SPECULATION ON THE HELICITY OF A VIRTUAL (TIMELIKE) PHOTON EMITTED IN HADRON-HADRON COLLISIONS*

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We conjecture that the linear polarization of a large-mass timelike photon emitted in high-energy hadron-hadron collisions is predominantly longitudinal ($\lambda = 0$). The conjecture can be tested experimentally by studying the decay angular distribution of muon pairs produced in

$$p\ (\pi) + N \rightarrow \mu^+ + \mu^- + \text{hadronic system}.$$

A muon-pair production process

$$A + B \rightarrow \mu^+ + \mu^- + C + D + \cdots, \tag{1}$$

where A, B, C, D, $\cdots$, are hadrons, measures the matrix element

$$\langle C, D, \cdots | j_\mu | A, B \rangle. \tag{2}$$

In (2) j_μ is the hadronic part of the electromagnetic current operator, and the virtual photon associated with the current is timelike ($q^2 < 0$ in our metric). In this note we speculate on the helicity (λ) of the virtual (timelike) photon (or equivalently the spin orientation of the di-muon system) emitted in Reaction (1).

The amplitudes for the emission of $\lambda = \pm 1$ (transverse) and $\lambda = 0$ (longitudinal) photons with four-momenta q in process (1) are given by

$$\langle C, D, \cdots | \epsilon^{(\pm 1)} \cdot j | A, B \rangle = \mp 2^{-1/2} \langle C, D, \cdots | (j_x \pm i j_y) | A, B \rangle, \tag{3a}$$

$$\langle C, D, \cdots | \epsilon^{(0)} \cdot j | A, B \rangle = [(-q^2)^{1/2}/q_0] \langle C, D, \cdots | j_z | A, B \rangle, \tag{3b}$$

where the z axis is taken in the direction of the virtual photon momentum, and $\epsilon_\mu^{(\pm 1, 0)}$ stand for the photon polarization vectors with $\lambda = \pm 1, 0$ satisfying $\epsilon^2 = 1$, $\epsilon \cdot q = 0$. To study the q^2 dependence of (3a) and (3b) it is convenient to define [as suggested by a formalism based on the current-field identity[1] $j_\mu = (m_\rho^2/f_\rho)\rho_\mu$]

$$\langle C, D, \cdots | J_\mu^{(\rho)} | A, B \rangle = [(m_\rho^2 + q^2)/m_\rho^2] f_\rho \langle C, D, \cdots | j_\mu | A, B \rangle. \tag{4}$$

In terms of the ρ source density $J_\mu^{(\rho)}$ we can rewrite (3a) and (3b) as

$$\langle C, D, \cdots | \epsilon^{(\pm 1)} \cdot j | A, B \rangle = \mp (m_\rho^2/f_\rho)(m_\rho^2 + q^2)^{-1} 2^{-1/2} \langle C, D, \cdots | (J_x^{(\rho)} \pm i J_y^{(\rho)}) | A, B \rangle, \tag{5a}$$

$$\langle C, D, \cdots | \epsilon^{(0)} \cdot j | A, B \rangle = (m_\rho^2/f_\rho)(m_\rho^2 + q^2)^{-1} [(-q^2)^{1/2}/q_0] \langle C, D, \cdots | J_z^{(\rho)} | A, B \rangle. \tag{5b}$$

If we ignore the finite width of the ρ meson, the matrix elements $\mp 2^{-1/2} \langle C, D, \cdots | (J_x^{(\rho)} \pm i J_y^{(\rho)}) | A, B \rangle$ and $(m_\rho/q_0) \langle C, D, \cdots | J_z^{(\rho)} | A, B \rangle$ evaluated at $q^2 = -m_\rho^2$ are just the hadronic amplitudes for the emission of $\lambda = \pm 1$ and $\lambda = 0$ (real) ρ mesons; barring dynamical accidents, they are likely to be of the same order of magnitude.[2] Suppose we invoke a mild form of vector-meson dominance: The invariant (off-shell) hadronic amplitudes $\langle C, D, \cdots | J_{x,y}^{(\rho)} | A, B \rangle$ and $(m_\rho/q_0) \langle C, D, \cdots | J_z^{(\rho)} | A, B \rangle$ are smooth in q^2 so that they continue to be comparable even if we go away from $q^2 = -m_\rho^2$. It then follows from the appearance of the extra $(-q^2)^{1/2}$ factor in (5b) that for $-q^2 \gg m_\rho^2$ the longitudinal amplitude dominates over the transverse amplitude.[3,4]

Our conjecture can be tested experimentally by examining the decay distribution of the di-muon system. With $\lambda = 0$, the muon distribution in the di-muon rest system should be $\sin^2\theta$ (rather than $1 + \cos^2\theta$ appropriate for $\lambda = \pm 1$) where θ is measured with respect to the direction of the timelike photon momentum (the "helicity frame").[5]

Experiments are in progress to study at the highest alternating-gradient synchrotron energy

$$p + \text{uranium block} \rightarrow \mu^+ + \mu^- + \text{anything} \tag{6}$$

with $m(\mu^+\mu^-) = (-q^2)^{1/2}$ in the range 1-6 GeV.[6,7] According to preliminary data reported at the Liverpool conference, the muon-pair mass distribution drops sharply with increasing di-muon mass, a feature in qualitative agreement with vector-meson dominance.[8] It appears desirable to examine the decay distribution of the muon pair to see whether the highly virtual (timelike) photon is indeed longitudinal, as conjectured in this brief note.

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[1]We ignore here the ω and φ contributions; they can easily be incorporated.

[2]Indeed, in reactions $\pi^- + p \to \rho^0 + n$ and $\pi^+ + p \to \rho^0 + \Delta^{++}$, where the density matrices have been determined experimentally, $2\rho_{11}^{(\mathrm{helicity})}$ is equal to $\rho_{00}^{(\mathrm{helicity})}$ within a factor of 2 or 3 except when $|t|$ is very small. See, e.g., R. Diebold and J. A. Poirier, Phys. Rev. Letters 20, 1532 (1968); A. M. Boyarski *et al.*, Phys. Rev. Letters 22, 148 (1969).

[3]At first sight it may appear that our smoothness assumption is dependent on the particular frame we have used and that a very different answer can be obtained by working in other frames (the di-muon rest frame, in particular). However, in the reaction $\pi^- + p \to \rho^0 + n$ where we can explicitly express the helicity amplitudes in terms of Ball's invariant amplitudes, it is possible to prove that the smoothness hypothesis must be applied to $\langle n | \epsilon^{(\pm 1)} \cdot J^{(\rho)} \times | \pi^- p \rangle$ and $(-q^2)^{-1/2} \langle n | \epsilon^{(0)} \cdot J^{(\rho)} | \pi^- p \rangle$ provided the Ball amplitudes are assumed to be slowly varying in q^2 [M. Le Bellac and G. Plaut, Nuovo Cimento 64A, 95 (1969); see especially their Eq. (2.3)]. Our extrapolation procedure appears to be reasonable also for elastic ρp scattering where one of the ρ's is taken off shell (H. Fraas and D. Schildknecht, to be published).

[4]The dominance of the longitudinal contribution conjectured here is reminiscent of a large longitudinal cross section predicted in the vector-meson dominance model for inelastic electron-proton scattering [J. J. Sakurai, Phys. Rev. Letters 22, 981 (1968)]. According to the Stanford Linear Accelerator Center—Massachusetts Institute of Technology experiment on inelastic ep scattering, the ratio of the longitudinal to the transverse cross section at $s^{1/2} = 2$-4 GeV, $q^2 = 4$ (GeV/c)^2 is considerably smaller than the vector-meson dominance prediction [R. E. Taylor, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969*, edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Lancashire, England, 1970), p. 251]. It appears that when $s \approx q^2$, we have to use a more sophisticated mass extrapolation procedure. In any case, even if simple-minded vector-meson dominance fails in the spacelike region, it may still work in the timelike region.

[5]See also R. J. Oakes, Nuovo Cimento 44A, 440 (1966), who discusses in detail the muon-pair distribution in the "Jackson frame."

[6]L. M. Lederman, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, England, 1969,* edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Lancashire, England, 1970).

[7]For earlier experiments on muon-pair production, see A. Wehmann *et al.*, Phys. Rev. Letters 17, 1113 (1966); B. D. Hyams *et al.*, Phys. Letters 24B, 634 (1967).

[8]However, alternative interpretations are also possible. See S. M. Berman, D. J. Levy, and T. L. Neff, Phys. Rev. Letters 23, 1363 (1969).