function of N gets more and more flat as y increases, so at very high intensity a large number of photon multiplicities become almost equally probable.

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[1]H. R. Reiss, Phys. Rev. A 1, 803 (1970).

[2]It is also possible to perform the derivation in Ref. 1 in such fashion as to achieve a closed-form result directly. This work, and further details of the present paper, will be published elsewhere.

[3]y^2 is more significant physically than y, since field intensity is proportional to y^2.

[4]We wish to emphasize the dynamical origin of this zero transition probability, which distinguishes it from zeros which arise from conservation and symmetry principles, such as angular momentum and parity.

[5]A complete vanishing of the transition probability would not be observed experimentally because a focused laser beam is not truly a plane-wave field as assumed here, and because of approximations inherent in the intense-field method employed here.

[6]When speaking of large values of N, one must keep in mind that the results have meaning only when the constraint $N\omega + \omega' = E_{2s} - E_{1s}$ is satisfied, i.e., $N\omega < E_{2s} - E_{1s}$.

Time-Dependent Photoelectric Counting Statistics for a Q-Switched Laser Near Threshold*

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(Received 24 August 1970)

The time-dependent variation of the photoelectric counting probability has been measured for a Q-switched single-mode laser, whose steady state is in the neighborhood of the threshold of oscillation. The mean and the variance of the instantaneous light intensity, at various times and for various pump parameters, are derived and found to be in very good agreement with the values predicted by Risken and Vollmer.

The problem of the time evolution of the optical field produced when a laser is switched on has been the subject of a number of theoretical[1-5] and experimental[6-11] investigations. Of the experiments, only one was directly concerned with the time dependence of the laser statistics.[9] But while most of the theoretical treatments are applicable to situations where the final steady state is not too far above the threshold of oscillation, very few dynamical measurements appear to have been carried out in this region. Yet it is in the neighborhood of the threshold that the results are likely to be most sensitive to the predictions of the theory. We wish to report the first measurements of the time development of the photon counting statistics, for a laser which is switched from a point far below to a point in the neighborhood of the threshold of oscillation. Within the statistical errors, the results are in very good agreement with the calculations of Risken and Vollmer,[2] which are based on a rotating-wave van der Pol–oscillator model of the laser.

The light source for these experiments was a 10-cm long, single-mode He:Ne laser, whose cavity length could be controlled by a mirror mounted on a piezoelectric ceramic, with the help of a feedback arrangement, as described previously.[12-14] The optical pumping was held constant throughout. By varying the cavity length we were able to vary the atomic amplification and thereby maintain the laser in any steady state from well below to far above the threshold of oscillation. In order to extinguish the laser we introduced a third, external mirror, aligned so as to form a stable Fabry-Perot cavity with the laser mirror nearest to it. A Pockels cell optical shutter was placed between the laser and this third mirror. By controlling the spacing of the external Fabry-Perot cavity, it was possible to extinguish the laser when the shutter was opened and to switch the laser on when the shutter was closed.[15] The effective switching time (about 50 nsec) turned out to be negligible compared with the characteristic rise time of the laser towards its steady state, which was of order 100 μsec. The laser was allowed to dwell in its steady state for about 20 msec following the turnon, when it was again extinguished, and the switching cycle was repeated about 30 times per sec. The feedback loop controlling the laser intensity was operative only during the 20-msec dwell time in the steady state.

The light beam produced by the laser was allowed to fall on a cooled photomultiplier tube, and the number of photoelectric pulses n registered during a short time interval from t to $t+T$ following the turnon of the laser was counted. This number was used as an address in a 100-channel analyzer, where the data generated in each cycle were accumulated in the usual manner.[9,14] The counting time interval T was 2 μsec, which was short compared with the rise time of the laser. By changing the delay time t, and the steady-state operating point of the laser, as characterized by the pump parameter α, we were able to explore the variation of the photon counting probability $p(n,t,\alpha)$ with t and α.

For a time interval t to $t+T$ which is sufficiently short that the instantaneous light intensity $I(t,\alpha)$ does not change significantly, the counting probability $p(n,t,\alpha)$ is simply related to the probability density $P(I,t,\alpha)$ of I, by the formula[16]

$$p(n,t,\alpha)=\int_0^\infty (1/n!)(\alpha I T)^n \times\exp(-\alpha I T)P(I,t,\alpha)dI, \qquad (1)$$

in which α is a constant characteristic of the detector. It follows immediately from this equation that the moments are related by

$$\langle n(t,\alpha)\rangle=\alpha T\langle I(t,\alpha)\rangle, \qquad (2)$$

and

$$\langle n(t,\alpha)[n(t,\alpha)-1]\rangle=\alpha^2T^2\langle I^2(t,\alpha)\rangle. \qquad (3)$$

Risken and Vollmer[2] have expressed $P(I,t,\alpha)$ in terms of the eigenvalues and eigenfunctions of a certain one-dimensional Schrödinger equation,[17] and have computed explicit forms of $P(I,t,\alpha)$, $\langle I,t,\alpha)\rangle$, and $\langle[\Delta I(t,\alpha)]^2\rangle$ for certain values of the pump parameter α ($\alpha=0,4,8$). Their results are expressed in terms of dimensionless intensities $\tilde{I}(\tilde{t},\alpha)$ and dimensionless times $\tilde{t}$, which may be defined, in terms of the pump parameter α, by[18]

$$\tilde{t}\equiv 0.171t/[T_c]_{\alpha=0},$$

$$\tilde{I}(\tilde{t},\alpha)\equiv\frac{I(\tilde{t},\alpha)}{\langle I(\infty,\alpha)\rangle}\left[\alpha+\frac{2}{\sqrt{\pi}}\frac{\exp(-\alpha^2/4)}{1+\Phi(\alpha/2)}\right], \qquad (4)$$

where T_c is the intensity correlation time,[16] and $\Phi(x)$ is the Gaussian error integral. We have taken advantage of these calculations to make measurements, and comparison with the theory, for the same values of α.

In order to make contact with the theoretical expressions, it was first necessary to identify the threshold of our laser ($\alpha=0$), and to determine the corresponding correlation time T_c, which provides the time scaling according to Eq. (4). Fortunately, the threshold is easily identified by the predicted value of the steady-state normalized second factorial moment,[2,19]

$$\langle n(\infty,0)[n(\infty,0)-1]\rangle/\langle n(\infty,0)\rangle^2=\pi/2, \qquad (5)$$

which is obtained from the measurements. With the laser adjusted to operate at threshold, the mean number $\langle n(\infty,0)\rangle$ was noted, and the correlation time T_c (about 40 μsec) was then determined from an auxiliary experiment. This involved measuring the steady-state normalized second-order factorial moment for various values of the counting time interval T, from which the correlation time T_c can be obtained by a straightforward linear extrapolation procedure, as has recently been described.[14,20] When the laser was to be operated at other values of the pump parameter α, the feedback control circuit was adjusted until the steady-state mean number of counts $\langle n(\infty,\alpha)\rangle$ satisfied the relation[2] implied by Eq. (4),

$$\frac{\langle n(\infty,\alpha)\rangle}{\langle n(\infty,0)\rangle}=\frac{\alpha\sqrt{\pi}}{2}+\frac{\exp(-\alpha^2/4)}{1+\Phi(\alpha/2)}. \qquad (6)$$

Three corrections to the experimental data were necessary in practice, in order to account for dead-time (~10 nsec) effects in the counting circuits, for background light from the laser gas discharge, and for the finite counting time interval. As regards the first two effects, if n_e and n refer to the experimentally observed and to the corrected number of counts, respectively, the corrections to the first two moments can be shown to be of the form[14,21-23]

$$\langle n\rangle=\langle n_e\rangle+\langle n_e(n_e-1)\rangle\delta-\langle n_B\rangle, \qquad (7)$$

$$\langle n(n-1)\rangle=\langle n_e(n_e-1)\rangle[1+\delta(3-2\langle n_e\rangle)] +2\delta\langle n_e(n_e-1)(n_e-2)\rangle +\langle n_e\rangle(1-\langle n_e\rangle)-\langle n_B\rangle. \qquad (8)$$

Here δ is the ratio of dead time to counting time (5×10^{-3}), and $\langle n_B\rangle$ is the mean number of counts due to the background light alone, as determined from a separate measurement. As regards the correction for the finite counting time T, this can be shown[24] to require an increase of the normalized second factorial moment of $4T/3\langle\tilde{T}\rangle$ (at most 0.03 in practice). The corrected moments of $n(t,\alpha)$ were then used to determine the first two moments of $I(t,\alpha)$ from Eqs. (2) and (3).

The experimentally derived values of $\langle\tilde{I}(\tilde{t},\alpha)\rangle$

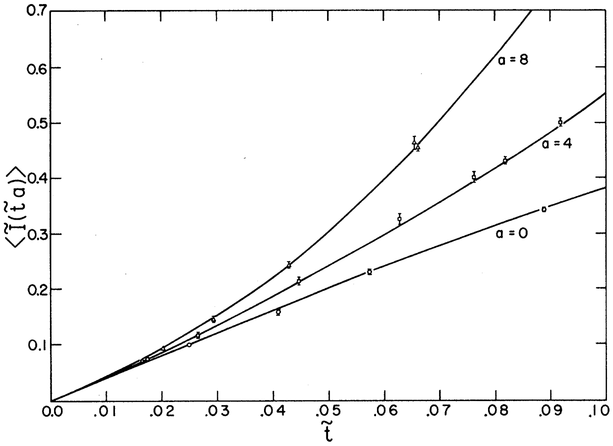

FIG. 1. Variation of the mean light intensity with time, in normalized units, for three different values of the pump parameter α. The statistical uncertainties of the experimental points are too small to show. The full curves are computed by Risken and Vollmer (Ref. 2).

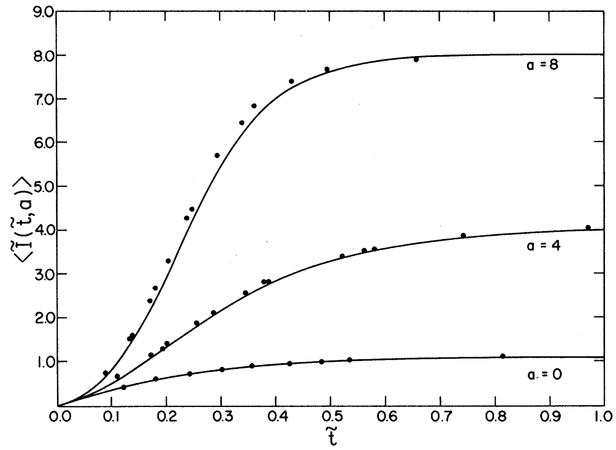

FIG. 2. Expanded display of the low-$\tilde{t}$ portion of Fig. 1, with statistical uncertainties of the experimental points shown. The full curves are those from Ref. 2.

for pump parameters $\alpha = 0, 4, 8$, together with the curves of Risken and Vollmer,[2] are shown in Figs. 1 and 2. The laser was switched between 10 000 and 25 000 times for each experimental point. In addition to the indicated statistical uncertainties of the derived values of $\langle \tilde{I}(\tilde{t}, \alpha)\rangle$, there is a statistical uncertainty of about ±2% in each $\tilde{t}$ value, due to the uncertainty[20] of $[T_c]_{\alpha=0}$. For short times $\tilde{t}$ the predicted form of $\tilde{I}(\tilde{t}, \alpha)$ is given by

$$\langle \tilde{I}(\tilde{t}, \alpha)\rangle = (2/\alpha)[\exp(2\alpha\tilde{t}) - 1], \tag{9}$$

which converges to $4\tilde{t}$ for sufficiently short $\tilde{t}$ for all α values, and this is seen to be well borne out in practice (see Fig. 2). However, it was found that the extinction of the laser produced by the external mirror was not quite complete at $\tilde{t} = 0$, although the initial slope of the curve was as predicted from Eq. (9). In other words, the light intensity behaved as if it had started to rise from $\langle \tilde{I}(\tilde{t}_0, \alpha)\rangle = 0$ at some slightly earlier time $\tilde{t}_0 < 0$. We therefore determined the normalized average intensity before the laser was switched on, and divided this by 4 to arrive

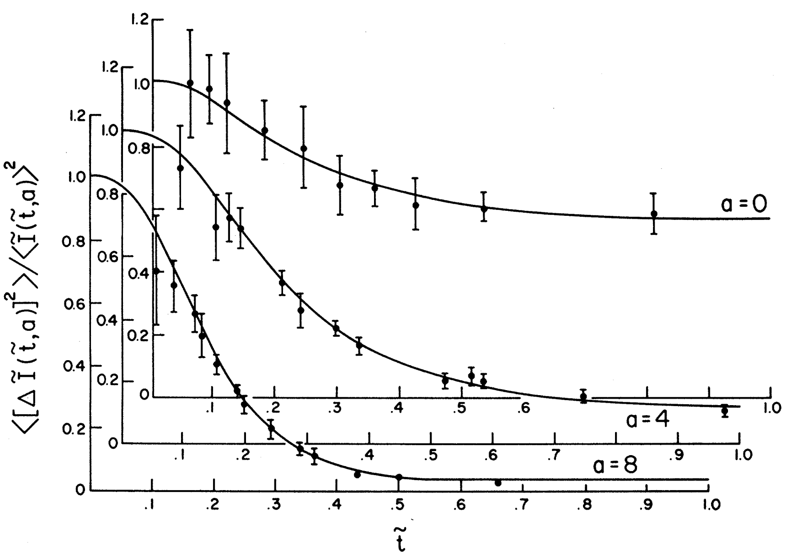

FIG. 3. Variation of the normalized variance of the light intensity with time, in normalized units, for three different values of the pump parameter α. The error bars represent statistical uncertainties of the experimental points. The full curves were computed by Risken and Vollmer (Ref. 2).

at a normalized time $|\tilde{t}_0|$ (~0.02), which was added to the experimental delay, in order to make $\langle \tilde{I}(\tilde{t}, \alpha)\rangle = 0$ at $\tilde{t} = 0$.

The experimentally derived time dependence of the normalized variance $\langle[\Delta \tilde{I}(\tilde{t}, \alpha)]^2\rangle / \langle \tilde{I}(\tilde{t}, \alpha)\rangle^2$, together with the computed curves of Risken and Vollmer,[2] is shown in Fig. 3. Inevitably, the statistical uncertainties associated with the second moments are much greater than those in Figs. 1 and 2.

It will be seen that, within the expected uncertainties, the measured and computed values are in very good agreement. We would like to emphasize that no scaling parameters have been arbitrarily adjusted to produce this agreement. We conclude that the dynamical predictions of the nonlinear oscillator theory, no less than the steady state ones, are very well confirmed.

We would like to express our appreciation to W. Davis for considerable help with the computations.

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*Work supported by the U. S. Air Force Office of Scientific Research.

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[20]Because of some drifts in the laser parameters between runs on different days, corresponding to different pump parameters α, the correlation time $[T_c]_{\alpha=0}$ was redetermined for each run.

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