Anomalous Damping of Large-Amplitude Electron Plasma Oscillations*

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With computer experiments we investigate the behavior of large-amplitude, high-phase-velocity plasma oscillations in an electron-ion plasma. The experiments show that such a wave, when above a given threshold amplitude, produces an instability that results in its anomalous damping. The results are in agreement with theory. This is one mechanism by which large-amplitude electrostatic oscillations generated by an instability (for example, by the two-stream instability) can be dissipated.

Recent work[1] has shown the existence of an anomalous resistance for a plasma driven by a large-amplitude externally imposed electric field oscillating near the electron plasma frequency. Here we show with numerical experiments that large-amplitude plasma oscillations in an electron-ion plasma are subject to this anomalous damping. This behavior can be very important in determining the wave energy spectrum in a plasma with large waves (for beam plasma interactions, etc.). The damping has a simple theoretical explanation in terms of an instability excited by the plasma oscillation.

The computer model has been described elsewhere[2] and is similar to several models in common use.[3,4] Basically, we follow the self-consistent motion of a large number of finite-size particles. For these experiments we use 10^4 ions and 10^4 electrons in a system 256 electron Debye lengths across. The ions are 100 times heavier than the electrons and have a temperature 30 times lower. We excite a large-amplitude wave with phase velocity ~$40v_{Te}$ by applying a sinusoidal force at the desired wave number and frequency for a time of $12.6\omega_p^{-1}$. We then shut off the driver and watch the evolution of the wave spectrum.

A typical result is shown in Fig. 1. Here $E_0^2/4\pi nkT = 0.8$, a large wave to illustrate the effect (yet no trapping takes place since the phase velocity is so large). The solid curve is the energy of the excited wave ($k\lambda_D = 0.025$); the dashed curve is the energy in the other plasma oscillations (higher k), and the dot-dashed curve is the square of the ion-density fluctuations

$$\sum_k |n_i(k)|^2/|n_i(0)|^2.$$

We see that the large wave excites an instability; both other plasma oscillations and the ion-density fluctuations begin to grow. When these waves and fluctuations reach a level sufficiently large, the main wave experiences a strong damping. Similar behavior has been observed in actual experiments.[5]

A large-amplitude plasma oscillation can excite several instabilities in an electron-ion plasma.[6-11] The one we here observe has been called the "oscillating two-stream instability."[8] In this

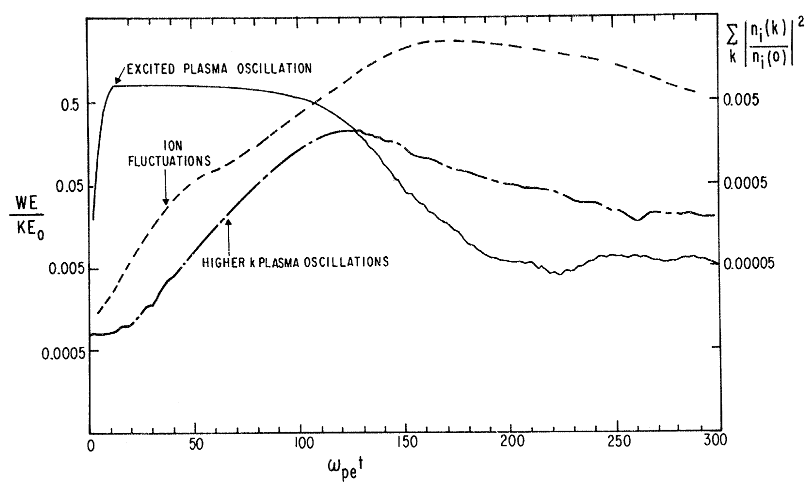

FIG. 1. The time evolution of the energy in the excited plasma oscillation ($k\lambda_D = 0.025$), the energy in higher-wave-number plasma oscillations ($k\lambda_D = 0.05$ to 0.25), and the magnitude of the ion density fluctuations.

case, the large wave drives unstable both a plasma oscillation at higher wave number and a purely growing ion oscillation. Using Nishikawa's results,[6] we can outline the basic features of this instability. First, there exists a threshold amplitude for instability to occur: $E_{th}^2/4\pi nkT = 4\nu/\omega_p$, where ν is the electron-ion collision frequency (or a Landau damping rate). When beam-type instabilities produce the high-frequency oscillations, this threshold can easily be exceeded. The maximum growth rate for fields well above threshold is

$$\gamma \simeq (e^2k^2E_0^2/8m_e m_i \omega_p)^{1/3}$$

for the wave number

$$k\lambda_D \simeq [\tfrac{2}{3}(\omega_0 - \omega_{pe} + \gamma)]^{1/2}.$$

Here ω_0 and E_0 refer to the large wave. Nishikawa's analysis assumes that the wave number of the large field exciting the instability is zero, but his results should give reasonable estimates when the wave number of the large wave is much less than that of the unstable waves. For the example shown in Fig. 1, the predicted maximum growth rate is $\sim 0.03\omega_{pe}$. This is in reasonable agreement with the observed value of $0.035\omega_{pe}$. It should be noted that a large wave can also drive unstable a plasma oscillation with lower wave number, due to a resonant-mode-coupling parametric instability.[10] Growth rates for this instability are similar to those for the oscillating two-stream instability, when the wave is well above threshold. We choose our large wave to be the first mode in the system, since this excludes the other instability[6] and allows a simpler experiment on a specific instability.

Several features of the observed instability are very significant. First, plasma oscillations with higher wave numbers grow. This means that the energy of the large wave moves to lower phase velocities and hence can more readily feed energy into the particles. For example, Fig. 2 shows the electron distribution function at $\omega_{pe}t = 250$. The energetic tail clearly shows that there has been significant energy transfer to the particles. This heating is due to the transfer of wave energy from the large wave to lower phase velocity plasma oscillations, as shown by the fact that there are still no particles with velocities equal to the phase velocity of the large wave. Indeed we choose our large wave to have a very high phase velocity ($\sim 40v_{Te}$) in order to exclude direct damping of it by the particles and hence to clearly illustrate the transfer of energy to other waves. Figure 1 shows that the large wave exhibits only a very weak damping due to electron-ion collisions until the unstable waves grow large. Then sizable energy transfer to the unstable waves sets in. These lower phase velocity waves in turn efficiently pull particles out of the main distribution when their amplitudes become sufficiently large. The net energy in the superthermal tail so produced is essentially the energy originally in the high phase velocity large wave.

The second important feature of the instability is that it drives up the ion-density fluctuations. This is very significant because the ion-density fluctuations enhance the high-frequency resistivity, as shown in a theoretical calculation by Dawson and Oberman[12] and in recent numerical simulations by Kruer, Kaw, Dawson, and Oberman.[1] The enhanced dissipation is due to the excitation of electron plasma oscillations by the interaction of the oscillating electron current with the ion-density fluctuations. Hence, the large wave can experience a sudden onset of "collisional" damping when the ion fluctuations become sufficiently large. This is in agreement with observations shown in Fig. 1. Using the Dawson-Oberman calculation[12] and the observed ion density, we estimate an effective damping rate of $\sim 0.1\omega_{pe}$ for the energy of the large wave. This is in reasonable agreement with the observed value of $\sim 0.07\omega_{pe}$.

Finally, the observed saturation of the instability is readily understood. Once the ion fluctuations reach a large value, they lead to a strong

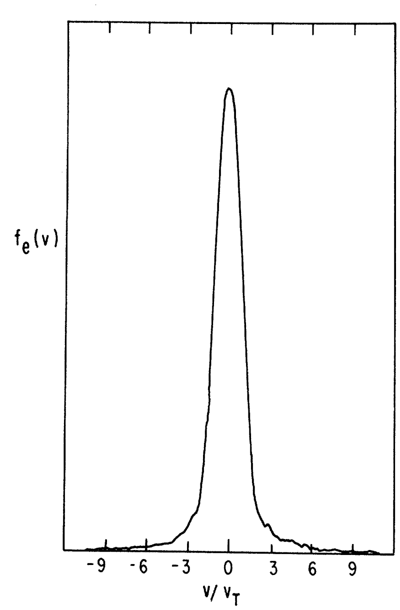

FIG. 2. The electron distribution function at $\omega_{pe}t = 250$.

anomalous damping of the large wave. In other words, they lead to an efficient coupling of energy out of the large wave into other plasma oscillations. Soon the large wave has lost so much energy that it no longer drives the instability. The growing waves saturate and indeed begin to lose energy (since, as we have seen, they are interacting with the particles and there is no longer the large original wave maintaining them at their high level).

In conclusion, we have shown with numerical experiments that large-amplitude plasma oscillations are subject to an electron-ion instability. This instability results in an enhanced resistivity, and hence in an anomalous damping of the wave. The results are consistent with theory and recent numerical calculations of anomalous resistivity. Such anomalous damping at large amplitudes may occur for other types of plasma waves (cyclotron waves, etc.), where large relative velocities exist between two species of particles.

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Theory of the Structural Transition in NbSn and V_3Si

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A dynamic theory, based on the electron-phonon interaction, of the cubic-to-tetragonal transformation in the β-tungsten structure is presented. The theory explains why Nb_3Sn behaves differently from V_3Si and shows that the lattice softening does not influence the superconducting transition.

The cubic-to-tetragonal phase transitions in the intermetallic compounds A_3B of the β-tungsten structure, such as Nb_3Sn and V_3Si, are induced by the band analog of the Jahn-Teller effect. The distortion splits the triply degenerate d bands into a singlet and a doubly degenerate band lying, respectively, below and above the bands in the undistorted structure. The energy gained by increased occupation of the lower band is balanced by the increase in the elastic energy. These transitions have been described in terms of a one-dimensional linear chain model to calculate the d-band structure in the tight-binding approximation.[1] The coupling of the electronic system to the elastic strain was discussed using a free-energy approach. A first-order phase transition was obtained.

In this paper a calculation is presented based on the electron-phonon interaction, considering coupled modes of acoustic phonons and electron density fluctuations. For a linear coupling of the elastic strain with the electron density a second-order transition is obtained. The soft mode is the acoustic shear mode propagating in the (110) direction, polarized in the $(1\bar{1}0)$ direction.

The Hamiltonian for the noninteracting d electrons will be written

$$H_d = \sum_{n=1}^{3} \sum_{k} \epsilon_n^{0}(k) a_n^{\dagger}(k) a_n(k), \quad (1)$$

where the index n refers to the three d bands, degenerate at $k=0$. The operators $a_n(k)$ and