POSSIBLE EXISTENCE OF SPIN-0 $W^{\pm}$ AND SOME OF ITS EXPERIMENTAL CONSEQUENCES*

T. D. Lee
Columbia University, New York, New York 10027
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The theoretical possibility that there may exist charged spin-0 weak intermediate bosons, in addition to the usual spin-1 intermediate bosons, is discussed. Some of its experimental consequences are analyzed.

In several different recent attempts[1,2] to reduce divergences contained in the weak-intermediate-boson theory, one is led to theoretical models in which, in addition to the usual charged spin-1 bosons $W_1^{\pm}$, there should also exist charged spin-0 bosons $W_0^{\pm}$. We note that in all such models the masses of spin-0 and spin-1 bosons are independent parameters. If, among these masses, the lowest one is of spin 1, then, as is well known, its decay into leptons offers a particularly useful tool in the experimental search for intermediate bosons. However, if the intermediate boson with the lowest mass is of spin 0, then, at least for a range of boson masses, an effective way to search for these bosons might be through their nonleptonic decay modes, rather than through their leptonic decay modes.

Let m_J be the mass of a charged spin-J intermediate boson $W_J^{\pm}$. In all these models, for $J=0$, if one neglects charged lepton masses as compared with m_0, then

$$W_0^{\pm} \not\rightarrow l^{\pm}+\nu_l \text{ (or } \bar{\nu}_l), \tag{1}$$

and therefore the hadronic decay modes of $W_0^{\pm}$ become the dominant ones. Furthermore, if $m_1 > m_0$, the γ transition

$$W_1^{\pm} \rightarrow W_0^{\pm}+\gamma \rightarrow \text{hadrons}+\gamma \tag{2}$$

can occur. As we shall see, in general, the usual minimal electromagnetic interaction would require the transition rate of (2) to be simply proportional to the fine structure constant α. Thus, if the mass difference m_1-m_0 is not too small, (2) could become the dominant decay mode of the spin-1 intermediate boson.

In such cases, searches for weak intermediate bosons based on lepton detections become relatively ineffective, especially if these intermediate boson masses are in the range of several GeV. In order to observe $W_0^{\pm}$ (or $W_1^{\pm}$), one may consider, for example,

$$\nu_\mu+p \rightarrow \mu^-+p+W_J^{\pm} \rightarrow \mu^-+p+\text{hadrons } (+\gamma). \tag{3}$$

The existence of $W_J^{\pm}$ may be detected by searching for possible threshold effects in the initial neutrino energy, or by analyzing the invariant-mass distributions of various hadron channels, or (in the case of $J=1$) by measuring the γ-energy distribution. The same methods can, of course, also be applied to other production processes of $W_J^{\pm}$.

If one assumes that $W_0^{\pm}$ is coupled to the divergence of the usual SU(3) octet hadron current,[3] then the isospin selection rule

$$|\Delta\vec{I}| = \tfrac{1}{2} \text{ and } 1 \tag{4}$$

should be valid for the hadronic decay modes of $W_0^{\pm}$; in addition, because of the conserved vector current hypothesis,[4] the $|\Delta\vec{I}|=1$ decays of $W_0^{\pm}$ can lead only to final hadron states of parity -1. Thus, in the absence of the electromagnetic interaction, one expects

$$W_0^{\pm} \not\rightarrow \pi^{\pm}+\pi^0 \tag{5}$$

and

$$W_0^{\pm} \not\rightarrow \pi^{\pm}+\eta^0. \tag{6}$$

Other hadron decay modes such as $K\pi$, 3π, $K\bar{K}\pi$, etc., are, of course, allowed.

In the following, we shall briefly review various theoretical models that require the existence of $W_0^{\pm}$. These models are (i) a renormalizable weak-intermediate-boson theory, (ii) a finite weak-intermediate-boson theory, and (iii) the theory of Gell-Mann, Goldberger, Kroll, and Low.

(i) We first discuss the case of a simple renormalizable weak-interaction boson theory. The Lagrangian density for the semiweak interaction is given by

$$\mathcal{L}(x) = g[J_\lambda{}^h(x)+J_\lambda{}^l(x)]W_\lambda(x)+\text{adjoint}, \tag{7}$$

where $J_\lambda{}^h(x)$ is the hadron current; $J_\lambda{}^l(x)$ is the lepton current, related to the charged-lepton field ψ_l and the neutrino field ψ_{ν_l} by

$$J_\lambda{}^l(x) = \sum_{l=e,\mu} i\psi_l{}^\dagger(x)\gamma_4\gamma_\lambda(1+\gamma_5)\psi_{\nu_l}(x); \tag{8}$$

and $W_\lambda(x)$ is a local field operator which is assumed to have no strong interaction. All presently observed weak transitions are second order in

g^2, transmitted by the covariant W propagator

$$D_{\mu\nu}(k^2)=\delta_{\mu\nu}\int\frac{\sigma_1 dM}{k^2+M^2}+k_\mu k_\nu\int\frac{(\sigma_1+\sigma_0)M^{-2}dM}{k^2+M^2}, \tag{9}$$

where all integrals are assumed to be convergent, and the integration extends from a positive definite lower limit to infinity. In order that the theory may correspond to a renormalizable one, we require

$$\int(\sigma_1+\sigma_0)M^{-2}dM=0. \tag{10}$$

The simplest solution is

$$\sigma_1=\pm\delta(M-m_1)$$

and

$$\sigma_0=\mp(m_0/m_1)^2\delta(M-m_0); \tag{11}$$

therefore,

$$D_{\mu\nu}(k)=\pm\left[\frac{\delta_{\mu\nu}}{k^2+m_1^{\,2}}+\frac{k_\mu k_\nu}{m_1^{\,2}}\left(\frac{1}{k^2+m_1^{\,2}}-\frac{1}{k^2+m_0^{\,2}}\right)\right], \tag{12}$$

which corresponds to a charged spin-1 boson $W_1^{\pm}$ of mass m_1 and a charged spin-0 boson $W_0^{\pm}$ of mass m_0. These two bosons are of opposite metric. The upper signs in (11) and (12) imply that $W_1^{\pm}$ is of positive metric and $W_0^{\pm}$ of negative metric, and the lower signs imply the opposite. The Fermi constant G is related to g^2 by

$$\pm g^2/m_1^{\,2}=G/\sqrt{2}, \tag{13}$$

where

$$G\cong10^{-5}/m_N^{\,2}. \tag{14}$$

In order that the electromagnetic interaction of these charged intermediate bosons also be renormalizable, we assume that the same W propagator (12) should appear in all Feynman diagrams involving electromagnetic interactions as well as weak interactions. Let the three-point vertex for $W_\nu\to W_\mu+\gamma$ be $[V_\lambda(k',k)]_{\mu\nu}$, where λ, μ, and ν denote the appropriate space-time indices, and k and k' are, respectively, the four-momenta of the initial W_ν and the final W_μ. Because of current conservation, $[V_\lambda(k',k)]_{\mu\nu}$ must satisfy the Ward identity

$$D_{\mu\nu}^{\;-1}(k')-D_{\mu\nu}^{\;-1}(k)=e^{-1}(k'-k)_\lambda(V_\lambda)_{\mu\nu}, \tag{15}$$

where, on account of (12),

$$D_{\mu\nu}^{\;-1}(k)=\pm[\delta_{\mu\nu}(k^2+m_1^{\,2})-m_0^{\,-2}(m_0^{\,2}-m_1^{\,2})k_\mu k_\nu]. \tag{16}$$

For simplicity, we assume charge-conjugation invariance and space-time reversal invariance for the electromagnetic interaction; therefore,

$$[V_\lambda(k',k)]_{\mu\nu}=-[V_\lambda(-k,-k')]_{\mu\nu}=[V_\lambda(k,k')]_{\mu\nu}. \tag{17}$$

Furthermore, in accordance with the principle of minimal electromagnetic interaction, $[V_\lambda(k,k')]_{\mu\nu}$ is assumed to be a linear function of k and k'. Thus, one finds

$$[V_\lambda(k',k)]_{\mu\nu}=\pm e\{\delta_{\mu\nu}(k+k')_\lambda+[(m_1/m_0)^2+\kappa][\delta_{\lambda\mu}k_\nu+\delta_{\lambda\nu}k_\mu']-(1+\kappa)[\delta_{\lambda\mu}k_\nu'+\delta_{\lambda\nu}k_\mu]\}, \tag{18}$$

where κ is a constant related to the magnetic moment M of the charged spin-1 boson $W_1^{\pm}$ by

$$M=(e/2m_1)(1+\kappa)(\text{spin}). \tag{19}$$

Formally, the theory is identical to the ξ formalism[5] of the charged vector-boson theory, where ξ is related to the two boson masses by

$$m_0^{\,2}/m_1^{\,2}=\xi^{-1}. \tag{20}$$

In the present case, ξ is always >0. By using the above vertex $[V_\lambda(k',k)]_{\mu\nu}$, one can easily evaluate the γ-transition rate of Reaction (2). For $m_1 > m_0$, one finds

$$\text{width}(W_1^\pm \to W_0^\pm\gamma) = \tfrac{1}{3}\alpha q(q/m_1)^2(1-\kappa)^2, \tag{21}$$

where $\alpha \cong (137)^{-1}$ and $q=(2m_1)^{-1}(m_1^2-m_0^2)$ is the γ energy in the rest system of $W_1^\pm$. The ratio of the partial widths $W_1^\pm \to l^\pm\nu$ and $W_1^\pm \to W_0^\pm\gamma$ is given by ($l = e$ or μ)

$$\frac{\text{width}(W_1^\pm \to l^\pm\nu)}{\text{width}(W_1^\pm \to W_0^\pm\gamma)} = 2\sqrt{2}(\pi\alpha)^{-1}(1-\kappa)^{-2}\left[1-\left(\frac{m_0}{m_1}\right)^2\right]^{-3} Gm_1^2, \tag{22}$$

which is only ~7% if one sets arbitrarily $m_1 = 5$ GeV, $m_0 = \frac{1}{2}m_1$, and $\kappa = 0$.

The neutrino production rate for $W_0^\pm$ can be calculated in the same way as that[6] for $W_1^\pm$. Here, we give only the differential cross section for the neutrino reaction

$$\nu_\mu + Z \to \mu^- + Z^* + W_0^+ \quad \llcorner\!\to \text{hadrons} \tag{23}$$

in the limit of zero muon mass, where Z denotes a target proton or nucleus, and Z^* is its final state. We find

$$d\sigma(\text{Reaction (23)}) = (4\pi^3)^{-1}g^2\alpha^2|T|^2d^3\mu d^3q\delta(E_\mu + E_W + E_{Z^*} - E_Z - E_\nu), \tag{24}$$

where[7]

$$T = q^{-2}[m_1^2 - 2(\nu\cdot\mu)]^{-1}E_W^{-1/2}m_1^{-1}\{(1-\kappa)[(q\cdot W)(j\cdot v)-(q\cdot j)(W\cdot v)] + q^2(v\cdot j)\}, \tag{25}$$

the four-momenta of ν_μ, μ^-, and W_0^+ are denoted, respectively, by ν, μ, and W, the four-momentum transfer to the target is

$$q = \nu - \mu - W,$$

and the fourth components of ν, μ, W, and q are, respectively, $iE_\nu = i|\vec{\nu}|$, $iE_\mu = i|\vec{\mu}|$, $iE_W = i[\vec{W}^2 + m_0^2]^{1/2}$, and $i(E_{Z^*} - E_Z)$. The Fourier transform of the four-current (in unit e) of the target nucleus is denoted by v; e.g., for a coherent process in which the target nucleus recoils as a whole, we may take, in the system $q_4 = 0$ (which is, to a very good approximation, the same as the laboratory system),

$$\vec{v} = 0 \text{ and } v_4 = iZF_Z(q^2),$$

where Z is the nuclear charge and $F_Z(0) = 1$. Clearly, in general $v\cdot q = 0$. The four-vector j is defined to be

$$j_\lambda = u_\mu^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)u_\nu,$$

where u_ν and u_μ are the appropriate Dirac spinors, normalized so that $u_\mu^\dagger u_\mu = u_\nu^\dagger u_\nu = 1$. In the limit of zero muon mass and if the z and x axes are chosen to be parallel to $\vec{\nu}-\vec{\mu}$ and $\vec{\mu}\times\vec{\nu}$, respectively, then j_λ is given by

$$j_x = 2\sin\tfrac{1}{2}\theta, \quad j_y = -2i\sin(\varphi + \tfrac{1}{2}\theta),$$

$$j_z = -2i\cos(\varphi + \tfrac{1}{2}\theta), \quad j_4 = i(E_\nu - E_\mu)^{-1}|\vec{\nu}-\vec{\mu}|j_z, \tag{26}$$

where

$$\theta = \angle(\vec{\nu}, \vec{\mu}) \text{ and } \varphi = \angle(\vec{\nu}, \vec{\nu}-\vec{\mu}).$$

Equations (24)-(26) give the explicit differential cross section for arbitrary kinematic configurations of $\vec{\nu}$, $\vec{\mu}$, and $\vec{q}$. In particular, in the forward direction, $\vec{\nu} \parallel \vec{\mu} \parallel \vec{q}$, and for the coherent production process of a heavy target nucleus of charge Z, (25) becomes simply

$$T = 2iE_W^{-1/2}m_1^{-3}\kappa ZF_Z(q^2), \tag{27}$$

which would vanish if κ happens to be zero.

(ii) In order to have a finite weak-intermediate-boson theory, we must require $D_{\mu\nu}(k^2) \sim O(k^{-4})$ as $k^2 \to \infty$. Therefore, in addition to (10), the spectral functions σ_1 and σ_0 in the W propagator should satisfy

$$\int \sigma_1 dM^2 = \int \sigma_0 dM^2 = 0. \tag{28}$$

In place of (13), the coupling constant g^2 is related to G by

$$g^2 \int M^{-2}\sigma_1 dM^2 = G/\sqrt{2}. \tag{29}$$

The simplest solution of (28) is

$$\sigma_1 = \pm[\delta(M-m_1) - \delta(M-m_1')]$$

and

$$\sigma_0 = \mp\eta[\delta(M-m_0) - \delta(M-m_0')], \tag{30}$$

which corresponds to two spin-1 bosons of opposite metric and two spin-0 bosons of opposite metric. Furthermore, because of (10), one finds

$$\eta = [m_1^{-2} - (m_1')^{-2}]/[m_0^{-2} - (m_0')^{-2}]. \tag{31}$$

The coupling constant g is now given by

$$\pm g^2[m_1^{-2} - (m_1')^{-2}] = G/\sqrt{2}. \tag{32}$$

Except for some straightforward changes due to the presence of more parameters, identical arguments given in the preceding section are applicable to the present case.

(iii) In the weak-interaction theory of Gell-Mann et al., in contrast to the previous two cases, the metric is positive definite. The infinities contained in high-order weak-interaction processes can be reduced only in the so-called "off-diagonal" channels by introducing a set of changed spin-0 bosons. The resulting theory is, however, not a renormalizable one. Two explicit models were given in their paper, and in both models, there are quadratic expressions in the (free) Lagrangian which contain derivative couplings between these charged spin-0 and spin-1 boson fields. The minimal electromagnetic interaction can be obtained, as usual, by replacing $\partial/\partial x_\mu$ by $\partial/\partial x_\mu - ieA_\mu$ in these explicit Lagrangians. It is easy to see that this would lead, in general, to a direct γ-transition matrix element between these charged spin-0 and spin-1 bosons. By using this minimal electromagnetic interaction, one can derive results similar to Eqs. (21) and (24). So far as detection of $W_J^\pm$ is concerned, the experimental consequences of this case are essentially identical to those given for the two previous cases.

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[7]The scalar product between any two four-vectors A and B is $A\cdot B = \vec{A}\cdot\vec{B} + A_4B_4 = \vec{A}\cdot\vec{B} - A_0B_0$.