ENERGIES OF QUARTET STRUCTURES IN EVEN-EVEN $N=Z$ NUCLEI

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(Received 7 August 1970)

Mass relationships are used to compute the energy of quartet excited states in $N=Z$ even-even nuclei for ^{12}C up to ^{52}Fe. The states obtained are quasibound up to excitation energies of about 40 MeV and could account for the narrow structures recently observed in heavy-ion transfer experiments.

Reactions have been recently performed in which α, ^{8}Be, or ^{12}C are transferred to even-even $N=Z$ nuclei.[1] They have two characteristics in common: (a) They preferentially excite a few states up to 30-40 MeV, although the density of nuclear states is very large, and (b) they excite states of very narrow width [for example,[2] 20 keV in $^{12}C(^{16}O, \alpha)^{24}Mg$]. These results indicate that the structure of these states involves only a few degrees of freedom in spite of the fact that many nucleons have to be excited to account for the narrow widths.

We shall use mass relationships between nuclei to show that a picture based on the strong interaction in a "quartet" made of 2 protons and 2 neutrons occupying a fourfold degenerate single-particle state (l, m orbit in LS coupling or jm, $j-m$ orbits in jj coupling) yields a small multiplicity of quasibound states in the range of energies covered in the above experiments. The present tabulation should be useful for interpreting and planning heavy-ion experiments and for suggesting further theoretical investigation of these states.

The separation energy of a nucleon in an even-even $N=Z$ nucleus is much larger than that of an α particle. For example, in ^{16}O the neutron threshold is at 15.7 MeV while the α threshold is at 7.2 MeV; likewise in ^{24}Mg they are at 16.2 and 9.3 MeV, respectively. This means that the last nucleon interacts strongly with the three others which make up the extracted α particle and much more weakly with all the remaining nucleons. More precisely in ^{16}O a proton or a neutron of the $p_{1/2}$ orbit has about 1.9 or 5.0 MeV interaction, respectively, with the ^{12}C core as computed from particle separation energy in ^{13}N and ^{13}C. On the other hand, the last proton or neutron of ^{16}O has ~10 MeV interaction with the remaining nucleons of the $p_{1/2}$ orbit. This effect is empirically obvious from the behavior of the nuclear masses of $4n$ nuclei which is a linear function of n, as compared with the behavior of the masses of $4n+x$ nuclei ($x=1, 2, 3, 4$) which are a quadratic function of x. This well known fact, which was the motivation of the Wigner supermultiplet theory,[3] is understood from the exchange nature of nuclear forces. Also the Pauli principle restricts the angular overlap of nucleons in the same j shell and the interaction between nucleons falls off rapidly for low overlap in angular coordinate space. These features of nuclear systems lead to paired and aligned structures the existence of which has been tested in the SU(4) model,[3] deformed orbital calculations,[4] and the stretch scheme.[5]

On the other hand, the shell model describes approximately the ground states of light nuclei by the states in which the lowest single-particle orbits are occupied. The low-lying excited nuclear states involve mostly valence nucleons and only a few nucleons excited from the core. The energies of states with large number of particle-hole pairs are very high, while many of the low-lying valence nucleon states have no specific features to warrant their selective excitation in transfer reactions. However, the experimental consequences of the weak nature of the overall nuclear field as compared with the strong internal binding of quartets are not accounted for in this model.

Because of the strong internal binding energy of quartets, and the weak interaction between quartets, we must expect states corresponding to the breaking of bonds between quartets leaving essentially unaffected the strong coupling inside each quartet. Already the first excited $J=0^+$, $T=0$ states of both ^{16}O and ^{40}Ca have been interpreted as states with a quartet excited from the ($0p$) shell to the ($0d$, $1s$) shell and a quartet from the ($0d$, $1s$) to the ($0f$, $1p$) shell, respectively.[6]

We enumerate the quartet states with $T=0$ in the even-even $N=Z$ nuclei from ^{12}C to ^{52}Fe, restricting the quartets to the ($0p$), ($0d$, $1s$), and ($0f$, $1p$) shells. We take into account exactly the self-energy and the interaction between quartets within a given shell from the observed binding energies of the nuclei whose ground states contain those quartets. The interaction between quartets in different shells is expected to be weak and is calculated as follows. The interaction between two quartets across the $0p$ and $(0d, 1s)$ shells, and across the $(0d, 1s)$ and $(0f, 1p)$ shells is fixed by the positions of the first 0^+ of ^{16}O and ^{40}Ca, respectively. The interaction across the $0p$ and $(0f, 1p)$ shells is left unspecified but must be small due to the small radial overlap. The only assumption is that these elementary quartet particle, quartet hole interactions are constant, independent of mass number and of the number of excited quartets. The angular orientation of quartets, which removes the total angular momentum degeneracy and which leads to rotational states, is ignored. The energy due to angular orientation is expected to be small in conformity with the assumed weakness of quartet-quartet coupling. Also the range of angular momenta will be limited by the usual shell-model limits.

We begin with the quartet states built with ^{16}O as the core. We designate x quartets in the $N=1$ major shell as $Q_p{}^x$. For y quartets in the $N=2$ major shell we have $Q_{(sd)}{}^y$. The interaction energy among the x quartets in the $N=1$ shell is known from the ground-state energy $E_0(A, Z)$ of the nucleus with $A=4x+4$ and $Z=2x+2$ with respect to the ground state of ^{16}O. The interaction energy among the y quartets in the $N=2$ shell is known from the energy of the nucleus with $A=4y+16$ and $Z=2y+8$ with respect to ^{16}O. The interaction energy between the $N=2$ and the $N=1$ quartets is given by $(3-x)yV_{p,(sd)}$, where $V_{p,(sd)}$ is the interaction between one $N=2$ quartet $(y=1)$ and one $N=1$ quartet hole $(x=2)$. Hence the excitation energy of the $Q_p{}^x Q_{(sd)}{}^y$ quartet structure in the nucleus with $A=4x+4y+4$ and $Z=2x+2y+2$ with respect to the ground-state energy of that nucleus is

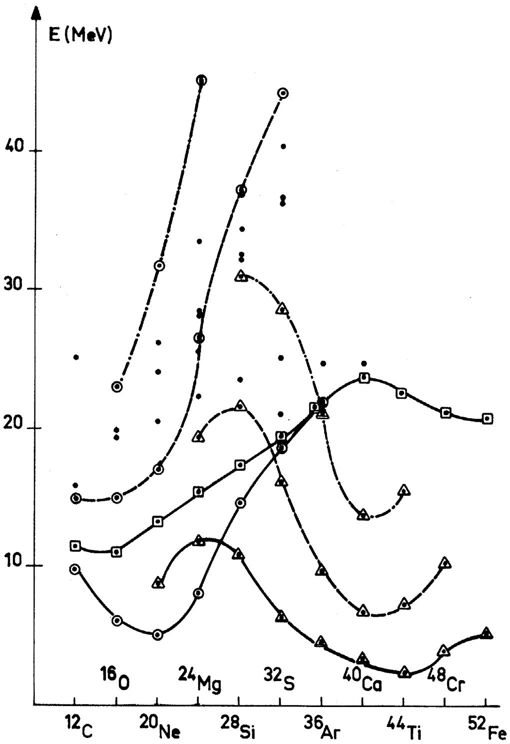

FIG. 1. The excitation energies E^* of quartet states are plotted as a function of mass number for the even-even $N=Z$ nuclei from ^{12}C to ^{52}Fe. The circles denote states with quartets excited from the $(0p)$ to the $(0d, 1s)$ shell, the triangles denote states with quartets excited from the $(0d, 1s)$ to the $(0f, 1p)$ shell, and the squares denote states with one quartet excited from the $(0p)$ to the $(0f, 1p)$ shells. A solid line connects states in which one quartet of a particular type is excited, a dashed line connects states in which two quartets of a particular type are excited, and the dash-dot line connects states in which three quartets of a type are excited. Only states with at most three quartets excited from one major shell to the next major shell are plotted, and states with only one quartet excited from the $(0p)$ to the $(0f, 1p)$ shells are plotted.

$$E^*(Q_p{}^x Q_{(sd)}{}^y) = E_0(4x+4, 2x+2) + E_0(4y+16, 2y+8) - E_0(4(x+y)+4, 2(x+y)+2) - E_0(16, 8) + (3-x)yV_{p,(sd)}. \quad (1)$$

The values for the masses $E_0(A, Z)$ come from the Mattauch, Thiele, and Wapstra mass tables.[7] The interaction $V_{p,(sd)}$ is fixed by setting the one-quartet-one-quartet-hole excitation energy equal to the excitation energy of the first excited $J=0^+$ state in ^{16}O; i.e., $E^*(Q_p{}^2 Q_{(sd)}{}^1) = 6.06$ MeV. This gives $V_{p,(sd)} = 3.63$ MeV. The energies of the quartet structures calculated with the above formula are tabulated in Table I and plotted as a function of mass number in Fig. 1.

Towards the end of the $(0d, 1s)$ shell the states in which a quartet is excited from the $0p$ shell to the $(0d, 1s)$ shell begin to rise in excitation energy. However, near ^{40}Ca the possibility of exciting a quartet from the $(0d, 1s)$ shell to the $(0f, 1p)$ shell becomes possible. In a similar way we estimate the excitation energies of such states, assuming that the $N=1$ major shell is closed:

$$E^*(Q_p{}^3 Q_{(sd)}{}^y Q_{(pf)}{}^z) = E_0(4y+16, 2y+8) + E_0(4z+40, 2z+20) - E_0(4(y+z)+16, 2(y+z)+8) - E_0(40, 20) + (6-y)zV_{(sd),(pf)}. \quad (2)$$

Table I. The excitation energies E^* for the $N=Z$ even-even nuclei from ^{12}C to ^{52}Fe are given for states in the nucleus with $A=4+4(x+y+z)$ which have x quartets in the $0p$ shell, y quartets in the $(0d, 1s)$ shell, and z quartets in the $(0f, 1p)$ shell. The interaction between a quartet in the $(0f, 1p)$ shell and a quartet hole in the $0p$ shell is denoted by V and left unspecified.

	[xyz]	E* (MeV)		[xyz]	E* (MeV)		[xyz]	E* (MeV)		[xyz]	E* (MeV)
^{12}C	[200]	0.0	^{24}Mg	[320]	0.0	^{32}S	[340]	0.0	^{40}Ca	[360]	0.0
	[110]	9.9		[230]	8.1		[331]	6.4		[351]	3.35
	[101]	11.4+2V		[311]	11.8		[322]	16.2		[342]	6.8
	[020]	15.0		[221]	15.4+V		[250]	18.7		[333]	13.8
	[011]	16.0+3V		[302]	19.5		[241]	19.5+V		[324]	24.7
	[002]	25.2+6V		[212]	22.5+2V		[232]	21.2+2V		[261]	23.7+V
^{16}O	[300]	0.0		[140]	26.6		[223]	25.2+3V		[252]	22.4+2V
	[210]	6.06		[131]	25.7+2V		[313]	28.7		[243]	24.0+3V
	[201]	11.1+V		[203]	28.3+3V		[142]	36.7+4V		[234]	27.1+5V
	[120]	15.0		[122]	28.3+4V		[133]	36.4+6V		[162]	45.0+4V
	[111]	19.5+2V		[113]	33.5+6V		[304]	39.2		[153]	41.8+6V
	[102]	20.0+4V		[050]	45.3		[151]	40.5+2V		[144]	40.9+8V
	[030]	23.1		[041]	38.9+3V		[214]	39.2+4V	^{44}Ti	[361]	0.0
	[021]	23.1+3V		[032]	28.3+6V		[160]	44.4		[352]	2.3
	[012]	23.1+6V		[023]	34.0+9V		[124]	40.1+8V		[343]	7.5
	[003]	21.5+9V		[014]	36.7+12V	^{36}Ar	[350]	0.0		[334]	15.6
^{20}Ne	[310]	0.0		[104]	44.0+6V		[341]	4.5		[262]	22.7+2V
	[220]	5.1	^{28}Si	[330]	0.0		[332]	9.8		[253]	26.7+3V
	[301]	8.8		[321]	10.9		[323]	21.4		[244]	25.8+4V
	[211]	13.3+V		[240]	14.7		[260]	21.9		[163]	33.6+6V
	[130]	17.0		[231]	17.4+V		[251]	21.6+V		[154]	31.5+8V
	[202]	17.4+2V		[312]	21.7		[242]	21.4+2V	^{48}Cr	[362]	0.0
	[121]	20.7+2V		[222]	23.7+2V		[152]	19.1+4V		[353]	4.0
	[112]	24.2+4V		[303]	31.1		[143]	17.0+6V		[344]	10.4
	[103]	26.3+6V		[213]	32.5+3V		[134]	17.9+8V		[263]	21.2+3V
	[040]	31.7		[141]	34.5+2V		[233]	24.8+3V		[254]	22.8+4V
	[031]	27.2+3V		[132]	32.5+4V		[161]	24.0+2V		[164]	42.4+8V
	[022]	26.1+6V		[150]	37.2		[314]	35.0	^{52}Fe	[363]	0.0
	[013]	27.7+9V		[123]	36.8+6V		[224]	32.1+4V		[354]	5.2
	[004]	27.3+12V		[204]	39.3+4V		[062]	59.7+6V		[264]	21.0+4V
				[033]	40.3+9V		[053]	52.8+9V			
				[114]	43.2+8V		[044]	48.3+12V			
				[024]	40.2+12V						
				[042]	44.1+6V						
				[051]	51.6+3V						
				[060]	59.1						

The interaction $V_{(sd),(pf)}$ between a quartet in the $N=3$ shell and a quartet hole in the $N=2$ shell is fixed by setting the excitation energy for the structure of one-quartet–one-quartet-hole to be that of the first 0^+ in ^{40}Ca; i.e., $E^*(Q_p{}^3Q_{(sd)}{}^5Q_{(pf)}{}^1)=3.35$ MeV. This gives $V_{(sd)(pf)}=1.54$ MeV. We have tabulated these excitation energies up to six holes in the $N=2$ shell (that is for $0\leq y\leq 5$). The states with more than four quartets in the (pf) shell are not calculated since the masses of $N=Z$ nuclei with $A>56$ are not known.

There is also the possibility that a quartet is excited from the $(0p)$ shell across the $(0d, 1s)$ shell into the $(0f, 1p)$ shell. There is no way of estimating the repulsive interaction V between a $N=3$ quartet and a $N=1$ quartet hole. However we expect it to be small, less than 1 MeV, because of the small radial overlap. Further, we can combine the different excitations so that we can have one quartet excited from the $N=1$ to the $N=3$ shell and one from the $N=2$ to the $N=3$ shell and so on. The general formula for the excitation energies of such states is

$$E^*(Q_p{}^x Q_{(sd)}{}^y Q_{(pf)}{}^z) = E^*(Q_p{}^3 Q_{(sd)}{}^y Q_{(pf)}{}^z) + E^*(Q_p{}^x Q_{(sd)}{}^{y+z} Q_{(pf)}{}^0) + z(3-x)(V - V_{p,(sd)}). \quad (3)$$

In Table I these energies are given for states in which at most four quartets are excited; the interaction V is left unspecified. In Fig. 1 the energies of only those states in which one quartet is excited from the $N=1$ to $N=3$ are plotted against mass number with $V=0$. Thus this curve gives a lower limit for these more complicated type of excitations. The excitations given in Eq. (3) are always higher than the one quartet excitations of Eqs. (1) and (2), but they do compete in energy with the former type in which two quartets are excited.

There are certain salient features of the quartet structures which are worth emphasizing. In the beginning of the $(0d, 1s)$ shell there are rather low excitations into the $(0f, 1p)$ shell. As an example, there is a state with a quartet excited to the $(0f, 1p)$ shell at about 12 MeV in ^{16}O and 9 MeV in ^{20}Ne. Likewise at the end of the $(0d, 1s)$ shell and the beginning of the $(0f, 1p)$ shell quartets excited from the $0p$ shell come around 20-23 MeV. States with many quartets excited are also relatively low. Indeed in ^{16}O the states with two and three quartets excited from the $0p$ to $(0d, 1s)$ shell come at 15 and 23 MeV, respectively. In the $(0f, 1p)$ shell there are very low states with a quartet excited from the $(0d, 1s)$ shell; at 2.3 MeV in ^{44}Ti, 4.0 MeV in ^{48}Cr, and 5.2 MeV in ^{52}Fe.

With respect to the known 0^+ states, let us note that in ^{20}Ne, ^{24}Mg, and ^{36}Ar there exist at 7.0, 6.4, and 5.2 MeV states which cannot be accounted for by standard shell-model calculations[8,9] but could be associated with an excited quartet structure.

The observation of quartet structures at energies as high as 40 MeV depends on their widths. These widths should be small if the quartets are quasibound, i.e., if the energy per excited quartet is smaller than the energy at the top of the Coulomb barrier for emitting an α particle. This energy is about 15 MeV in light nuclei. Thus states with one, two, and three quartets excited are quasibound up to 15, 30, and 45 MeV, respectively. The coupling of these states to the open channels (nucleon, gamma, etc.) is small because of the large difference in structure.

The numerical values quoted in this paper are to be considered as a rough guide. The accuracy of the model is limited by the assumption of shell closure for quartets at $A = 16$ and $A = 40$ and by the neglect of the variation with mass number of the internal structure of quartets and of the interaction between quartets in different shells.

Two of the authors (A.A. and J.G.) are grateful to Professor C. Bloch for his hospitality. We are thankful to Mrs. Nicole Tichit for her diligent computational help.

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