PRECISE DETERMINATION OF THE K_L-K_S MASS DIFFERENCE BY THE GAP METHOD (UNIVERSITY OF CHICAGO–UNIVERSITY OF ILLINOIS CHICAGO CIRCLE COLLABORATION)*

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The K_L–K_S mass difference, $\Delta m \equiv (M_L - M_S)c^2/\hbar$, has been redetermined in a high-statistics experiment performed at the zero-gradient synchrotron of Argonne National Laboratory, using the "gap method" originally developed at Princeton. Two independent measurements, using B_4C and Cu, respectively, as regenerators, were performed and gave consistent results. Their mean is $\Delta m = (0.542 \pm 0.006)\times 10^{10}$ sec^{-1}. With this value, the result of our recent determination of arg η_{+-} by "vacuum regeneration" is recalculated to be $(44.7 \pm 5.0)°$, where the error allows for the uncertainty in Δm given above.

Interference between common decay channels of K_L and K_S mesons leads to a time-dependent term proportional to $\cos(\Delta m t - \Phi)$, where $\Delta m \equiv (M_L - M_S)c^2/\hbar$ and Φ is a phase. The accuracy with which the relevant parameters, in particular Φ, can be determined from experimental studies of such interferences is often seriously limited by the current uncertainty (±2.4 %)[1] in the magnitude of Δm. This is particularly the case in the recent "vacuum regeneration" experiments[2,3] designed to determine $\Phi = \varphi_{+-}$, the phase of the complex CP-nonconservation parameter η_{+-}; an uncertainty of 1% in Δm entails here an uncertainty of about 3.0° in φ_{+-}. For this reason an improvement in our knowledge of Δm is at present of considerable interest.

We present here a new measurement of Δm based on the "gap method," originally devised by Christenson, Cronin, Fitch, and Turlay.[4] In this method a K_L beam traverses successively two regenerators R_1 and R_2 (of thicknesses L_1 and L_2, $L_1 > L_2$) separated by a gap G, and one records the coherent $K \to \pi^+\pi^-$ rate behind the second regenerator as a function of G. Not only does this approach have a higher statistical power than those experiments[5-7] in which one studies the time distribution of $K \to \pi^+\pi^-$ events behind a single regenerator, but it also appears to be less subject to systematic uncertainties. Specifically, it is less sensitive (a) to the acceptance ϵ of the detection apparatus as a function of kaon momentum p and decay point Z, and (b) to the magnitude and phase of the regeneration amplitude $\rho_0(p)$ of the material used for the regenerators. In fact, whereas the single-regenerator experiments are based on the $K_S \to \pi^+\pi^-$ and $K_L \to \pi^+\pi^-$ interference, and hence depend on η_{+-}/ρ_0, the gap method exploits the interference between $K_S \to \pi^+\pi^-$ amplitudes from the two regenerators and hence would work even if $K_L \to \pi^+\pi^-$ were strictly absent ($\eta_{+-} = 0$). In that case, the gap method would be rigorously independent of ρ_0; in actual practice, only a mild dependence on η_{+-}/ρ_0 obtains, and the currently available knowledge of this parameter is no limitation for the present purposes. These remarks can be substantiated by considering the formal expression for the decay distribution (per incident K_L of momentum p) behind R_2:

$$dN_{+-}/dz \propto \exp[-N\sigma_t(L_1+L_2)]|\rho_0|^2|[\alpha_1 e^{-\beta(g+l_2)} + \alpha_2]e^{-\beta z} + \eta_{+-}/\rho_0|^2, \tag{1}$$

where z, l_1, l_2, and g are the corresponding (upper case) laboratory lengths in units of $\Lambda_s(p)$ = mean decay path of K_S; z is measured from the exit face of R_2, N = scattering centers per unit volume, and

$$\rho_0(p) = 2\pi N\Lambda_s \hbar i f_{21}/p, \quad \alpha_j = [1 - \exp(-\beta l_j)]/\beta, \quad \beta = -i\Delta m\tau_s + \tfrac{1}{2}\ (j = 1, 2).$$

The experimentally measured quantity, i.e., the number of $\pi^+\pi^-$ decays behind R_2 at a given gap G, for an incident K_L spectrum dN/dp can be written as

$$N_{+-}(G,\Delta m) \propto \int dp (d\tilde{N}/dp) \int dZ\, \epsilon(p,Z) |a(p,Z;G,\Delta m)|^2, \tag{2}$$

where $d\tilde{N}/dp = (dN/dp)|\rho_0|^2 \exp[-N\sigma_t(L_1+L_2)]$ is the "effective spectrum," $\epsilon(p,Z)$ the acceptance of the apparatus, and $|a(p,Z;G,\Delta m)|^2$ the last factor in Eq. (1). Note that while ϵ and $|a|^2$ are known functions of their arguments, $d\tilde{N}/dp$ must be determined experimentally, a point to which we shall return later.

To make the gap method statistically efficient and least prone to systematic errors, the following criteria are relevant: (1) The transmission-regenerated K_S events must be separated as cleanly as possible from the others (diffraction and incoherent regeneration, backgrounds, etc.); this can be achieved (a) by higher angular resolution and (b) by appropriate choice of regenerator material. (2) With the regenerator material selected, L_1 and L_2 are chosen so as to maximize the sensitivity of $N_{+-}(G,\Delta m)$ to Δm. (3) The incident flux (monitor) must be stable while G is being varied. (4) Gap-dependent effects in triggering, event selection, etc., must be absent or at least small and known.

Higher angular resolution (~2 mrad) is readily achieved with our wire-chamber spectrometer.[2] To obtain a clean separation between transmission and diffraction events it is advantageous to regenerate on small (light) nuclei, and to peak $|\rho_0|^2$ one needs to maximize their number density. On these grounds we chose B_4C (2.45 g/cm^3) as our prime regenerator material, adopting $L_1 = 12$ in. and $L_2 = 2$ in. for optimum sensitivity. However, in order to verify the presumed independence to material of the result, we also collected data using Cu regenerators ($L_1 = 4.3$ in., $L_2 = 0.8$ in.).

The setup used here is essentially the one described in Ref. 2. In the present experiment the external beam from the zero-gradient synchrotron (ZGS) (~2×10^{10} protons/pulse, pulse length ~600 msec) was focused on a diamond-shaped Hevimet target, and the sweeping magnet was followed by a uranium collimator that filled the "nose-cone" of Ref. 2. The spectrometer was modified in two respects: (a) The regions of the wire chambers traversed by the neutral beam were not neutralized; (b) a chamber (No. 0), rotated by 45° with respect to the others, was added upstream of chamber 1. These changes were primarily introduced to make ϵ flat and to minimize the percentage of "ambiguous" events in keeping with criterion (4) above.

While the thin regenerator R_2, covered by a $\frac{1}{16}$-in.-thick anticoincidence counter $\bar{1}$, was kept in a fixed position, R_1 was movable. It was in fact made to sweep continually through the range of interest ($0 \leq G \leq 57$ in.) in discrete steps of 3 in., these displacements occuring between ZGS pulses under computer control. For each pulse, the rates in three different monitors (two scatter monitors, one neutron monitor) were recorded together with the regenerator position. This "rapid cycling" essentially eliminates the

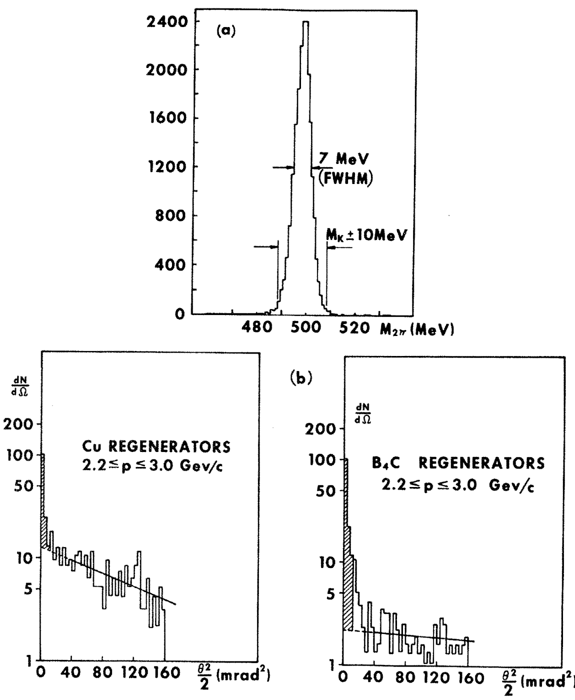

FIG. 1. (a) $M_{2\pi}$ distribution for the Cu data for $2.2 \leq p \leq 4.0$ GeV/c and $\frac{1}{2}\theta^2 \leq 12\times10^{-6}$. Fiducial volume cuts are also included. The peak width is ~7 MeV full width at half-maximum. Mass cuts used in the analysis are indicated. (b) Angular ($\frac{1}{2}\theta^2$) distribution of the Cu and B_4C data in the gap of maximum interference (G_0). It is seen that the background (diffraction plus incoherent regeneration) is smaller under the coherent peak in B_4C. This is the reason for which B_4C was chosen. (For purposes of comparison, the B_4C curve has been normalized to make the number of events in the first $\frac{1}{2}\theta^2$ bin equal to that in the Cu curve.) The shaded region ($\frac{1}{2}\theta^2 < 12\times10^{-6}$) is the coherent regenerated sample isolated by the subtraction technique described in the text.

effects of long-term monitor drifts. The "2π" trigger was essentially as discussed in Ref. 2, except that two large anticoincidence counters, A_R and A_L, were provided at the sides of $\overline{1}$ to suppress charged particles coming from upstream, in particular from R_1. The rates in these counters, and consequently the fraction of suppressed triggers, in fact varied with the position of R_1. To correct for this gap-dependent effect, neutron monitor counts gated by the same anticoincidence as the event triggers were also recorded for each pulse. This loss correction varied smoothly between 3 and 8%.

Event selection and data analysis.—The "kaon" mass ($M_{2\pi}$) and momentum were computed for all presumed 2π triggers, and the angle θ between the incident and regenerated kaon directions was calculated assuming the line connecting the decay vertex and the target as the incident direction. The data were then cut on the vertex distribution (retaining 0.0 in. $< Z <$ 18.0 in.) and the $M_{2\pi}$ distribution [to ± 10 MeV from the mass peak; see Fig. 1(a)], and grouped according to regenerator positions. For each G, a $\theta^2/2$ distribution [see Fig. 1(b)] was plotted and the number of "true" transmission events, $N_{+-}(G)$, was obtained by extrapolating a straight line fitted to the "diffraction" events under the peak at $\theta = 0$. While this conventional, purely empirical procedure needs little justification, we note that these angular distributions were found in good agreement with multiple-scattering theory[8] and reasonable nuclear parameters.[6,9]

A small fraction of the triggers lead to "ambiguous" events, i.e., to events which could not be fully reconstructed. The distribution of these events as a function of G was found to be the same as that of the "good" events. Adding the 45° chamber reduced the fraction of these events from 10 to 1% of the total triggers, and yielded a corresponding increase in the fraction of "good" 2π events. Thus the ambiguous events did not cause a G-dependent bias. A further check was provided by the K_{e3} decays in the fiducial volume. On physical grounds, the number of such events should be essentially unaffected by the presence of R_2. The number of reconstructed e events was indeed found to be independent of gap.

To obtain Δm one has to fit the data to the predicted distribution $N_{+-}(G, \Delta m)$, Eq. (2). In determining the "effective spectrum" $d\tilde{N}/dp$ of Eq. (2), we exploit the fact that at small gaps the observed spectrum,

$$(dN_{+-}/dp)_G \propto (d\tilde{N}/dp) \times \int dZ\, \epsilon(p,Z)|a(p,Z;G,\Delta m)|^2, \quad (3)$$

depends only weakly on Δm. In the present analysis we have used the first five gaps to determine the effective spectrum. The data of gaps six to twenty are put into a number distribution versus gap. The mass difference is determined by fitting this distribution with Eq. (2), which is recast as

$$N_{+-}(G,\Delta m) \propto \int dp \sum_{n=1}^{5}\left[\frac{(dN_{+-}/dp)_n}{\int dZ\, \epsilon(p,Z)|a(p,z;n,\Delta m)|^2}\right] \int dZ\, \epsilon(p,Z)|a(p,Z;G,\Delta m)|^2, \quad G>5. \quad (4)$$

Because of the higher rates at small gaps, approximately $\frac{3}{4}$ of the data were used to determine the spectrum, even though only $\frac{1}{4}$ of the running time was spent in the first five gaps. The virtue of this approach is that the statistical correlation between Δm and $|\rho_0|$ is exceedingly small (see Table I). In fact, Δm is primarily determined by the position G_0 of the interference minimum, while information on $|\rho_0|$ comes mainly from the ratio $N_{+-}(0)/N_{+-}(G_0)$.

Since no previous measurements of regeneration in B_4C appear to exist, we performed an auxiliary experiment to determine ρ_0/η_{+-} for this material. The relevant numbers are given in Table I.

The statistical power of the gap method is enhanced by grouping the data into momentum bins, because G_0 is momentum dependent (see Fig. 2). Thus the effect is "washed out" when data from a wide momentum band are grouped together. Accordingly, the present data have been analyzed in two bins, viz. $2.2 \leq p \leq 3.0$ and $3.0 \leq p \leq 4.0$ GeV/c. The results (in units of 10^{10} sec^{-1}), illustrated in Fig. 2, are, for B_4C,

at 2.2-3.0 GeV/c, $\Delta m = 0.544 \pm 0.007$,

at 3.0-4.0 GeV/c, $\Delta m = 0.542 \pm 0.010$;

and for Cu,

at 2.2-3.0 GeV/c, $\Delta m = 0.533 \pm 0.015$,

at 3.0-4.0 GeV/c, $\Delta m = 0.550 \pm 0.021$;

grand average, $\Delta m = 0.542 \pm 0.005$. (5)

Table I. External parameters used in fits, and their correlations with Δm. $\tau_s = (0.862 \pm 0.006)\times 10^{-10}$ sec,[a] $\eta_{+-} = (1.92 \pm 0.05)\times 10^{-3}$,[a] $f_{21} = f_A + f_B(p - p_0)$.[b]

Regenerator	Δp (GeV/c)	f_A (fm)	f_B [fm/(GeV/c)]	$\varphi_{+-}-\varphi_{if\,21}$ (deg)	C^{f}			
					τ_s	f_A	f_B	$\varphi_{+-}-\varphi_{if\,21}$
B_4C	2.2–3.0	4.3 ± 0.4[c]	0.8 ± 0.3[e]	86.4 ± 9.0[c]	1/2	−1/50	−1/900	1/11
	3.0–4.0	4.8 ± 0.5	0.8 ± 0.3	85.6 ± 8.2	1/2	−1/50	−1/900	1/16
Cu	2.2–3.0	14.4 ± 0.4[d]	2.6 ± 0.2[d]	88.5 ± 5.2[d]	1/2	−1/30	−1/900	1/10
	3.0–4.0	15.7 ± 0.4	2.6 ± 0.2	96.7 ± 4.9	1/2	−1/30	−1/900	1/15

[a] Ref. 10.

[b] p_0 = central momentum of each bin.

[c] Our own measurement. Experimental details to be published.

[d] Ref. 7.

[e] Assumed equal to carbon; see Gaillard *et al.*, Rutherford Laboratory Report No. RPP/H/35.

[f] E.g., $\delta(\Delta m)/\Delta m \equiv C\delta\tau_s/\tau_s$.

The results are based on a total of 59 157 events. Of these, 41 963 are with B_4C regenerators.

The fits were obtained with the values of external parameters given in Table I, which also lists the relevant error correlations. Allowing for the uncertainties in these parameters, we finally obtain

$$\Delta m = (0.542 \pm 0.006)\times 10^{10}\ \mathrm{sec}^{-1}. \tag{6}$$

It was found that regeneration in the air in the gap and also in the anti counter $\bar{1}$ had a negligible influence on the value of Δm. This value may be compared with the weighted average,[1] $(0.542 \pm 0.013)\times 10^{10}$ sec^{-1}, of recent determinations; that average does not include the unpublished result, $(0.542 \pm 0.006)\times 10^{10}$ sec^{-1}, of a new CERN experiment.[10] Thus a new weighted aver-

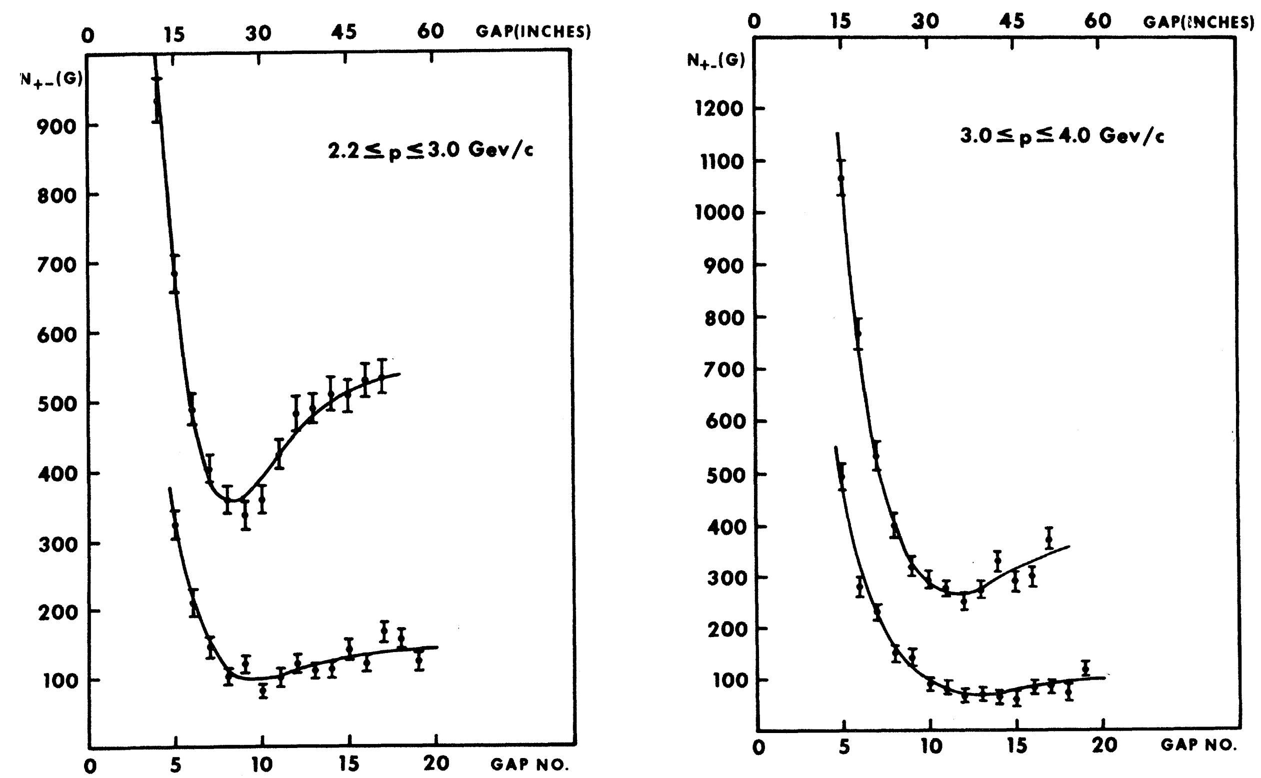

FIG. 2. The number of events versus gap for B_4C (upper points) and Cu (lower points) in each momentum bin. The smooth curves represent the following best fits: B_4C (2.2–3.0 GeV/c), $\Delta m = (0.544 \pm 0.007)\times 10^{10}$ sec^{-1}, $\chi^2 = 14/12$ d.f., 7058 events; Cu (2.2–3.0 GeV/c), $\Delta m = (0.533 \pm 0.015)\times 10^{10}$ sec^{-1}, $\chi^2 = 15/13$ d.f., 2137 events; B_4C (3.0–4.0 GeV/c), $\Delta m = (0.542 \pm 0.010)\times 10^{10}$ sec^{-1}, $\chi^2 = 12/11$ d.f., 5481 events; Cu (3.0–4.0 GeV/c), $\Delta m = (0.550 \pm 0.021)\times 10^{10}$ sec^{-1}, $\chi^2 = 18/13$ d.f., 2115 events.

age can be calculated,

$$\Delta m = (0.542 \pm 0.004) \times 10^{10} \text{ sec}^{-1}. \qquad (7)$$

We can now re-evaluate the "vacuum regeneration" experiments[2,3] using (7). Reference 2 now yields

$$\varphi_{+-} = (44.7 \pm 4.0)^\circ, \qquad (8)$$

while Ref. 3 gives $(49 \pm 12)^\circ$; the main effect is that the uncertainty due to Δm (not included in the errors just quoted!) is now reduced to $\pm 2^\circ$. Combining these values and taking into account the error due to uncertainties in Δm, we get

$$\varphi_{+-} = (45.2 \pm 4.0)^\circ.$$

The value is consistent with those theories which predict

$$\varphi_{+-} \simeq \tan^{-1} 2\Delta m \tau_s = (43.2 \pm 0.4)^\circ,$$

in particular the superweak theory, for which this prediction is exact.

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PHOTON SPLITTING IN A STRONG MAGNETIC FIELD

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We determine the absorption coefficient and polarization selection rules for photon splitting in a strong magnetic field, and describe the possible application of our results to pulsars.

Recent work on pulsars suggests the presence of trapped magnetic fields within an order of magnitude in either direction of the electrodynamic critical field $B_{cr} = m^2/e = 4.41 \times 10^{13}$ G.[1] (Here m and e are, respectively, the electronic mass and charge.) In such intense fields, electrodynamic processes which are unobservable in the laboratory can become important. One such process, for photons with energy $\omega > 2m$, is photopair production, for which both the photon absorption coefficient and the corresponding vacuum dispersion have been calculated by Toll.[2] For $\omega < 2m$ the photopair process is kinematically forbidden, and the only[3] photon absorption