Now more information about background reactions is available, and more recent experiments[2-6] favor rather broad (several hundreds of MeV wide) $\pi\pi$ resonances or enhancements in this mass region.

In this paper we report the results of a reanalysis of our data from Reaction (1). The $2\pi^0$ mass spectrum is presented with explicit corrections for geometry and detection inefficiencies and with only a small amount (<9%) of background contamination. Following Sonderegger and Bonamy,[6] we have introduced a form factor into the Chew-Low extrapolation formula,[7] and fitted the modified formula to the data in the physical region in order to obtain $\frac{1}{4}|\exp(2i\delta_0^{\,2}) - \exp(2i\delta_0^{\,0})|^2$, the square of the difference of the isospin-0 and -2 s-wave $\pi\pi$ scattering amplitudes. Here the phase shift for isospin I and orbital angular momentum l is denoted by $\delta_l^{\,I}$.

The experiment was performed with 10-GeV/c π^- from the Brookhaven National Laboratory alternating gradient synchrotron incident on a liquid-hydrogen target. [The virtues of a high-energy study of Reaction (1) have been discussed elsewhere.[8]] A brass-plate spark chamber was used to measure the directions of the four gammas from the $\pi^0 \to 2\gamma$ decays, but the recoil neutron was not detected. A detailed description of the apparatus may be found elsewhere.[8,9] The procedure for the kinematic reconstruction of the $2\pi^0$ events using only the directions of the four decay gammas is as described in our previous paper.[1] We have found that this procedure is equivalent to that of Carroll, Middlemas, and Williams,[10] who performed a similar experiment at lower incident momenta.[2] For our data both procedures give the same results.

Background contamination of our $2\pi^0$ mass spectrum from the following final states has been calculated: $3\pi^0 n$, $K_1^{\,0}(\to 2\pi^0)K_1^{\,0}n$, $K_1^{\,0}(\to 2\pi^0)K_2^{\,0}n$, $K_1^{\,0}(\to 2\pi^0)\Lambda^0(\to n\pi^0)$, and $K_1^{\,0}(\to 2\pi^0)\Sigma^0(\to \Lambda^0\gamma \to n\pi^0\gamma)$. Monte Carlo studies show that only two of these final states, $3\pi^0 n$ and $K_1^{\,0}K_2^{\,0}n$, can contribute measurably to our data and that the 4γ events from these final states have very broad distributions in t' when analyzed as $\pi^0\pi^0 n$. Here $t' \equiv t - t_{\rm min}$, where t is the square of the four-momentum transfer to the recoil neutron, and $t_{\rm min}$ is the minimum possible value of t for a particular di-pion mass value. Since the t' distribution for true $\pi^0\pi^0 n$ events peaks sharply at small t' values, restriction to small t' values cuts out much of the background.

In the subset of 547 events with $|t'| \leq 0.1$ (GeV/c)^2 from which we deduce the $2\pi^0$ mass spectrum, we calculate the contamination from the $3\pi^0 n$ final state to be <45 events by using a $3\pi^0$ spectrum synthesized from $\eta(\to 3\pi^0)$ and four-body phase space. From high-energy experiments which give the $\pi^+\pi^-\pi^0$ mass spectrum[11-15] there is no reason to believe that any other sharp resonances except for the η are present in the $3\pi^0$ mass spectrum; the 3π background in these $\pi^+\pi^-\pi^0$ spectra looks similar to that expected from four-body phase space. For $\eta(\to 3\pi^0)n$ we use a cross section 6.8 ± 1.0 μb, derived from our own data on $\eta(\to 2\gamma)n$[9] and η decay branching ratios.[16] For $3\pi^0 n$ phase space we use <200 μb based upon our own sample of 5γ and 6γ events; this is also a generous upper limit to an extrapolation from lower energies.[17] The $K_1^0K_2^0$ spectrum and a 7-μb $K_1^0K_2^0n$ cross section are synthesized from data on the $K_1^0K_1^0n$ final state[18]; from this we calculate that <3 events from $K_1^0K_2^0n$ contaminate the same subset of 547 events.

In the corrections for geometry and detection inefficiencies and analysis cuts we use the simplest one-pion–exchange (OPE) model to describe the formation of the di-pion system, giving the di-pion–decay angular distribution as $[P_J(\cos\theta_{\pi\pi})]^2$, where P_J is the Jth Legendre polynomial, J is the spin of the di-pion system, and $\theta_{\pi\pi}$ is the angle between the incident π^- and one of the decay π^0's in the di-pion rest frame. Within statistics, events in the "low" mass region $0.27 \leq M_{\pi\pi} \lesssim 1$ GeV are isotropic in the Treiman-Yang angle and in $\cos\theta_{\pi\pi}$ so we take $J=0$ for this mass region.[1,8]

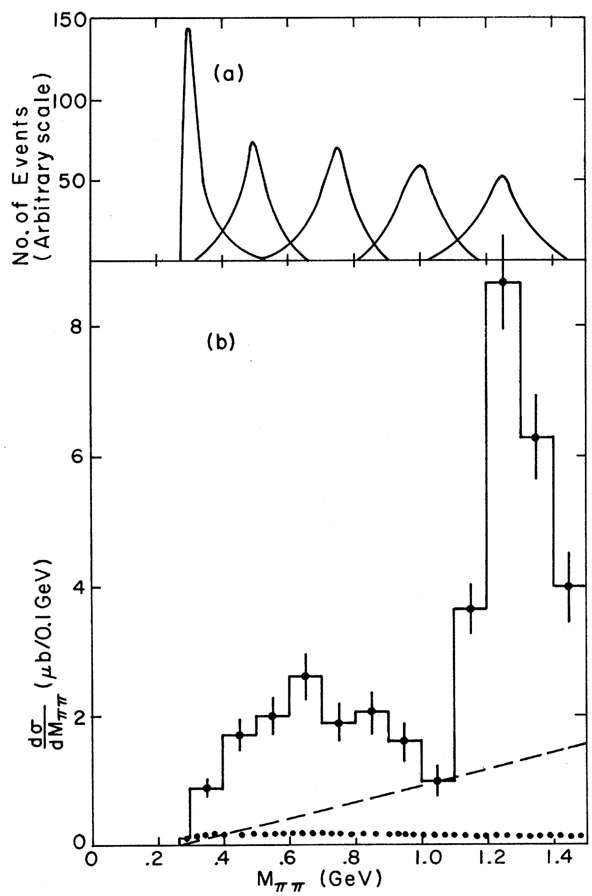

FIG. 1. (a) Monte Carlo mass resolution curves for di-pion masses of 0.3, 0.5, 0.75, 1.0, and 1.25 GeV. (b) The $2\pi^0$ mass spectrum, corrected for geometry and detection inefficiencies and analysis cuts. For comparison, the dashed line shows an upper limit to phase space given by normalizing to the valley at 1.05 GeV between the low-mass enhancement and the f^0 meson, and the dotted line is the calculated upper limit for contamination from other reactions. Errors shown are statistical only.

The $2\pi^0$ mass spectrum, corrected for geometry and detection inefficiencies and cuts in analysis, is shown in Fig. 1(b). Neither the three-body phase space (dashed line) nor background (upper limit shown by dotted line) can account for the enhancement in the low-mass region. The t distribution for these events (Fig. 2) is fitted by

$$[|t|/(t-\mu^2)^2]e^{A't}$$

with $A' = 7\pm 2$ (GeV/c)$^{-2}$, and, within statistics, is the same for smaller di-pion mass intervals.

We now discuss the low-mass enhancement in terms of $\pi\pi$ scattering, following Sonderegger and Bonamy.[6] The amplitude for the $\pi\pi$ scattering reaction

$$\pi^- + \pi^+ \to \pi^0 + \pi^0, \qquad (2)$$

which occurs at the upper vertex of the OPE diagram describing Reaction (1), is proportional to the difference of the isospin-2 and -0 $\pi\pi$ scattering amplitudes. Assuming that only s-wave $\pi\pi$ scattering occurs for $M_{\pi\pi} \lesssim 1.0$ GeV, the cross

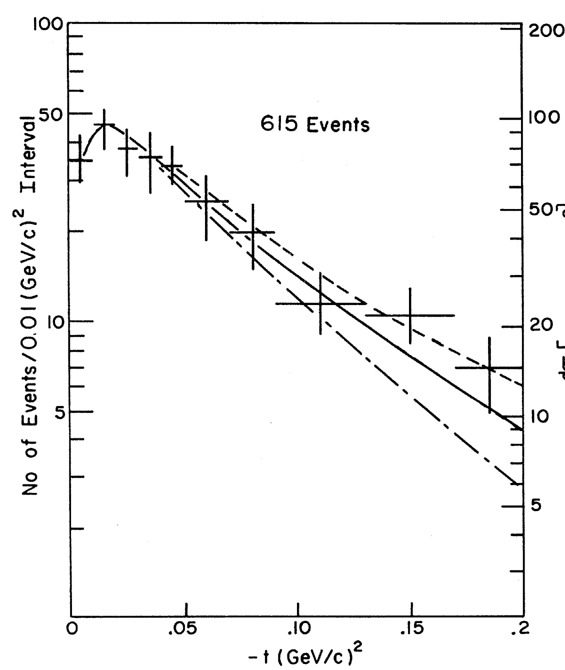

FIG. 2. Differential production cross section for the di-pion mass region $0.27 \leq M_{\pi\pi} \leq 1.0$ GeV as a function of t. The curves are fits to the data (by eye) using $[|t|/(t-\mu^2)^2]e^{A't}$ with $A' = 5$ (GeV/c)$^{-2}$ (dashed line), 7 (GeV/c)$^{-2}$ (solid line), and 9 (GeV/c)$^{-2}$ (broken line). Errors shown are statistical only.

section for Reaction (2), $\sigma_{\pi\pi}$, is given by

$$\sigma_{\pi\pi}=\frac{2}{9}\frac{4\pi}{k^2}\left|\frac{\exp(2i\delta_0{}^2)-\exp(2i\delta_0{}^0)}{2i}\right|^2$$

$$=\frac{2}{9}\frac{4\pi}{k^2}\sin^2(\delta_0{}^2-\delta_0{}^0), \qquad (3)$$

where k, the pion momentum in the di-pion rest frame, is given by

$$k=(M_{\pi\pi}{}^2/4-\mu^2)^{1/2};$$

here μ is the pion mass.

In principle, $\sigma_{\pi\pi}$ can be obtained from the experimentally measured cross section for Reaction (1) by a rigorous Chew-Low extrapolation[7] to the pion pole, but this requires a very large number of events, which is unavailable from this experiment. Thus, a form factor has been put into the Chew-Low extrapolation formula to give

$$\frac{d^2\sigma}{dtdM_{\pi\pi}}=\frac{16M_{\pi\pi}{}^2f^2}{9\mu^2p^2k}\frac{|t|}{(t-\mu^2)^2}\times\sin^2(\delta_0{}^2-\delta_0{}^0)F^2(t), \qquad (4)$$

where p is the incident π^- laboratory momentum, $f^2=0.081$, and σ is the production cross section for Reaction (1). The form factor used by Sonderegger and Bonamy[6] and also used here is given by $F^2(t)=\exp[A'(t-\mu^2)]$, and has been chosen for its quite adequate description of the experimental t distribution (see Fig. 2). The normalization of $F^2(t)$ is such that $F^2(\mu^2)=1$, and A' is determined from the data. This modified Chew-Low formula (4) is fitted to the data in the physical region to give $\sin^2(\delta_0{}^2-\delta_0{}^0)$ as a function of $M_{\pi\pi}$. Using $A'=7$ (GeV/c)$^{-2}$, the results of the fit to our data are shown in Fig. 3 along with the "5"-GeV/c results[22] of Sonderegger and Bonamy,[6] who also use $A'=7$ (GeV/c)$^{-2}$ in a fit to their data. As an examination of Fig. 2 shows, the choice of $A'=7$ (GeV/c)$^{-2}$ from our t distribution is not a clearcut one. Since the value of A' determines the normalization of $\sin^2(\delta_0{}^2-\delta_0{}^0)$, we have indicated on Fig. 3 the effect of using either $A'=5$ (GeV/c)$^{-2}$ or $A'=9$ (GeV/c)$^{-2}$.

As shown in Fig. 3, the values of $\sin^2(\delta_0{}^2-\delta_0{}^0)$ from our experiment at 10 GeV/c are in excellent agreement with the values for the same quantity from the similar experiment of Sonderegger and Bonamy[6] at "5" GeV/c. Note that the results of both experiments fail to reach the unitarity limit using $A'=7$ (GeV/c)$^{-2}$; $A'\cong 11$ (GeV/c)$^{-2}$ would be required. We will discuss the normalization further, below.

Since $\delta_0{}^2$ is small and negative in the di-pion mass region under consideration,[21] the main contribution to $\sin^2(\delta_0{}^2-\delta_0{}^0)$ comes from $\delta_0{}^0$, suggesting the existence of a very broad isospin-0, s-wave $\pi\pi$ resonance centered at $M_{\pi\pi}=600\pm100$ MeV. Such a broad resonance would be consistent with the broad scalar resonance proposed by Lovelace, Heinz, and Donnachie[23] in order to describe backward πp elastic scattering using dispersion relations and also with the predictions of Wagner[24] and Kang[25] using the Veneziano mod-

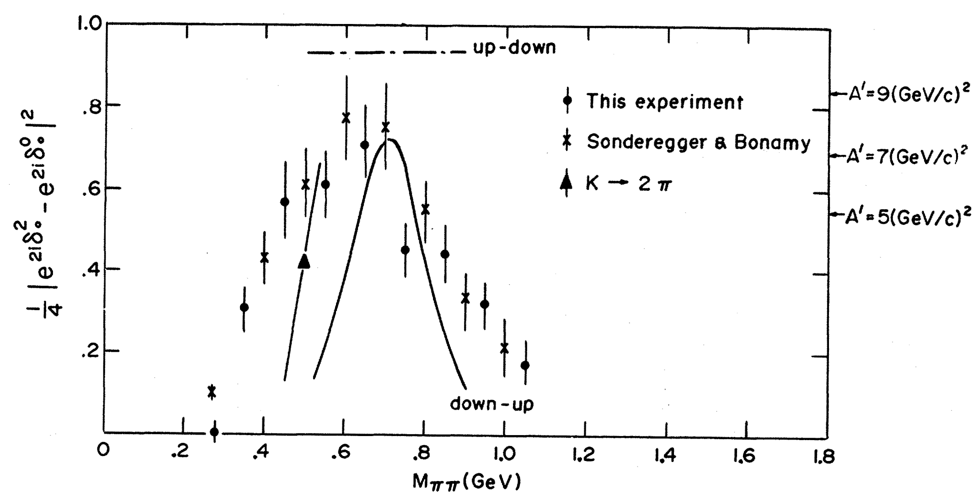

FIG. 3. Values of $\frac{1}{4}|\exp(2i\delta_0{}^2)-\exp(2i\delta_0{}^0)|^2[=\sin^2(\delta_0{}^2-\delta_0{}^0)]$ with $A'=7$ (GeV/c)$^{-2}$ as a function of di-pion mass for these data and those of Ref. 6. For these two sets of data errors shown are statistical only. Since A' determines the normalization, the peaks reached with $A'=5$, 7, and 9 (GeV/c)$^{-2}$ are shown by the arrows on the ordinate. Also shown are the following: a point corresponding to the "best" value for $(\delta_0{}^2-\delta_0{}^0)$ of $\pm(40^{+15}_{-20})^\circ\pm180^\circ$ from $K^0\to2\pi$ decays (Ref. 19), and two Monte Carlo curves giving the "down-up" and "up-down" solutions for $\delta_0{}^0$ [$\delta_0{}^0$ is from Marateck *et al.* (Ref. 20) and $\delta_0{}^2$ is from the effective-range formula fit of Baton and Laurens (Ref. 21)].

el. However, because of our mass resolution, our results do not exclude the existence of two (or more) narrow isospin-0, s-wave $\pi\pi$ resonances in the low-mass region.

Our results and those of Sonderegger and Bonamy[6] do not agree with the $\delta_0{}^0$ phase-shift solutions from studies of the reaction[20, 26, 27]

$$\pi^- + p \to \pi^+ + \pi^- + n. \qquad (5)$$

Plotted with our data on Fig. 3 are the "down-up" and "up-down" solutions for $\delta_0{}^0$ of Marateck et al.[20] Our data lie between the "up" and "down" branches of $\delta_0{}^0$ on both sides of $M_{\pi\pi} \simeq 0.7$ GeV.

The slope of $\sin^2(\delta_0{}^2 - \delta_0{}^0)$ at threshold is 3 ± 1 times greater than given by Weinberg's current-algebra scattering lengths[28] $a_0 = 0.02\mu^{-1}$ and $a_2 = -0.06\mu^{-1}$. Here we have taken into account our limited mass resolution which spills some events into the threshold region.

We now return to the question of absolute normalization of cross sections, noting first that there are several possible theoretical reasons why the points in Fig. 3 need not reach the unitarity limit for a pure s-wave resonance; among them are inadequacy of the OPE parametrization of Eq. (4) and inelasticity.

In the low-mass region our corrections for geometry and detection inefficiencies should be reliable because we observe with fair efficiency all regions of the decay angular distribution; at $M_{\pi\pi} = 600$ MeV the detection efficiency changes <25% over the whole region of $\cos\theta_{\pi\pi}$. Our normalization is confirmed by very good agreement of our $\pi^0 n$ and $\eta(\to 2\gamma)n$ cross sections, derived from the same set of data pictures, with those of the Saclay-Orsay group (see Ref. 9).

However, our absolute cross section for $f^0(\to\pi^0\pi^0)n$ is 20 ± 5 μb, lower by a factor of about 3 than both what is indicated in Ref. 6 and what we deduce from an interpolation of cross sections for $f^0(\to\pi^+\pi^-)n$ at 8 GeV/c[29] and 11 GeV/c.[30] In this higher mass region our geometric and detection inefficiency corrections depend strongly on the assumption that the f^0 decay angular distribution is $[P_2(\cos\theta_{\pi\pi})]^2$, which fits our data very well in the limited region of $\cos\theta_{\pi\pi}$ we observe [$|\cos\theta_{\pi\pi}| \lesssim 0.75$; see Fig. 3(b) of Ref. 1 for our experimentally observed f^0 decay angular distribution]. If it peaks more sharply near $|\cos\theta_{\pi\pi}| = 1.0$, we have underestimated the f^0 cross section. However, to gain such a large factor in the f^0 normalization would require an unusually peaked decay angular distribution.

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†Present address: Department of Physics, Northeastern University, Boston, Mass. 02115.

‡Present address: Lawrence Radiation Laboratory, University of California, Berkeley, Calif. 94720.

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Dimensions of Currents and Current Commutators*

M. A. B. Bég
The Rockefeller University, New York, New York 10021

and

J. Bernstein†
Stevens Institute of Technology, Hoboken, New Jersey 07030

and

David J. Gross‡
Princeton University, Princeton, New Jersey 08540

and

R. Jackiw‡
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

A. Sirlin†
New York University, New York, New York 10003
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We study the scale dimensions of current components and Schwinger terms. We state and prove necessary and sufficient conditions for the spatial and temporal components of hadronic currents to have the same dimension. The incompatibility of well-defined c-number Schwinger terms with universal dimensionality of current components is demonstrated. Experimental implications are discussed.

The discovery of scaling in deep inelastic electron-proton scattering has engendered renewed interest in the notion that strong interactions become scale invariant at small distances.[1] Recent work of Wilson[2] suggests that it may be possible to implement this idea within the framework of quantum field theory provided one adheres to a meaningful definition of the scale dimensionality d of field operators. For a spin-zero field $\varphi(x)$, an operational definition for d is the following: If the renormalized propagator $\Delta_F'(q^2)$ behaves as $(-q^2)^{d-2}$ as $q^2\to-\infty$, the scale dimension of the field is d. (Very similar definitions can be given for fields with other spins.)

It is likely that, in general, d is anomalous, i.e., $\neq 1$.[3] Furthermore, this asymptotic behavior of the propagator will follow from considerations of broken scale invariance[4] if d arises by commuting at equal times $\varphi(x)$ with D, the generator of infinitesimal scale transformations:

$$i[D(x_0),\varphi(x)]=x^\mu\partial_\mu\varphi(x)+d\varphi(x). \quad (1)$$

In this note we study the scale dimensions of current components and Schwinger terms. We state and prove necessary and sufficient conditions for the temporal and spatial components of currents to have the same dimension. We further demonstrate the incompatibility of well-defined