mel'nitsyn,[18] who suggested that a similar effect could possibly occur in a toroidal system if the plasma were nonuniform.

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NEW RELAXATION EFFECT IN THE SOUND ATTENUATION IN LIQUID ^{4}He UNDER PRESSURE*

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Observation of the first-sound attenuation in liquid ^{4}He under pressure has revealed a new relaxation mechanism which is not predicted by any present theory.

In previous work we have shown that certain aspects of the theory of sound attenuation in liquid ^{4}He are not in agreement with experiment at very low temperature.[1] In the collisionless regime ($\omega\tau_{pp} \gg 1$, where τ_{pp} is the wide-angle phonon-phonon relaxation time) the calculated attenuation due to a three-phonon process is given by[2]

$$\alpha(\omega, T) = \frac{\pi^2}{60}\frac{(u+1)^2}{\rho}\frac{k_B{}^4}{\hbar^3 c^6}\omega T^4 \times[\tan^{-1}(2\omega\tau_{pp}) - \tan^{-1}(3\gamma\bar{p}^2\omega\tau_{pp})], \quad (1)$$

where ρ and c are the density and sound velocity, respectively, and $u \equiv (\rho/c)\partial c/\partial\rho$. The dispersion constant, γ, usually thought to be positive, is defined through the relation $\epsilon = cp(1-\gamma p^2)$, where ϵ and p are the energy and momentum, respectively, of an elementary excitation and $\bar{p} \equiv 3k_B T/c$ is the average thermal phonon momentum. The measured attenuation was always found to be greater than that predicted by Eq. (1). A recent precise determination of u has eliminated the possibility that an uncertainty in that parameter might account for the discrepancy.[3] The change of the sound velocity with temperature also disagreed with theory although in this case the measured change was smaller than the predicted value.[2] Moreover, this change was observed to be smaller at higher frequencies, i.e., opposite to that predicted by theory.[4] The calculation of the velocity shift has been extended recently[5] and is now in better agreement; however, the frequency effects still remain at variance with experiment.

It has recently been pointed out[6] that a negative value of γ would lead to twice the attenuation in the limit $|3\gamma\bar{p}^2\omega\tau| \gg 1$ (rather than to zero attenuation as would be the case for positive γ). Recent inelastic neutron scattering experiments[7] indicate that the magnitude of γ must be smaller than 10^{36}. The longest τ, on the other hand, would be the "size-effect limited" value, i.e., d/c where d is some cell dimension. For a typical cell dimension and for the lower temperatures and frequencies the inequality is then only marginally satisfied. Also, the higher-frequency data clearly show that the attenuation is increasing less than linearly (with frequency) whereas a negative value of γ would require a higher than linear dependence. Thus we feel that the explanation for the extra attenuation likely lies elsewhere, especially in view of the observation of the very rapid relaxation process to be discussed shortly.

In an effort to shed more light on this problem we have extended our studies to include the pressure dependence of the sonic attenuation. Previous studies[8,9] of the pressure dependence showed that the attenuation decreased as the pressure increased; this is in qualitative agreement with Eq. (1) since the sound velocity increases with pressure. These studies were limited to relatively high temperatures and low frequencies, however, and did not reveal the features to be discussed here. We have measured the temperature dependence of the sound attenuation for several values of the hydrostatic pressure and at frequencies of 15, 45, 105, and 256 MHz. These measurements were performed using a ^3He-^4He dilution refrigerator which regularly cools to 17 mK. Such low temperatures permit an unambiguous normalization of the attenuation (and velocity) to its zero temperature value. The Cu sonic cell employed was similar in construction to those used in previous measurements[10] except for the addition of a cylinder of sintered Cu, in contact with the liquid, to insure heat transfer. Also, the length of the sonic path was increased to 2.02 cm. Sound was generated and received by 15-MHz unloaded x-cut quartz transducers. The ultrasonic comparator used has been described previously.[10] All temperatures were measured using a cerium magnesium nitrate magnetic thermometer.

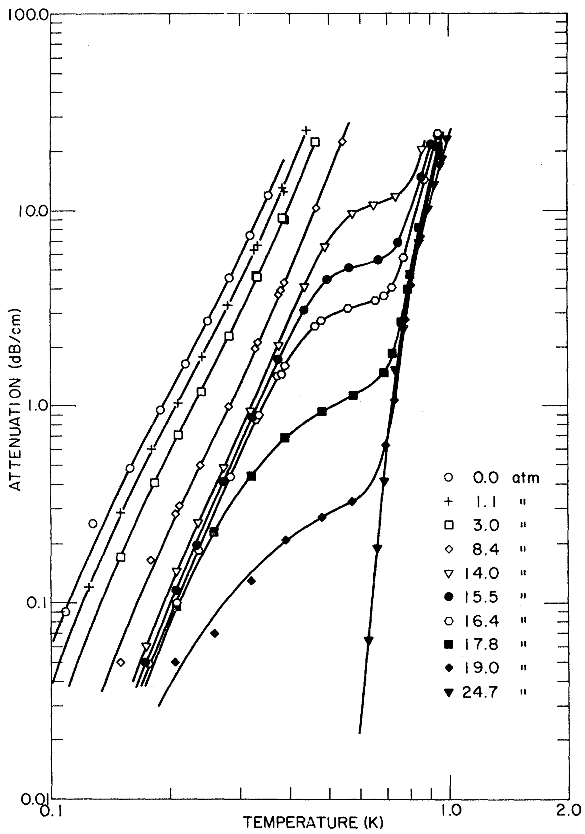

FIG. 1. The temperature dependence of the measured attenuation for a frequency of 105 MHz at various values of the hydrostatic pressure. The solid lines are smooth curves through the data. A T^4 temperature dependence would be a straight line of slope 2 on this graph.

Figure 1 shows the measured temperature dependence of the sonic attenuation at 0.0, 1.1, 3.0, 8.4, 14.0, 15.5, 16.4, 17.8, 19.0, and 24.7 atm for a frequency of 105 MHz. Shown for comparison in Fig. 2 are some of the corresponding pressures as calculated from the theory of Khalatnikov and Chernikova[11] and Van de Meijdenberg, Taconis, and De Bruyn Ouboter.[12] Comparison of Figs. 1 and 2 immediately shows that theory and experiment are in striking disagreement at high pressure. For pressures up to 8.4 atm we see a temperature dependence similar to the T^4 dependence predicted by Eq. (1) although the magnitude of the measured attenuation is again larger than that predicted by theory. For pressures between 14.0 and 19.0 atm we observe that this initial temperature dependence is followed by a shoulder after which the attenuation rises steeply. For $P = 24.7$ atm the attenuation rises steeply at all temperatures with no sign of

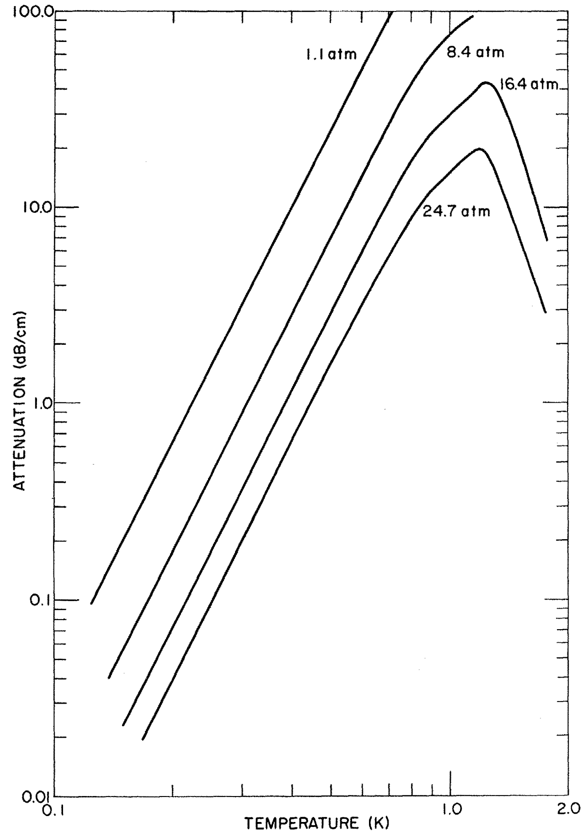

FIG. 2. The temperature dependence of the attenuation as calculated from the theory of Khalatnikov and Chernikova for 105 MHz and for pressures corresponding to some of the experimental values of Fig. 1.

a T^4 component. The other frequencies studied show similar behavior. The shoulder observed at the intermediate pressures strongly suggests a relaxation effect where $\omega\tau$ goes through unity in passing from a collisionless mode of propagation to a hydrodynamic one. The theory of Khalatnikov and Chernikova contains two important relaxation times:

$$\tau_{pp}^{-1}=\frac{9\times 13!}{2^{13}}\frac{k_B^{\,9}}{h^7}\frac{(u+1)^4}{(\rho c^5)^2}T^9, \qquad (2)$$

arising from so called wide-angle phonon-phonon scattering, and

$$\tau_{pr}^{-1}=(2\pi)^{17/2}\Gamma\frac{k_B^{\,9/2}}{h^7}\left(\frac{p_0^{\,4}\mu^{1/2}}{\rho^2c^5}\right)T^{9/2}e^{-\Delta/k_BT} \qquad (3)$$

from elastic phonon-roton collisions, where

$$\Gamma=\frac{2}{9}+\frac{1}{25}\left(\frac{p_0}{\mu c}\right)^2+\frac{2}{9}\left(\frac{p_0}{\mu c}\right)A+A^2$$

with

$$A=\left(\frac{\rho^2}{p_0c}\right)\frac{\partial^2\Delta}{\partial\rho^2}+\left(\frac{p_0}{\mu c}\right)\left(\frac{\rho}{p_0}\frac{\partial p_0}{\partial\rho}\right)^2$$

and p_0, μ, Δ are the roton momentum, mass, and energy gap, respectively. The pressure dependence of the quantities entering these relaxation times are known[12] and computation shows that $\omega\tau_{pr}$ and $\omega\tau_{pp}$ are both much larger than unity in the temperature range of the shoulder.

Khalatnikov and Chernikova state that their theory is valid at very low temperature only so long as $t_{pp}\ll\tau_{pp}$, where t_{pp} is the scattering time characterizing the colinear four-phonon process. Eckstein and Jäckle[13] have shown that t_{pp} is given by

$$t_{pp}^{-1}=\frac{107.0}{192\pi^3}\frac{(u+1)^4k_B^{\,7}}{\rho^2c^8\hbar^7\gamma}T^7 \qquad (4)$$

and, since $\tau_{pp}^{-1}\propto T^9$, there will be a transition to the region $t_{pp}\geqslant\tau_{pp}$ below some temperature. Using γ from the neutron data we find that $t_{pp}\ll\tau_{pp}$ in the temperature range of interest, i.e., the theory of Khalatnikov and Chernikova should be valid.[14] (The theory of Eckstein and Varga[15] predicts $\gamma\propto c^{-1}\rho^{-1/3}$ and thus should not change radically with pressure.)

If we eliminate the phonon-phonon contribution to the attenuation by taking the limit $\omega\tau_{pp}\ll 1$, but keep $\omega\tau_{pr}\gg 1$, we find the attenuation is given by

$$\alpha=\frac{\pi^2k_B^{\,4}T^4}{45\hbar^3\rho^2c^6}\frac{1}{\tau_{pr}}\propto T^{17/2}e^{-\Delta/k_BT}.$$

(The direct Rayleigh-like acoustic phonon-roton scattering is negligible, since it is proportional to $\omega^4T^{1/2}e^{-\Delta/k_BT}$.) This temperature dependence agrees qualitatively with the data for temperatures which lie above the shoulder and this suggests that it is the phonon-phonon part of the theory which is in error. The data clearly show, however, that the shoulder moves to lower temperature as the pressure increases whereas all the mechanisms of phonon-phonon scattering give a τ^{-1} varying approximately as $(T/c)^n$, i.e., the shoulder would move to higher temperature since the velocity increases with increasing density.

A mechanism that has approximately the right pressure dependence is roton-roton scattering. Khalatnikov[16] shows that the scattering time, τ_{rr}, for this process goes as $e^{\Delta/T}$ so that the temperature where $\omega\tau_{rr}=1$ would vary with pressure in the same direction as Δ (decreasing with increasing pressure). The theory of Khalatnikov and Chernikova assumes $\omega\tau_{rr} \ll 1$ and under this condition predicts an ωT^4 component at low temperature. On the other hand our data show that the ωT^4 component disappears above the shoulder ($\omega\tau < 1$) and thus the shoulder cannot be identified with roton-roton scattering.

If we approximate the scattering time producing the shoulder (whatever its nature) by the form $\tau^{-1} = aT^n$, then to reproduce the shape of the curve a value of $n \cong 4$ is required. Furthermore, the shoulders at different frequencies (and pressures) require somewhat different exponents, thus indicating that a simple power law for the relaxation process is inadequate. (The exponent n appears to increase with increasing temperature.) In no case can a value of n approaching 9 account for the data, since such a steep temperature dependence would generally result in a well defined peak rather than a shoulder. The same holds true for an exponential temperature dependence. Clearly these data require a total reexamination of the theory of sound propagation in liquid ^{4}He at low temperature.

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