tion to the peripheral OPE amplitude is illustrated in Figs. 1 and 2. These data appear to be extremely sensitive to the value of g_π^2.

If, as an approximation, we consider the phase shifts for $l \leq 5$ to be phenomenologically determined, and treat g_π^2 as a free parameter which fixes only the phase shifts $l \geq 6$, we may observe the variation in $(d\sigma/dt)_{u=0}$ and β as g_π^2 is changed. Between 250 and 400 MeV we find that this variation is directly proportional to the change in g_π^2. Thus the back-angle data in this energy range serve as a measure of the pion coupling constant in n-p scattering.

After completion of this Letter, the authors learned that Phillips[8] had proposed the same mechanism (interference between the long-range OPE tail and a short-range amplitude) to explain a narrow peak observed in n-p charge exchange at ~2 GeV.[9] Byers[10] also calculated the high-energy n-p backward cross section using the coherent-droplet model with an OPE term dominating for large impact parameter.

The authors wish to thank Dr. J. E. Simmons for drawing their attention to the interesting features of the PPA data, Dr. R. Arndt for stimulating discussions and for the use of his *NN* analysis code, and T. Devlin and R. Mischke for an informative discussion.

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PION, KAON, AND ANTIPROTON PRODUCTION BETWEEN 10 AND 70 BeV*

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We present a new scaling law and empirical formulas for the double differential cross section of secondary production between 10 and 70 BeV. The 0° momentum spectrum is found to scale according to the square root of the incident momentum, thus reflecting the importance of the c.m. system in the pionization process. It is pointed out that the 70-BeV production data of the Serpukhov-CERN collaboration experiment are very likely too low by a factor of ~2.5.

The problem of secondary particle production is of great importance in two respects. First, the phenomenon of multiparticle production is so complex that no theory has yet been really successful in interpreting all the experimental data. Second, the knowledge of the secondary particle flux from high-energy accelerators is vital to the planning of experiments utilizing such secondary beams. Recently, considerable interest has been focused on the particle production data from the Serpukhov 70-BeV proton synchrotron. As will be discussed later, the prediction of the hypothesis of limiting fragmentation[1] disagrees with the data of the Serpukhov-CERN (SC) collaboration experiment[2] by a factor of 6. The thermodynamical model prediction disagrees with the SC data by a factor of 5.[3] The parton model prediction disagrees with the SC data by a factor of 3.5.[4] Liland and Pilkuhn[5] have described a scaling law for fast-pion production based on the SC data at 70 BeV and the CERN data at 19.2 BeV/c. In this paper, we present a new scaling law together with an empirical formula and show that the SC data are most likely too low by a factor of ~2.5.

In previous papers[6] we have constructed empirical formulas which well represent the pion, kaon, and antiproton production between 10 and 35 BeV/c, based on certain characteristics of the experimental data. The angular and momentum distribution of the secondaries in the laboratory system can be represented by

$$\frac{d^2\sigma}{dp\,d\Omega}=AP^B\left(1-\frac{P}{P_i}\right)\times\exp\left[-\frac{CP^D}{P_i^E}-F\theta(P-GP_i\cos^H\theta)\right], \qquad (1)$$

where P_i and P are the momenta of the incident protons and of the secondaries, respectively, θ is the production angle, and A through H are parameters to be determined by least-squares analysis. This formula was designed to reproduce the existing data between 10 and 35 BeV/c as quantitatively as possible with a number of parameters. The term $GP_i\cos^H\theta$ takes care of some detailed structure of the angular distribution at low secondary energies and can be omitted for our purpose here. It was found that $B\approx\frac{1}{2}$ and $D\approx E\approx 5/3$. Introducing new abbreviations $Y=d^2\sigma/dp\,d\Omega$ and $X=P/P_i$, we can rewrite the formula as

$$Y=AP_i^{1/2}F(X)\exp(-CP_t) \qquad (2)$$

with

$$F(X)=X^{1/2}(1-X)\exp(-BX^{5/3}), \qquad (3)$$

where A, B, and C are new parameters, $P_t=P\theta$ is the transverse momentum, and $F(X)$ is the "shape factor" which depends only on X. Note that $F(X)$ is an invariant of P_i and commands the main feature of the momentum distribution. For the same X and P_t Eq. (2) thus provides scaling law 1:

$$Y(X,P_t;P_i')=(P_i'/P_i)^{1/2}Y(X,P_t;P_i), \qquad (4)$$

which means that the particle yield for given X and P_t is scaled by the square root of incident momentum. In particular, the shape of the momentum spectrum at 0° ($P_t=0$) for a certain type of secondary particle is given by $F(X)$ while the yield is proportional to $P_i^{1/2}$. The exponential distribution of the transverse momentum is a well established fact at least to the first approximation. For the same X and P_t, it also provides scaling law 2:

$$\theta'=(P_i/P_i')\theta, \qquad (5)$$

which means that for given $X=P'/P_i'=P/P_i$, and $P'\theta'=P\theta$, the production angle is scaled by the inverse of the incident momentum. Scaling law 2 is identical with that of Liland and Pilkuhn and is illustrated in detail there.[5] Scaling law 1, nevertheless, differs from that of Liland and Pilkuhn by $P_i^{1/2}$, and differs also from that of the hypothesis of limiting fragmentation which predicts that the 0° momentum distribution scales linearly with the incident momentum.[7] Let us examine scaling law 1 by the experimental data. In Fig. 1(a) are plotted the 0° π^- production from p-Be collision between 11.8 and 23.1 BeV/c.[8-10] The 13.4-BeV/c data are obtained by extrapolating the data of Ref. 9 and normalized by a factor of 2. The necessity of normalizing these data was pointed out in our previous analysis[6] and subsequently so verified by a new experimant.[10] The 71-BeV/c p-Al data were converted into corresponding p-Be data by a factor of 2.08 in accord with absorption cross sections in the respective reactions, namely 472 and 227 mb.[11] It is seen that the 71-BeV/c data join smoothly with the

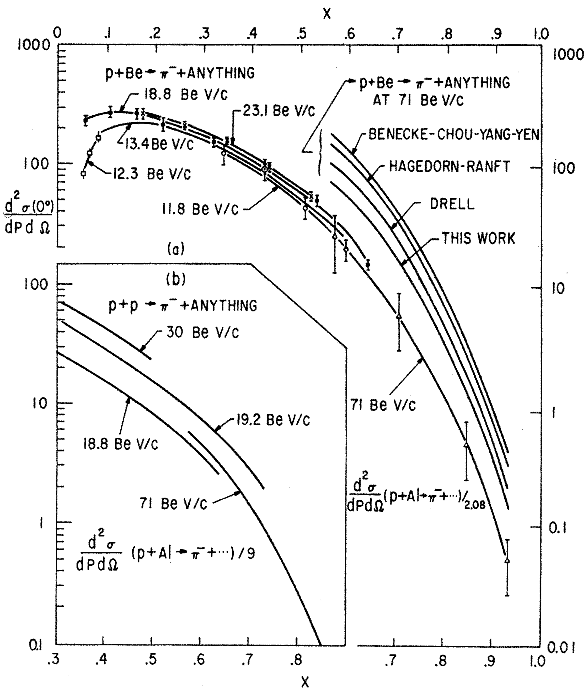

FIG. 1. (a) π^- production at 0° from p-Be collision between 11.8 and 23.1 BeV/c. 0° π^- production from p-Al collision at 71 BeV/c is converted into p-Be collision by a factor 2.08. Four curves predicted by various authors are also plotted. (Data from Refs. 2, 8, 9, and 10.) (b) 0° π^- production from p-p and p-Al collisions. The 19.2- and 30-BeV/c curves indicate an energy-dependent scaling law. It is very unlikely that the 71-BeV/c curve should lie below the 19.2- and 30-BeV/c curves (see text).

11.8-BeV/c data by accident. Four curves expected, respectively, by our scaling law, by the hypothesis of limiting fragmentation, by the thermodynamical model, and by the parton model are also plotted. They differ from the experimental data by factors 2.5, 6, 5, and 3.5, respectively. To provide further evidence of the energy-dependent scaling law we plot the π^- data from p-p collisions at 18.8,[8] 19.2,[12] and 30 BeV/c[13] [Fig. 1(b)]. The data at 19.2 and 30 GeV/c are obtained by extrapolating the data of Refs. 12 and 13. We note that there is definitely an energy dependence in the yield of pions. However, a meaningful scaling law cannot be obtained by comparing these data. We emphasize that the 18.8- and 19.2-BeV/c data disagree with each other by a factor of ~2. We have already mentioned that the data at 13.4[9] and 12.3 BeV/c[10] also disagree by a factor of ~2. It is important to point out that all these experiments were carried out by external targets and are supposedly more reliable as far as the absolute normalization is concerned. In light of these inconsistent examples, it would hardly be a surprise if the 70-BeV data turned out to be off by a factor of ~2.5 as predicted by our scaling law, especially because this experiment was carried out by an internal target with inherent uncertainty in the targeting factor, in addition to the already less favorable experimental conditions, such as the fringing field effect of the synchrotron leading to uncertainties in solid angles, constrained geometry, etc.

The scaling factor $P_i^{1/2}$ can be interpreted as follows. The c.m. differential cross section of secondary pions (or kaons, etc.) varies slowly as the incident energy, as can be judged from the slow increase of the mean pion (or kaon, etc.) energy in the c.m. system.[14] If we assume that the c.m. differential cross sections are identical for different P_i, the factor $P_i^{1/2}$ follows directly from the Lorentz transformation of the forward longitudinal momentum from the c.m. to the laboratory system. This reflects the importance of the c.m. system in the multiparticle production process. Also as a consequence, the scaling laws cannot be expected to hold for low-energy secondaries ($X \lesssim 0.2$) as can be seen from Fig. 1(a).

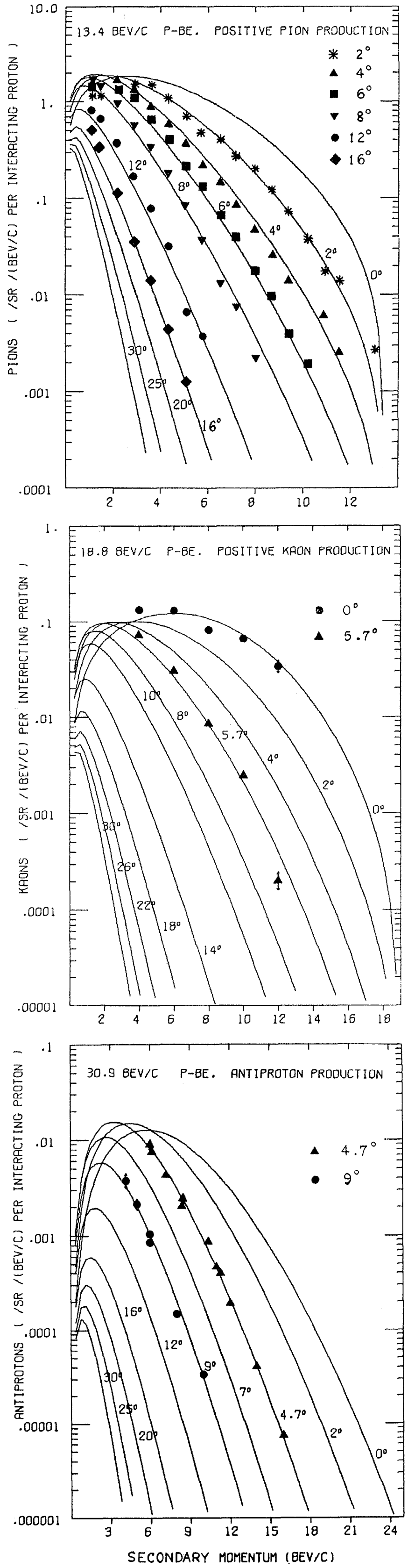

FIG. 2. Examples of secondary pion, kaon, and antiproton momentum spectra. Curves are calculated by formula (1) with appropriate parameters given in Ref. 6. (Data from Refs. 8, 9, and 16.)

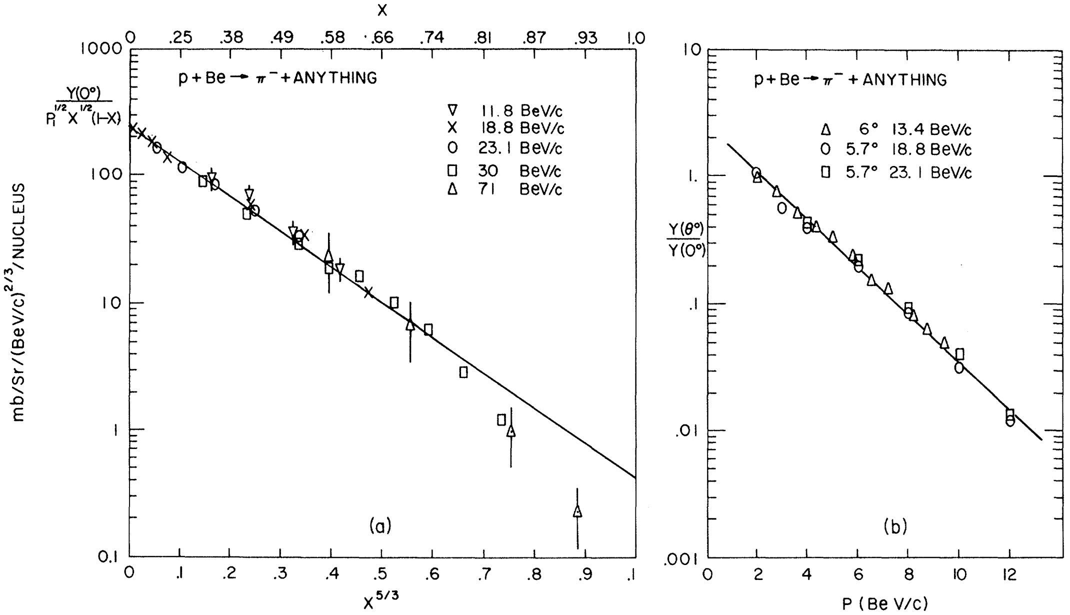

FIG. 3. (a) $Y(0°)P_t^{-1/2}X^{-1/2}(1-X)^{-1}$ plotted against $X^{5/3}$. The deviation of data from the line at $X \gtrsim 0.8$ is discussed in the text. 30-BeV/c (p-p) and 71-BeV/c (p-Al) data are properly normalized. (Data from Refs. 2, 8, and 13.) (b) Plot of $Y(\sim 5.7°)/Y(0°)$ against the secondary momentum P. Both (a) and (b) show how formula (2) can represent the experimental data. (Data from Refs. 8 and 9.)

Formula (1) proved to be able to represent well the pion, kaon, and antiproton production between 10 and 35 BeV.[6] Examples are shown in Fig. 2. There is no doubt that formula (2) can well represent pion, kaon, and antiproton production between 10 and 70 BeV. The parameter A is the normalization factor, B dictates the falloff slope of the 5/3-power exponential tail of the 0° momentum distribution, and C is related to the average of the transverse momentum distribution. Figure 3 shows how the existing data can fit to formula (2). Figure 3(a) is a plot of $Y(0°) \times P_t^{-1/2}X^{-1/2}(1-X)^{-1}$ against $X^{5/3}$. The dropoff of the experimental points from the straight line at $X \gtrsim 0.8$ reflects (1) the kinematical effect near the production threshold where only single particles can be produced, and (2) the excess of neutral pions at very low charged pion multiplicity.[15] In applying formula (2) such deviation has to be corrected in the region $X \gtrsim 0.8$. Figure 3(b) is a plot of $Y(\sim 5.7°)/Y(0°)$ versus the secondary momentum. Below $P \lesssim 2$ BeV/c, detailed fitting can be improved by modifying the angular dependence term to $\exp[-F\theta(p-G\cos^H\theta)]$. Rough estimates of A, B, and C can be obtained by means of Fig. 3, but detailed least-squares fitting to all data has yet to be carried out.

Our scaling laws are essentially based on about 300 $\pi^\pm$ data between 10 and 33 BeV.[6,8,9,16] It is reasonable to expect that the same scaling law should extend to a higher energy range covering a factor of 3 or even higher. In order to test various theoretical models as well as predict secondary particle fluxes from a future generation of high energy accelerators such as the Batavia 500 BeV machine, it is of utmost urgency to examine experimentally the various scaling laws up to 70 BeV with internally consistent data at several incident energies, say 70, 50, 30, 20, and 10 BeV.

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REMARKS ABOUT THE HYPOTHESIS OF LIMITING FRAGMENTATION*

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Remarks are made about the hypothesis of limiting fragmentation. In particular, the concept of favored and disfavored fragment distribution is introduced. Also a sum rule is proved leading to a useful quantity called energy-fragmentation fraction.

This paper contains a number of remarks about the recently proposed hypothesis[1] of limiting fragmentation.

Further experimental evidence.–Smith, Sprafka, and Anderson[2] have recently published a systematic study of the single-particle (π^- and π^+) distribution in pp collisions in a bubble chamber at five incoming energies from 13 to 28 BeV. This study remarkably confirms the hypothesis of limiting fragmentation. In fact, it seems that the single-particle distribution already approaches a limit in that region of incoming energies (see Fig. 1). [The main indication of this fact is that the coefficient $a_{\parallel}$ in Ref. 2 (for four to eight prongs) is experimentally proportional to $p_{\mathrm{inc}}^{-1/2}$.] We should emphasize that the Berkeley work[2] is the only published systematic comparative study of single-particle distributions for several incoming energies.

There have appeared[3] high-energy π^- spectrum measurements in p-Al collisions at 19 and 70 BeV/c. In the projectile system these represent slow "backward" π^- fragmented from the projectile proton (i.e., region A in Fig. 1). A comparison of the data at these two energies indicates that these backward π^--production differential cross sections fall with the incoming energy. We remark, however, that these differential cross sections are one or two orders of magnitude smaller than the main π^--production differential cross sections (which are "forward," i.e., in region B of Fig. 1, and form the bulk of the data of Ref. 2). That the approach to a limiting distribution is slow where the cross section is small is a general characteristic of all high-energy processes. [Cf. the approach to a limit of $d\sigma/dt$ for elastic processes, and Ref. 1, §3.]

It was emphasized many years ago[4] by Pal and Peters that the ratio μ^+/μ^- at sea level remains approximately 1.25 up to 100 BeV. They formulated from this fact a phenomenological model for high-energy collisions. Their model is in some essential respects consistent with the hypothesis of limiting fragmentation. (The main difference seems to be that in their model, the limiting distributions ρ_1, ρ_2, etc. exist, but their integrals are convergent rather