$\simeq\gamma(H_c-H_0)$ whose strength is characterized by $(b\pm\frac{1}{2}K)$. For $\omega<10^9$ the resonance condition reduces effectively to $H_0\simeq H_c$. The ultrasonic attenuation corresponding to this elastic mode shows precisely the dependence on the propagation direction expected on the basis of the finite-deformation theory.[4] The experimentally observed strong dependence of the attenuation on the orientation of the applied magnetic field is a consequence of the strong orientation dependence of the spin-wave frequencies themselves for $\omega/\gamma H_c \ll 1$.[14]

In conclusion, finite-deformation magnetoelastic theory is in excellent agreement with the experimental results presented here and also provides an alternate explanation independent of the existence of domains for the data of Shapira and Zak[4] near the spin-flop transition. The usual small-strain theory, on the other hand, does not even provide a correct qualitative description of either set of data. This provides conclusive evidence that the rotational component of a shear deformation cannot in general be neglected even to first order when considering elastic phenomena in gyroscopic media. In particular, whenever the appropriate anisotropy and magnetoelastic coupling constants are comparable in magnitude, significant first-order rotational effects can be expected.[16]

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Effect of Spin-Lattice Coupling on the Critical Resistivity of a Ferromagnet*

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Evidence is presented which demonstrates that the larger part of the anomaly above T_c in the c-axis electrical resistivity of gadolinium arises not from spin-disorder scattering but rather from the anomalous lattice contraction near T_c. The portion of the resistance anomaly attributable to spin-disorder scattering is monotonic in temperature and of the general form specified by the Fisher-Langer model.

Two general types of behavior have been observed for the temperature dependence of the electrical resistivity ρ of metallic ferromagnets: (1) a monotonic temperature dependence of ρ in the vicinity of T_c with a singularity in $d\rho/dT$ at T_c, or (2) a nonmonotonic temperature dependence of ρ with a maximum in the vicinity of T_c. Examples of behavior (1) are found in nickel,[1,2] iron,[3] gadolinium (a axis),[4] and various intermetallic compounds (e.g., $CdCo_2$)[5]; and examples of

behavior (2) are found in gadolinium (c axis)[4] and various other intermetallic compounds (e.g., $GdNi_2$ [6] and $GdPt_2$ [5]). Within the framework of the de Gennes–Friedel[7] and Fisher-Langer[8] models for spin-disorder scattering, it is possible, in principle, to account qualitatively if not quantitatively for either type of behavior. Behavior (1) corresponds to using for the two-spin correlation function $\Gamma(k,T)$ the form valid in the limit $k \gg \kappa(T)$,[8] where $\kappa(T)$ is the inverse correlation range. Behavior (2) corresponds to using the molecular field (Ornstein-Zernike) form for $\Gamma(k, T)$.[7]

The motivation for the present work is linked to the general question of whether the anomalies in electrical resistivity near the Curie point, in particular, the behavior (2), can be explained as resulting only from spin-disorder scattering. Gadolinium was chosen for the study since it exhibits both types of behavior (depending upon crystal orientation), the electronic structure and other properties have been reasonably well characterized, and high-purity single crystals of Gd[9] were available for the study. Two sets of measurements were performed on each crystal. Electrical resistivity or specific heat of the sample was measured simultaneously with the inductance of a coil wound around the sample. Since the inductance, as well as the specific heat, displays a sharp anomaly at T_c,[10] this method has made possible the precise determination of the position of T_c in relation to the anomaly in the functional form of the resistivity.

Electrical resistivity measurements were made on crystals of dimension $\sim 1 \times 1 \times 7$ mm^3, which were spark cut from a specially prepared crystal[9] with a resistance ratio of $\rho(292°\text{K})/\rho(4.2°\text{K}) = 103$. The results of the measurements along the a and c crystal axes are shown in Fig. 1. The behavior in the a direction has a familiar shape, qualitatively similar to that observed in Ni,[1,2] and is readily explained as resulting from spin-disorder scattering of the conduction electrons. An explanation solely in terms of spin-disorder scattering is not possible for the anomaly in the c direction for the following reasons:

(1) The maximum in the resistivity coincides with the maximum in specific heat ($19.35 \pm 0.10°$K) to within one-tenth of a degree. The Ornstein-Zernike form for $\Gamma(k,T)$ may not be applied for $k \gg \kappa(T)$ and this fact, as shown by Fisher and Langer,[8] leads inevitably to a maximum in the resistivity above T_c and a maximum in $d\rho/dT$ at T_c.

(2) Below T_c the value of $d\rho/dT$ increases as the temperature is lowered. This is inconsistent with both the de Gennes–Friedel and Fisher-Langer versions of the theory of spin-disorder scattering.

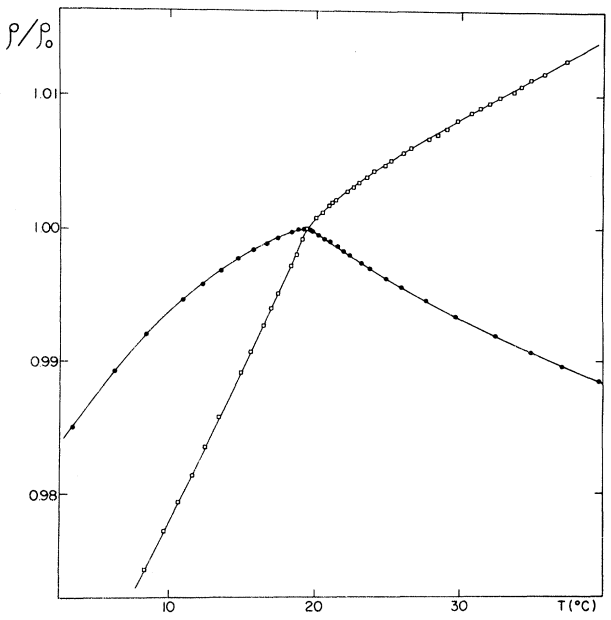

FIG. 1. Circles, resistivity of Gd in c direction; squares, resistivity of Gd in a direction.

As a possible explanation for the data of Fig. 1, we have examined the effects of the anomalous lattice expansion in Gd in the vicinity of T_c. The temperature dependence of the thermal expansion coefficient was measured by Bozorth and Wakiyama[11] and also by Cadieu and Douglass.[12] The anomaly in the c direction is pronounced and similar to that in the specific heat except of opposite sign. The effect in the a direction is more than an order of magnitude smaller. The resistivity in the i direction may be considered a function of the lattice constants L_j and temperature. For experiments at constant pressure, by taking partial derivatives it follows that

$$\left(\frac{\partial \rho_i}{\partial T}\right)_P = \sum_j \left(\frac{\partial \rho_i}{\partial L_j}\right)_T \left(\frac{\partial L_j}{\partial T}\right)_P + \left(\frac{\partial \rho_i}{\partial T}\right)_{L_j}. \quad (1)$$

Since the thermal expansion coefficients α_j are given by $\alpha_j = (\partial L_j/\partial T)_P/L_j$, one obtains the result that the temperature derivative of the resistivity has a term proportional to α. In a slightly more general form, this was first pointed out by Kontorovich.[13] However, this work did not consider the effects above T_c and predicted that $\partial\rho/\partial T$ will have a term proportional to the square of the magnetization. The values of $\partial\rho_i/\partial L$ are only weakly dependent on temperature and as a first

approximation these terms may be considered constant. We also take into account only the effect in the c direction which is the strongest. Using the functional form $10^6\alpha_c(T) = A\ln|\epsilon| + B$, where $\epsilon = (T-T_c)/T_c$, and the values $A_+ = 47.2 \pm 3.5$ and $B_+ = 9.6 \pm 2.0$ for $T > T_c$ and $A_- = 62.2 \pm 4.6$ and $B_- = 29.4 \pm 2.0$ for $T < T_c$, determined by Cadieu and Douglass,[12] we arrive at the following expression[14] for the component of the resistivity arising from the first term on the right-hand side of Eq. (1):

$$\rho_l = K[A\epsilon \ln|\epsilon| + (B-A)\epsilon] + K', \tag{2}$$

where K is to be identified with $LT_c(\partial\rho/\partial L)_T$ in the c direction and K' is a constant. The total resistivity is $\rho = \rho_l + \rho_{ph} + \rho_s$, where $\rho_{ph} \propto T$ is the phonon contribution and ρ_s is the contribution from spin-disorder scattering; we have neglected the residual resistivity which is ignorably small. In the limit $T \gg T_c$ the temperature dependence of ρ_s may be neglected and one has

$$\rho = K(A\epsilon \ln|\epsilon| + B'\epsilon) + K'', \quad T \gg T_c, \tag{3}$$

where $\rho_{ph} = K(B'-B+A)(\epsilon+1)$ and the constant K'' is given by

$$K'' = K' + K(B'-B+A) + \lim_{T \gg T_c} \rho_s.$$

In Fig. 2 we show a fit of Eq. (3) to the c-axis results in the region $T - T_c \gtrsim 20°K$, where the temperature dependence of the spin-disorder scattering is expected to be negligibly small.[15] Using the Cadieu-Douglass values for A and B, this procedure determines the quantities K and B' but not K'. The result for ρ_{ph} in the c direction is $\rho_{ph}^c/\rho_0 = 0.17\epsilon + 1$, where ρ_0 is the value of the measured resistivity at T_c. This may be compared with the slope of the linear part of the resistivity along the a axis above T_c, determined from Fig. 1 to be $\rho_{ph}^a/\rho_0 = 0.19\epsilon + 1$.

The average value of $L_c(\partial\rho/\partial L_c)_T/\rho$ can be approximately determined using the measurements of $\partial\rho/\partial P$ for polycrystalline Gd by Stager and Drickamer[16] and the measurements of $\partial P/\partial L_c$ by Wakabayashi et al.[17] This gives as an order-of-magnitude estimate $L_c(\partial\rho/\partial L_c)_T/\rho = 5.5$. This value can be compared with the value of 9.0 for $L_c(\partial\rho_c/\partial L_c)_T/\rho_c$ deduced from the fit in Fig. 2. Considering the roughness of this procedure any closer agreement would have to be considered fortuitous.

Finally, in Fig. 2 we plot the quantity $\rho - K[A\epsilon \ln|\epsilon| + (B-A)\epsilon]$. The similarity between this curve and the measured resistivity along the a axis is striking. Both curves describe the contribution to the resistivity arising mainly from spin-disorder and phonon scattering and are well described by the Fisher-Langer theory.

In conclusion, we have for the first time presented evidence that the dominant part of the anomaly in electrical resistivity near the Curie point in ferromagnets may in some cases arise

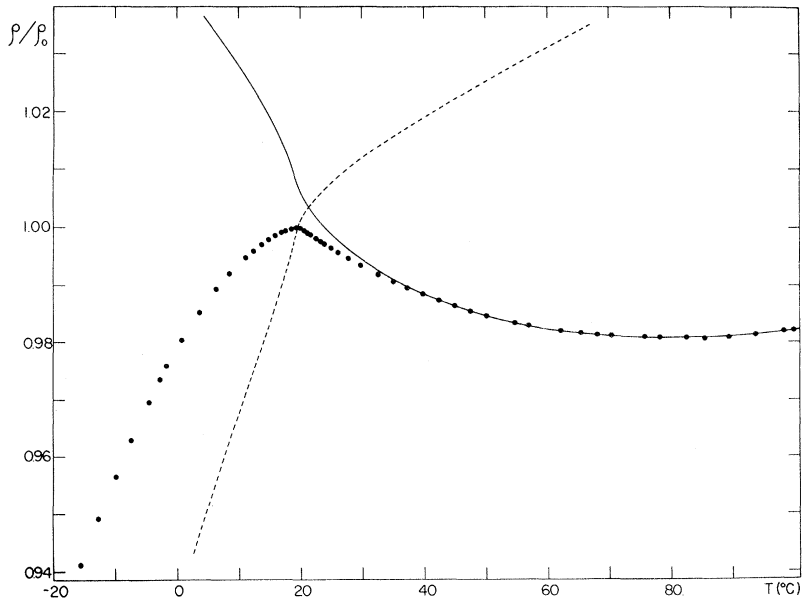

FIG. 2. Circles, normalized resistivity ρ/ρ_0 of Gd in c direction, where $\rho_0 = \rho(T_c)$; full curve, fit to Eq. (3) for $T - T_c \gtrsim 20°K$; dashed curve, $\{\rho - K[A\epsilon \ln|\epsilon| + (B-A)\epsilon]\}/\rho_0$.

from the anomalous lattice expansion near T_c. The question that remains, which can be answered only by further experimentation, is whether critical lattice effects play a dominant role in the explanation of the resistivity maximum near T_c observed in other systems (e.g., $GdNi_2$[6] and $GdPt_2$[5]).

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Magnon Heat Conduction and Magnon-Electron Scattering in Fe-Ni

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The thermal and electrical conductivities of several transition alloys have been measured at low temperatures at high magnetic fields. The Permalloys 70Ni-30Fe and 81Ni-19Fe show a 3% magnon conductivity at 4 K. The temperature dependence of the magnon conductivity for these two alloys implies that the magnons are scattered by the electrons through the s-d exchange.

Magnon thermal conductivity has already been observed in ferromagnetic and antiferromagnetic insulators[1-3] and there is some evidence of such conduction in the ferromagnetic rare earths,[4] but there is no similar evidence in transition metals. We have measured the thermal and electrical conductivities of several transition alloys in the temperature range 1.2-4.5 K and in magnetic fields up to 6 T. The results for the Permalloys 81Ni-19Fe and 70Ni-30Fe are consistent with a model for magnon thermal conduction which assumes s-d exchange scattering of magnons by the conduction electrons.

The apparatus is conventional, using carbon resistance thermometers with an ac phase-sensitive bridge. With it we have been able to make measurements accurate to about 0.2% in most cases.

For the 70Ni-30Fe alloy, the thermal conductivity was measured at zero external field ($B_E = 0$) and at three longitudinal fields above saturation. The curve showing the difference in the thermal conduction from $B_E = 0$ to $B_E = 0.781$ T (Fig. 1) can be predicted[5] from the corresponding variation of the electrical conductivity by assuming that the Wiedemann-Franz law is obeyed, i.e., that the change is an electronic effect. This law should hold exactly for these alloys since