There was no measurable change in the absorption, indicating that the frequency stability of the spin-flip Raman laser is at least as good as the estimated linewidth of Raman-laser emission, i.e., 0.03 cm^{-1} or ~1:3 $\times 10^4$. The ultimate resolution possible with the spin-flip Raman-laser spectrometer is expected to be much better than what we have shown so far. This is so because the emission linewidth of a quantum oscillator such as the spin-flip Raman laser is expected to be significantly narrower than the present upper limit of 0.03 cm^{-1} arrived at in the present paper.[4]

An additional advantage of the present spin-flip Raman-laser spectrometer lies in the fact that the output occurs in the form of ~30-nsec-wide pulses in normal operation (or ~3-nsec-wide pulses in mode-locked operation).[2] This should allow time-resolved spectroscopy in the 10-14-μm range which heretofore has not been possible with high resolution. The spin-flip Raman laser has the drawback of a limited tuning range. However, this disadvantage could be easily overcome by using different pump lasers, higher magnetic fields, and other narrower band-gap semiconductors (in place of InSb), such as $Pb_{1-x}Sn_xTe$ or $Hg_{1-x}Cd_xTe$ where the g value of the conduction electrons is larger than that in InSb.

In conclusion, the linewidth and fine-tuning measurements both indicate the superiority of the spin-flip Raman laser as the source in infrared spectroscopy as compared with conventional grating spectrometers. Further improvements in the measurement of the linewidth will require heterodyne spectroscopy to pin down the exact spectral width of the spin-flip Raman-laser output. We have experimentally shown the superiority of the spin-flip Raman laser as a source in absorption spectroscopy as compared with the conventional techniques. It is clear that the immediate applications of the spin-flip Raman laser in infrared spectroscopy and as a local oscillator in heterodyne spectroscopy are eminently reasonable. Further improvements in the characteristics of the spin-flip Raman laser will accrue from its cw operation.[9]

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FORMATION AND INTERACTION OF ION-ACOUSTIC SOLITONS*

H. Ikezi,† R. J. Taylor,‡ and D. R. Baker
Department of Physics, University of California, Los Angeles, California 90024
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The formation and propagation of ion-acoustic solitons are observed experimentally. The character of a solitary pulse is observed to follow the predictions of the Korteweg–de Vries equation with respect to the shape and velocity of the soliton. The interaction between two solitons is modified significantly by dissipation. However, the nonlinear nature of the interactions is confirmed for solitons moving in the same direction. Solitons moving in the opposite direction and colliding have very little effect on each other.

The formation and propagation of solitons is one of the most interesting results of the nonlinear analysis of dispersive waves in many media. Kennel and Sagdeev[1] showed that the formation of solitons is closely related to the wave structure of a laminar collisionless shock front. Sagdeev[2] showed, using the fluid equations, that in a plasma of hot isothermal electrons and cold ions, a single pulse (soliton), traveling slightly faster than the ion sound speed, can propagate without changing shape. Using the same equations Washimi and Taniuti[3,4] showed that small but finite amplitude ion waves can be described by the Korteweg–de Vries (K-dV) equation, which contains the lower order nonlinear and dispersive terms but of course neglects Landau damping. In recent years the character of the solution of the K-dV equation has been intensively studied. Using numerical techniques Zabusky and Kruskal[5] studied a periodic sinusoidal initial perturbation and found that it developed into many solitons, whose velocity increased with individual soliton amplitude. The solitons always regained their identity after they had interacted with each other.[6] It was also shown[7-11] that a single-humped initial perturbation will develop into a solitary pulse followed by smaller amplitude oscillations resembling a wave train.

The experiment we wish to report here was performed in the University of California at Los Angeles double plasma[12] device. Although some Landau damping by the ions is present in our plasma, under appropriate conditions we have been able to perform observations on the formation and interaction of ion-acoustic solitons.

In order to compare our results on the formation of solitons with the asymptotic solutions of the K-dV equations, we recall that the electron density profile in the solitary wave pulse is given by

$$\tilde{n} = \delta n \operatorname{sech}^2[(x-ut)/D], \tag{1}$$

where

$$u = v_s(1 + \tfrac{1}{3}\delta n/n_0), \tag{2}$$

and

$$(D/\lambda_D)^2 = 6n_0/\delta n. \tag{3}$$

Here, n_0 is the unperturbed plasma density, λ_D is the Debye length, and v_s is the ion-acoustic velocity.[13] Equations (1)-(3) correspond to the small-amplitude limit of Sagdeev's solitary pulse.[2] From these equations, it follows that as the soliton amplitude increases, the width D decreases, and the velocity u increases. It is also known that only compressional solitons appear in the ion-acoustic branch.[2]

In our experiments, the plasma parameters are as follows: density, $n_0 \sim 10^9$ cm^{-3}; electron temperature, $T_e = 1.5$-3 eV; ion temperature, $T_i \simeq 0.2$ eV; and argon gas pressure, $(2$-$5)\times 10^{-4}$ Torr. For these values of the parameters, ion-ion and ion-neutral collisions can be ignored. However, the electrons have many collisions with the wall sheath during the wave propagation and have been measured to have a Boltzmann distribution $[n_e \simeq n_0 \exp(e\varphi/KT_e)]$. Plasma diameter (30 cm) is much greater than the characteristic length of the wave (a few mm). The temperature ratio T_e/T_i is high enough so that Landau damping is small, and the fluid equations can be used to describe the main features of the wave propagation such as width, amplitude, and velocity.

The wave-excitation method, employed here, is similar to that used to excite collisionless shocks in this device.[14] Two plasmas, which are produced by independent discharges, are separated by a negatively biased mesh grid. Therefore the electrons are prevented from short circuiting the plasmas. Application of pulsed potential difference between the plasmas exites a compressional wave in one of the plasmas and a rarefaction wave in the other. Only the compressional wave forms solitons. In order to increase the spatial resolution of the detector, the wave-receiver probe is biased slightly above the plasma potential so that the probe sheath is small. The probe detects the electron saturation current which is proportional to the electron density.

Although collisionless shocks and solitons are closely related, there is a definite difference between the propagation and interaction of single solitons and those which are part of a shock structure. The shock structure is strongly affected by the fact that the steady-state density on either side of the shock front is different. In order to eliminate these effects from our investigation of the propagation and interaction of single solitons, in most cases we have excited them with pulses with a short (several μsec) time duration, as opposed to the ramp or step function which would be used to excite shock waves.

Figure 1 shows the propagation of the compressional wave excited by the applied potential pulse shown on the top trace. As the wave propagates, its front steepens, and an oscillatory structure starts to develop. Finally the initial single-humped wave is divided into a train of many peaks. The precursor is a group of streaming ions.[14] The calculations by Karpman[9] predict that seven or eight solitons should be formed from the perturbation shown by the second trace in Fig. 1. However, only the first peak in Fig. 1 satisfies the requirements on amplitude, width, and velocity given by Eqs. (2) and (3). Plots of width and velocity versus amplitude are shown in Fig. 2. The oscillations following the first peak cannot be identified as solitary waves and may

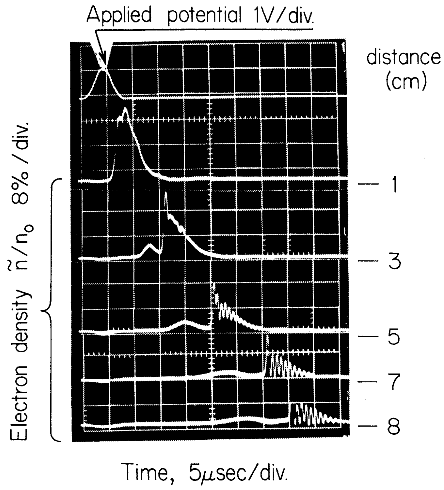

FIG. 1. Typical plot of perturbed electron density versus time with distance as a parameter. Top trace shows potential applied between the two plasmas. Plasma parameters are $T_e=2$ eV, $T_i=0.2$ eV, $n_0=5.0\times10^9$. The wavelength of the oscillations is approximately $10\lambda_D$.

correspond to the wave train discussed in Refs. 7, 8, and 9. Apparently finite ion-temperature effects such as ion reflection by the wave potential and Landau damping severely limit the number of solitons produced below that predicted by the lossless K-dV equation.

It should be noted that the formation of the soliton is sensitive to the value of T_e/T_i and also to contamination by light ions. However, when the value of T_e/T_i is high enough ($\gtrsim 10$) and there is no contamination of light gases (low base pressure) the compressional soliton is observed to be formed from almost any shape of initial wave form, e.g., square waves, sinusoidal waves, and compressional waves with ramp shapes.[15] In our experiment the Landau damping due to electrons[16] is estimated to be small compared with that due to ions.

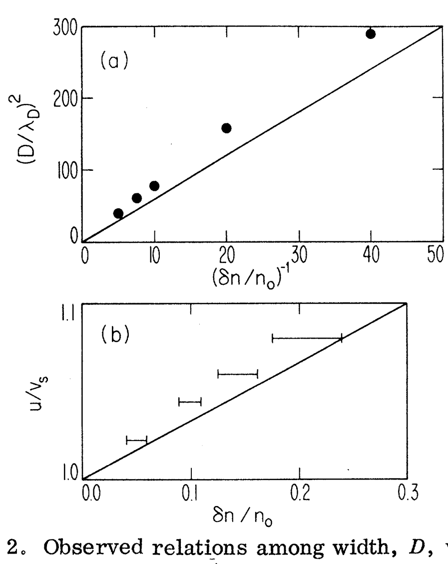

FIG. 2. Observed relations among width, D, velocity, u, and amplitude, $\delta\tilde{n}/n_0$, of soliton. Solid lines indicate Eqs. (2) and (3). Dots and bars are experimental data points.

The interaction of two solitons was investigated in the following two cases: (a) two different-amplitude solitons propagating in the same direction, with one overtaking the other, and (b) two solitons propagating in opposite directions to each other.

Case (a) is realized by applying two consecutive voltage pulses between the two plasmas with amplitudes such that the first pulse generates a small-amplitude soliton and the second pulse produces a larger amplitude one. Since the larger amplitude soliton propagates faster, it will overtake the smaller one. Figure 3(a) shows the resulting interaction. The wave profile is shown as a function of distance for different times after the excitation of the first pulse. The interaction is depicted in the wave frame such that the small pulse is initially stationary. The time difference between each adjacent two curves is 10 μsec. Because of the velocity of the waves the interaction shown takes place over 15 cm, resulting in considerable damping. The difference in velocity between the two pulses is much less than the wave

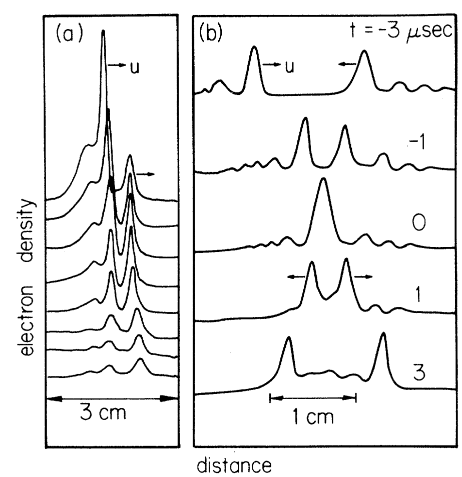

FIG. 3. Interactions of two solitons. (a) Two solitons propagate in the same direction in the laboratory frame. The figure is depicted in the wave frame such that the smaller soliton is initially stationary. Time differences between each adjacent two curves are 10 μsec. (b) Two solitons propagating in opposite directions to each other, depicted in the laboratory frame. $\lambda_D \approx 2\times10^{-2}$ cm.

velocity so that not much motion is evident in the wave train, but some features of the nonlinear interaction described by the loss-free K-dV equation are evident. As the larger pulse overtakes the small pulse, the amplitude of the front pulse increases as the amplitude of the large pulse decreases. Finally the amplitude of the front pulse is larger than the pulse in back. There is some separation of the pulses after the interaction but this may be due to the natural increase in width due to the decrease in amplitude due to damping. At no time do the peaks coalesce into one as might be expected from a linear superposition. If a linear interaction had occurred the higher initial velocity of the second pulse would have allowed it to overtake the first pulse at about the fifth trace in Fig. 3(a).

It is not possible to explain all of the features of this interaction by a lossless theory. The details of the interaction are very sensitive to finite ion-temperature effects. Damping due to dissipation is the major cause of the decrease in amplitude of the larger pulse [(Fig. 3(a)]. Damping also limits the distance over which the interaction can be observed. In addition, ions reflected off the large potential of the second pulse can interact with the first pulse causing it to grow or damp. These dissipative effects must be included in any theoretical description of soliton interaction in our experiment.

For case (b) a third independent discharge is put at the opposite end of the plasma in which the waves are detected. Then a soliton is excited at each end of the main plasma at the same time. The solitons propagate toward each other and interact near the center of the main plasma. The plots in Fig. 3(b) are obtained in the same manner as in Fig. 3(a), by sampling the probe signal at fixed times while sweeping the probe position. The experimental results show no observable nonlinear behavior upon interaction. The two peaks add up linearly when they overlap and penetrate through each other. No deformation of the trajectories of the solitons is observed under the condition $\delta n/n_0 \leqslant 0.2$. In order to test for higher order effects in the head-on collisions of two solitons, which is demonstrated by numerical computation[17,18] of the fluid equations, it may be necessary to excite higher amplitude solitons.

To summarize, we have observed that in a high-temperature-ratio plasma an ion-acoustic pulse will break up into a soliton and a trailing wave train. The amplitude and the width of the soliton is related to its velocity in close agreement with theoretical predictions. A nonlinear interaction (two peaks always remain separate) is observed when one soliton overtakes another. An apparent linear interaction (two peaks temporarily merging into one larger peak) is observed when the two solitons pass through each other from opposite directions.

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†On leave from Institute of Plasma Physics, Nagoya University, Nagoya, Japan.

‡Present address: TRW Systems, Redondo Beach, Calif. 90278.

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