Similar results are obtained in this case. Ion-density fluctuations grow and lead to an enhanced resistivity.

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SIMULATION OF COUNTERSTREAMING PLASMAS WITH APPLICATION TO COLLISIONLESS ELECTROSTATIC SHOCKS*

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A one-dimensional sheet-charge model is used to simulate homogeneous conterstreaming plasmas with no magnetic field. We demonstrate that the only electrostatic instability capable of dissipating a substantial fraction of the relative ion-drift energy and thus generating a turbulent, collisionless shock is the ion-ion two-stream instability. Furthermore, it is shown that such a shock cannot occur at high Mach numbers.

The objective of this study is to determine the conditions for the occurrence of turbulent, collisionless, electrostatic shocks; the magnetic field is thus taken to be zero. Following Sagdeev,[1] we distinguish turbulent shocks from laminar shocks, in which instabilities are absent (e.g., Montgomery and Joyce[2]) or weak. The laminar wave breaks for Mach numbers M greater than some critical value M_c (e.g., for the nonlinear ion acoustic wave,[1] $M_c = 1.6$). The electrostatic shocks recently observed in the laboratory[3] had $M \leq 1.2$ and were apparently laminar. In turbulent shocks, presumably $M > M_c$, and significant counterstreaming occurs.[4,5] In order to follow the nonlinear behavior of the resulting electrostatic instabilities, we resort to computer simulation.

(I) The model.—As an idealization of the shock-generation problem, consider two plasmas initially separated in space which encounter each other at a plane normal to the relative velocity $2v_0$ (nonrelativistic). The region of interpenetration will be bounded by two fronts, each advancing into one of the plasmas. Instabilities in this region will heat the plasma there to some extent. Long after the initial encounter, we can focus on the region just behind one of the fronts where the plasma (labeled "1"), moving with the front, streams against the cooler incoming plasma ("2"). If a shock forms, the front will be a shock front; then this description is similar to the Mott-Smith shock model.[4,6]

Due to limitations of computer time and memory, we study this spatially inhomogeneous problem by means of a model of 1-d (one dimensional) homogeneous counterstreaming plasmas. The plasmas are of equal density. We thus simulate the dissipation mechanism in the shock rather than the detailed structure (for further discussion, see below).

The relation between the Mach number M of the shock and the relative velocity $2v_0$ of the counterstreaming plasmas can be determined from Tidman's analysis.[4] Let $\alpha = v_0[(T_e + T_i)/(m_i + m_e)]^{-1/2}$, where T is the temperature in ergs; then

$$M = \alpha(16/15)^{1/2} + (1 + 16\alpha^2/15)^{1/2}. \qquad (1)$$

For $T_e \gg T_i$ and neglecting m_e/m_i, we note that $\alpha = v_0/v_s = \mu^{1/2} v_0/v_{Te}$, where $v_s = (T_e/m_i)^{1/2}$, $\mu = m_i/m_e$, and $v_{Te} = (T_e/m_e)^{1/2}$.

We recapitulate some of the results of the linear theory of identical counterstreaming plasmas.[7] The e-e (electron-electron) two-stream instability occurs for $v_0 \gtrsim 1.3 v_{Te}$ ($M \gtrsim 2.6\mu^{1/2}$). The e-i (electron-ion) instability occurs for $1.55 v_{Te} \gtrsim v_0 \gtrsim 5.6 v_{Ti}$ and requires $T_i/T_e \gtrsim 0.3$. The i-i (ion-ion) instability arises when $v_0 \lesssim v_s$ ($T_e \gtrsim m_i v_0^2$; M not very large) and also requires $v_0 \gtrsim 1.3 v_{Ti}$ and $T_i/T_e \lesssim 0.3$. For the case in which the two plasmas are not identical, we refer to the electrons in plasma 1 (the warmer plasma) as beam $e1$, etc. For $T_{e1} \geqslant T_{e2}$ and $T_{e1} \gg m_e v_0^2$, we find that the e-e instability is very weak and that there are now two cases for the e-i instability between beams $e2$ and $i1$: (1) the e-i two-stream instability if $2v_0 \gg v_{T_{e2}}$; and (2) the ion-wave, or ion-sound, instability (not the i-i instability) if $2v_0 \ll v_{T_{e2}}$. The latter has a lower growth rate and is much weaker than the former.[8] Since the e-e instability can only heat the electrons to a few times $m_e v_0^2$ and is ineffective at stopping the ions, we focus on the e-i and i-i instabilities as dissipation mechanisms.

(II) Comments on computer simulation. —We adopt the 1-d "cloud-in-cell" model of a plasma.[9] The trajectories of particles of finite thickness are followed in time by solving the equation of motion for each particle and Poisson's equation for the electric field; periodic boundary conditions are imposed.

A real plasma will usually be far more nearly collisionless than our model because we are restricted to using a relatively small number of particles per Debye length, $n\lambda_D$. However, to keep collisional effects at an acceptable level, we make $n\lambda_D$ large enough (often ~250) that the wave-damping time $2n\lambda_D/\omega_{pe}$ due to e-i collisions[10] is comparable with the running time of the experiment; we focus on phenomena occurring on a shorter time scale.

(III) Electron-ion instability. —Four experiments were performed, two at mass ratio $\mu = 100$ (Expts. 1 and 4) and two at $\mu = 1836$ (Expts. 2 and 3). We focus on the $\mu = 1836$ experiments, which were run with length $L = 32 v_0/\omega_{pe}$ and a density of each beam $n = 256\omega_{pe}/v_0$. Initial temperatures were $T_{e1} = 4.0 m_e v_0^2$; $T_{i1} = 0.1 m_e v_0^2$ (Expt. 2) and $T_{i1} = 1.0 m_e v_0^2$ (Expt. 3); and $T_{e2} = T_{i2} = 0.01 m_e v_0^2$. These conditions correspond to very large initial Mach numbers; taking plasma 2 to represent the unshocked plasma in accordance with the discussion above, we find $M \simeq 15\mu^{1/2}$; but if we focus on the interaction between beams $e1$ and $i2$ and take plasma 1 to represent the unshocked plasma, then $M \simeq \mu^{1/2}$. Experiments at much lower Mach numbers are discussed in Sec. IV.

The evolution of the instability is shown in Fig. 1. Since T_{e2} was initially small, beams $e2$ and $i1$ interacted via the e-i two-stream instability. T_{e2} increased $3.1 m_e v_0^2$, while T_{i1} rose $0.15 m_e v_0^2$ and $0.4 m_e v_0^2$ in Expts. 2 and 3, respectively. (Here T is the spatially averaged kinetic energy of the particles in the coordinate system comoving with the beam; it thus includes oscillatory energy, which dissipates by collisionless processes rath-

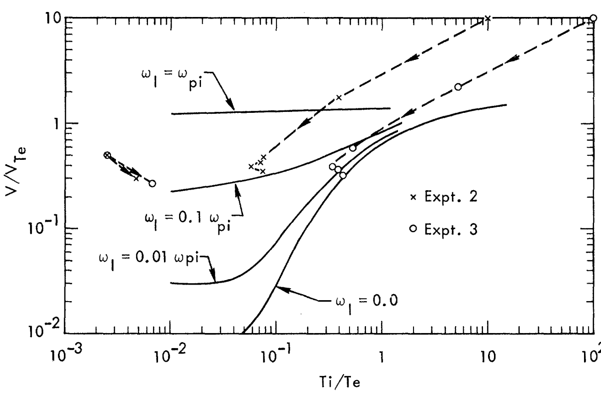

FIG. 1. Evolution of the e-i (electron-ion) instability in expts. 2 and 3. Half the e-i relative velocity divided by the electron thermal velocity is plotted against the temperature ratio. The solid lines (obtained from Ref. 7) are lines of constant growth rate ω_I for the e-i instability in a current-carrying plasma; $\omega_I = 0$ divides the stable and unstable regions. The experimental points on the long tracks at the right are for beams $e2$ and $i1$, which interacted via the e-i two-stream instability. For example, the abscissa in this case is T_{i1}/T_{e2}. The time interval between the points is $100\omega_{pe}^{-1}$, except that Expt. 3 skips from $400\omega_{pe}^{-1}$ to $750\omega_{pe}^{-1}$. Only the initial and final points are shown for beams $e1$ and $i2$ (short tracks at left), which interacted via the ion-wave instability.

er slowly in 1-d, but fairly rapidly in 2-d and 3-d).[11] Beams $e1$ and $i2$ interacted via the ion-wave instability and were not significantly heated: $\Delta T_{e2} \lesssim 0.4 m_e v_0^2$ and $\Delta T_{i2} \lesssim 0.02 m_e v_0^2$. Clearly the heating is inadequate to bring the system into the region where the i-i instability can occur ($T_e \gtrsim m_i v_0^2$). Note that the higher initial value of T_{i1} in Expt. 3 forced the system to remain closer to the region of stability. The electric field energy rose 2 orders of magnitude above the thermal level to about 20% of the electron-drift energy. Experiments 1 and 4 at $\mu = 100$ were run until $t_f = 40\omega_{pi}^{-1}$, a larger number of ion plasma periods than was possible at $\mu = 1836$; the results were consistent with the high mass-ratio runs. In all cases the reduction in the ion momentum was less than μ^{-1}. These results are consistent with the work of Davidson et al.[12] Our experiments give no evidence that the e-i instability can dissipate a significant fraction of the ion-drift energy.

(IV) Ion-ion instability. —Conditions for the occurrence of this instability are given in Fig. 2. The requirement $v_0/v_s \leq 1$, where $v_s = (T_e/m_i)^{1/2}$, is set by the condition that the wavelength of the fastest growing mode be less than a Debye length: $k\lambda_D \gtrsim 1$. In 2-d and 3-d the linearly unstable region has been shown to be considerably larger[13]; but if the initial value of v_0/v_s lies well above the unstable region in Fig. 2 ($T_e \ll m_i v_0^2$), the instability will be cut off by ion Landau damping when T_i approaches T_e (since then $k\lambda_{Di}$ also $\gtrsim 1$). Lampe has confirmed this in a 2-d quasilinear calculation.[14] Since T_i remains well below $m_i v_0^2$, the dissipation is small. We conclude that the conditions under which the i-i instability can dissipate a significant part of the relative ion-drift energy are roughly the same in 1-d as in 3-d.

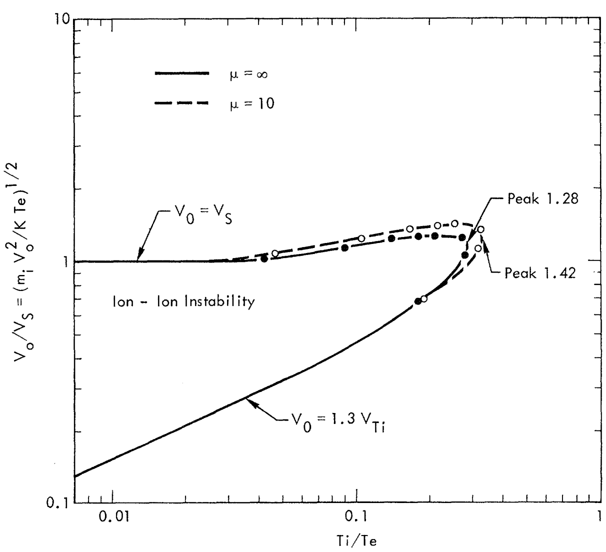

FIG. 2. Conditions for the occurrence of the ion-ion instability between identical counterstreaming plasmas for two values of the mass ratio μ (generalized from Ref. 7). The curve for $\mu = \infty$ obtains whenever $\mu \gg 1$ and is indistinguishable from the curve for $\mu = 1836$. The peak values are the maximum values of v_0/v_S for instability; they correspond to Mach numbers 3.0 for $\mu = \infty$ and 3.4 for $\mu = 10$.

Table I gives the results of six experiments on the i-i instability. A strong instability develops when the initial electron temperature exceeds a minimum value corresponding to the peak value of v_0/v_s in Fig. 2 (i.e., $M \lesssim 3$). This is shown by the large increase in T_i and the significant reduction in the ion relative velocity under these conditions (Expts. 6, 8-10). The instability becomes weaker as T_{i1} is increased (Expt. 9). The electric field energy grew to $(0.02\text{-}0.06) m_i v_0^2$. On the other hand, at higher Mach numbers ($M > 3$; Expts. 5, 7) no i-i instability occurs; furthermore the e-i instability in this region ($M \ll \mu^{1/2}$) is unable to heat the electrons to the point where the i-i instability can occur. The observed heating in this case is primarily due to e-i collisions.

(V) Conclusions. —To relate our results to shocks, we note that in a shock all the relative drift energy ($2m_i v_0^2$ per ion) goes into heat. Additional heating is due to work done by the thermal pressure of the incoming plasma, but at the Mach numbers we are considering ($M \gtrsim 1.6$) this heating is less than half the total. In our homogeneous model a heating $\Delta T_i \simeq m_i v_0^2$ is equivalent to significantly reducing the ion-drift energy, since that is the only source of free energy. If an instability observed in our model substantially reduces the counterstreaming of the ions, then it is likely that a shock will develop between colliding plasmas (see Sec. I) under analogous conditions; otherwise the system would approach the 1-d homogeneous case, which is unstable. The shock thickness will probably not be much greater than the e-folding length λ_{max} of the most unstable waves[4] in the homogeneous case, even if the shock structure is determined by a different, nonelectrostatic instability: Shock gradients would have to be significant in a distance of order λ_{max} to dominate or stabilize the electrostatic instability. On the other hand, if electrostatic instabilities observed in our model do not dissipate a substantial fraction of the ion drift energy, then they are unlikely to determine the shock structure.

We have found that only the i-i instability can

Table I. Results of numerical experiments on the ion-ion instability.

Mass ratio		10		40			160[a]
Expt. [Stable (S) or unstable (U)]		5(S)	6(U)	7(S)	8(U)	9(U)	10(U)
Mach number	M	4.9	2.5	3.7	2.9	2.5	2.2
Density	$n v_o/\omega_{pe}$	128	128	64	64	32	32
Length	$L \omega_{pe}/v_o$	64	64	128	128	256	256
Running time	$t_f \omega_{pe}$	500	400	600	500	500	600
Temperature	$T_{e1} = T_{e2}$	2.0	10.0	15.0	27.0	40.0	240.0
$T = T(t = o)$	$\Delta T_{ei} \simeq \Delta T_{e2}$	0.7	1.8	1.0	5.2	2.5	36.0
$\Delta T = T(t=t_f) - T(t=o)$	T_{i1}	0.1	2.5	10.0	10.0	27.0	1.0
Units : $m_e v_o^2$	ΔT_{i1}	0.16	6.7	0.6	25.6	13.8	146.0
	T_{i2}	0.1	0.1	0.1	0.1	0.1	1.0
	ΔT_{i2}	0.16	6.6	0.36	31.4	10.3	74.0
Final ion relative velocity $/2v_o$		0.989	0.46	0.992	0.44	0.82	0.17
Growth rate	ω_I/ω_{pi}	-	0.13	-	0.08	0.07	0.13

[a]This experiment ended before the instability saturated.

significantly slow the ions, and in our 1-d model it can occur only for $v_0 \lesssim v_s$ ($M \lesssim 3$). If the system is initially outside the region in which the i-i instability can occur, our results show that the e-i instability is unable to move the system into it. We conclude that electrostatic shocks cannot occur at high Mach numbers. Even allowing for the somewhat larger region of instability in 3-d (see above), it is unlikely that an electrostatic shock can occur for $v_0 \gtrsim$ a few times v_s, or $M \gtrsim 6$.

Our conclusion is altered if a mechanism not involving electrostatic instabilities can cause an increase in the electrostatic potential across the shock front at Mach numbers $M \gtrsim 6$. For example, Montgomery and Joyce[2] have discussed such a shock model in which there is no dissipation and no upper bound on M. (Presumably M would be limited by instabilities, which were not included in their model.) Our results preclude neither a shock based on their model nor one in which an analogous mechanism (one not involving electrostatic instabilities) reduces v_0 to the point that the i-i instability can occur. In such a shock we conclude that the energy dissipation and ion slowing could not exceed that given by the usual jump conditions[4] at $M \simeq 6$.

Our results conflict with those of Colgate and Hartman.[5] In their computer simulation they found that an electrostatic shock developed between colliding plasmas even when $T_e \ll m_i v_0^2$ ($M \gg 3$). However, we have shown that because of the small value of $n\lambda_D$ which they used ($n\lambda_D \sim 4$), electron-ion collisional effects dominated their results; Dawson, Papadopoulos, and Shanny[15] have come to the same conclusion.

On the other hand, our results are basically consistent with Tidman's theory, in which quasilinear theory is applied in the context of the Mott-Smith shock model[6] to determine the properties of an electrostatic shock based on the i-i instability. From the dispersion relation for spatially growing waves between beams of unequal density, Tidman found several necessary conditions for the shock to occur; one of them [his Eq. (79)] can be shown to require $M \lesssim 6$. In addition, his estimate of the shock thickness L_s agrees with the crude guess we can make from our results: From the observed growth rate $\omega_I \simeq 0.1\omega_{pi}$, which often persisted until the instability saturated, we estimate $L_s \simeq 2v_0/\omega_I \simeq 10(2v_0/\omega_{pi}) = \text{const} \times U/\omega_{pi}$. Here $U > 2v_0$ is the shock velocity, so that the "const" is somewhat less than 10.

After the completion of this research, it was learned that similar results have been obtained by the Naval Research Laboratory group.[12,16] In particular, they did not find any significant difference between their 1-d results and their 2-d results.

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FAST TIME-RESOLVED SPECTRA OF ELECTROSTATIC TURBULENCE IN THE EARTH'S BOW SHOCK*

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We present time-resolved spectra of electrostatic turbulence in the earth's bow-shock structure. Spectral details on scales for a few Debye lengths indicate that single modes or groups of single modes dominate the turbulent spectrum. These modes are probably ion-acoustic or Buneman instabilities of short wavelength ($k\lambda_D \sim 1$) which are generated in parts of the shock microstructure containing diamagnetic drift currents.

In a previous note,[1] evidence for the detection of electric field turbulence in the earth's collisionless bow shock was presented. At that time, only narrow-band filter and broad-band frequency-time analyses of this turbulence were available. We have recently subjected the broad-band analog electric field data (1-22 kHz) from our OGO-5 experiment[2] to a fast-time-resolution spectral analysis which allows a complete turbulence spectrum over a selected passband to be formed each 12.5 msec. During a 12.5 msec interval, the spacecraft moves through a distance comparable with the plasma Debye length, or some 20-40 m. Thus the time-resolved spectra allow examination of very fine details of shock turbulence. We have chosen a fairly typical example of such a time-resolved spectrum of turbulence in a bow-shock structure observed near $0^h46^m54^s$ UT on 12 March 1968. We believe that these spectra are the first ever presented showing the microscopic details of electrostatic wave turbulence. As such, they should be of interest not only to the understanding of the collisionless shock dissipation mechanism, but also to the descriptions of plasma turbulence by such tools as quasilinear theory.

In any single satellite measurement, the length scales inferred from measurements must always involve some assumption about the convection of the plasma disturbance relative to the spacecraft. The upstream conditions at the time of shock encounter were approximately: ion density $n \sim 10$ cm^{-3}; flow speed $U_0 \sim 380$ km/sec; ion temperature $T_i \sim 6.3\times10^4$ °K; electron temperature unknown, but probably $T_e \sim 10^5$ °K; interplanetary field $B_0 \sim 7\times10^{-5}$ G; satellite orbital speed $V_s \sim 1.9$ km/sec. From these parameters we conclude $\omega_{pi}/2\pi \sim 650$ Hz, $\omega_{pe}/2\pi \sim 28$ kHz, $c/\omega_{pe} \sim 1.7$ km, $c/\omega_{pi} \sim 73$ km, $\omega_{ce}/2\pi \sim 200$ Hz, $\omega_{ci}/2\pi \sim 0.11$ Hz, and $\lambda_D \sim 7$ m.