10^{-7} to 10^{-5}, and for the Crab pulsar 10^{-6} to 10^{-4}. Clearly, a change in the value of M and especially of R would affect strongly these limits. It should be added that the calculated oblateness of pulsars is very small so that the sudden adjustments in the shape are of the order of a centimeter for the Vela and tenths of a millimeter for the Crab pulsar. Thus it is hard to believe that the strength of the crust or the frequency of its quakes is seriously affected by details of its external and internal shapes.

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EVIDENCE AGAINST A_1 PRODUCTION IN HIGH-ENERGY K^+p INTERACTIONS*

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The production of the A_1 has been reported at 12.8 GeV/c in the reaction $K^+p \to Kp(3\pi)$. It has also been reported at 9.0 GeV/c in the reactions $K^+p \to Kp(3\pi)$ and $K^+p \to Kp(4\pi)$. At 12.0 GeV/c, with five times the data of the 12.8-GeV/c experiment and three times the data of the 9.0-GeV/c experiment, we see no evidence for A_1 production in any of these reactions.

The A_1 enhancement[1] has been seen mainly in the reaction $\pi^\pm p \to A_1^\pm p$, $A_1^\pm \to \rho^0\pi^\pm$, $\rho^0 \to \pi^+\pi^-$, where its interpretation as a resonance has been questioned because Deck or other diffraction processes may be present.[2] In K^+p interactions, where A_1 simulating effects of this type are probably not so prominent, the observation of the A_1 would greatly favor the resonance interpretation independently of the concept of duality.[2]

Recently, observations of the A_1 have been reported in the reactions

$$K^+p \to K^+p\pi^+\pi^-(\pi^0) \tag{1}$$

(Berlingieri et al.[3]),

$$K^+p \to K^0p\pi^+\pi^+\pi^- \tag{2a}$$

(Ref. 3 and Alexander, Firestone, and Goldhaber[4]),

$$K^+p \to (K^0)p\pi^+\pi^+\pi^- \tag{2b}$$

(Ref. 4), and

$$K^+p \to K^0p\pi^+\pi^+\pi^-(\pi^0) \tag{3}$$

(Ref. 4), in bubble-chamber experiments at 12.8[3] and 9.0[4] GeV/c (the brackets indicate a particle not detected). In our experiment at 12.0 GeV/c, we have studied these reactions with a path length corresponding to about 35 events/μb—at least three times as great as in either of the other experiments.[5]

All our events were measured on a spiral reader, which, in addition to the coordinate measurements, gave track ionization density information useful in selecting among possible kinematic hypotheses. For each kinematic fit a bubble-density chi square (χ_B^2) was calculated. Fits to Reactions (1), (2b), and (3), which have one constraint, were considered only if their χ_B^2 was within 3 of the lowest χ_B^2 of all hypotheses tried, including missing-mass calculations.[6] The most probable reaction was defined to be that reaction with the lowest sum of kinematics χ^2 and bubble density χ^2.

Reaction (1).—To make comparisons with other bubble-chamber experiments more meaningful we feel that it is necessary to state the criteria used in selecting our sample of events for the study of Reaction (1). We required that (a) there be no four-constraint fit to the event with a confidence level greater than 10^{-5}; (b) Reaction (1) be the most probable reaction and have a kinematic confidence level of at least 10^{-3}; (c) in case of a final-state ionization ambiguity between the identities of the K^+ and the proton, the proton be taken to be that track with the lower momentum[7]; (d) the square of the four-momentum transferred between the target and final-state proton, $|t_{pp}|$, be less than 1.0 GeV^2.

The main ambiguities resulting from this selection involved the identities of the $\pi^{+,0}$ and $K^{+,0}$. Some events of Reaction (2b) were included in the sample of events of Reaction (1), whereas some events of Reaction (1) were excluded from our sample and called events of Reaction (2b). In addition, the identities of the π^+ and K^+ were interchanged in some of the events in our sample of Reaction (1). By taking events of Reaction (2a), ignoring the existence of the visible K^0 decay, and refitting them kinematically, we were able to estimate that 8% of the events in our sample of Reaction (1) came from Reaction (2b). By passing Monte Carlo-generated events[8] through our reconstruction and kinematic fitting programs, we found that for 10 to 20% of the events of Reaction (1) the wrong permutation of the identities of the final-state π^+ and K^+ was used.[9] We have examined the effects of these ambiguities and believe that they do not qualitatively affect the shapes of any of the distributions in our data.

We believe that the evidence presented for the production of the A_1 in Reaction (1) in the University of Rochester experiment at 12.8 GeV/c (Ref. 3) (hereafter referred to as UR) consists mainly of three observations as presented in their text and their figures: (i) the observation of a peak in the three-pion mass spectrum between 950 and 1150 MeV; (ii) the confinement of this peak to events selected from the periphery of the three-pion Dalitz plot and its further great enhancement when a ρ-meson selection is applied; (iii) the presence of a $\rho^\pm\pi^\mp$ signal and absence of a strong $\rho^0\pi^0$ signal in the vicinity of the A_1 and no apparent ρ signal of either type in the sum of two control regions, above and below the A_1. We consider each of these three topics in turn.

The three-pion mass spectrum for the events passing our selection criteria is shown in Fig. 1(a). We observe a broad plateau extending from 1.0 to 1.3 GeV, but no narrow peak at 1.0 GeV corresponding to that seen by UR.[10] Our results may be quantitatively compared with those of UR [Fig. 1(b) of Ref. 3; $|t_{pp}| \leq 1.0$ GeV^2] by evaluating the χ^2 sum[11]

$$\chi^2 \equiv \sum_i \frac{(A_iR-B_i)^2}{A_iR+B_iR},$$

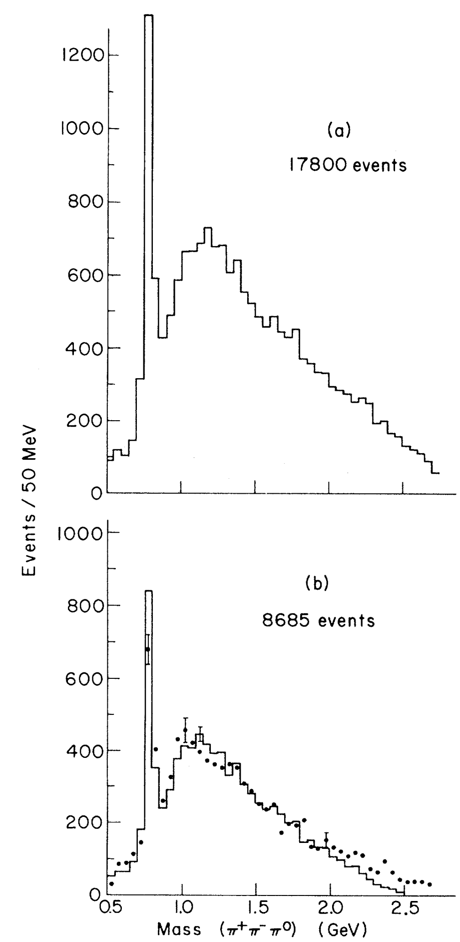

FIG. 1. Three-pion mass spectrum for events in Reaction (1) with (a) $|t_{pp}| \leq 1.0$ GeV^2, (b) $|t_{pp}| \leq 0.3$ GeV^2. The dots represent the data of Fig. 1(b) of Ref. 3 multiplied by 2.6.

where A_i is the number of events in the ith bin of the $M(3\pi)$ histogram of UR, and B_i is the number of events in the corresponding bin of the $M(3\pi)$ histogram for this experiment. The parameter R is the ratio of the total numbers of events in the two histograms. For the region $0.85 \leq M(3\pi) \leq 2.0$ GeV, $R = 4.15$ and $\chi^2 = 98.4$ for 22 degrees of freedom.[12] The disagreement comes from two regions: $0.9 \leq M(3\pi) \leq 1.1$ GeV and $1.65 \leq M(3\pi) \leq 2.0$ GeV. The discrepancy at higher mass is most likely the result of a scanning selection by UR against higher-momentum protons.[13]

To approximate better the conditions of UR, Fig. 1(b) shows the events of Fig. 1(a) with the restriction that $|t_{pp}| \leq 0.3$ GeV^2. The dots in Fig. 1(b) represent the data of UR multiplied by 2.6. The agreement between the two sets of data in the region $1.3 \leq M(3\pi) \leq 2.0$ GeV is striking. The disagreement beyond $M(3\pi) = 2.0$ GeV is probably because we excluded all our events with $|t_{pp}| > 0.3$ GeV^2 from Fig. 1(b). For the region $0.85 \leq M(3\pi) \leq 2.0$ GeV, $R = 2.59$, and $\chi^2 = 23.5$ for 22 degrees of freedom—a confidence level of 0.37. Even in the small region around the A_1, $0.9 \leq M(3\pi) \leq 1.2$ GeV, we have $R = 2.54$ and $\chi^2 = 7.18$ for five degrees of freedom—a confidence level of 0.21. This means that the UR data are in statistical agreement with an experimental distribution containing more than two and a half times the data with no peak at the A_1 mass. Note that our sample of events with $|t_{pp}| \leq 1.0$ GeV^2 [Fig. 1(a)] contains more than five times the number of events in the UR sample and also shows no peak at the mass of the A_1.

Selection by UR of events from the periphery of the 3π Dalitz plot[14] resulted in enhanced production of $\pi^+\pi^-\pi^0$ states in the vicinity of the A_1. The corresponding graph for our data is shown in Fig. 2(a) ($\lambda \leq 0.006$) where a 250-MeV-wide plateau is observed from 950 to 1200 MeV. Selection by UR of events from the center of the Dalitz plot ($\lambda \geq 0.0065$) showed no excess of events around the A_1. We see a similar effect in our data (not shown). These observations can be understood by considering the effect of any ρ production present on the separation of events into those in the center and those on the periphery of the Dalitz plot. Figure 2(c) shows a normalized Dalitz plot on which the locations of the contour $\lambda = 0.006$ and the $\rho^\pm$ bands [$0.67 \leq M(2\pi) \leq 0.86$ GeV] for $M(3\pi) = 1.0$ GeV have been drawn. As $M(3\pi)$ increases, the $\rho^\pm$ bands decrease in width and move across the plot in the direction shown by the arrows. If we assume that the fraction of

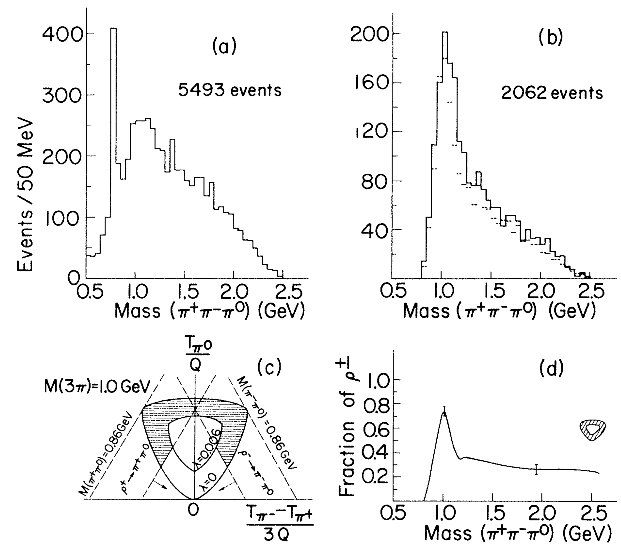

FIG. 2. (a) Three-pion mass spectrum for those events of Fig. 1(b) with $\lambda \leq 0.006$. (b) Three-pion mass spectrum for those events of (a) in which at least one ($\pi^\pm\pi^0$) combination has a mass between 0.67 and 0.86 GeV. The dashed lines are a prediction assuming a completely uniform Dalitz plot (see text). (c) Normalized Dalitz plot showing the contour $\lambda = 0.006$ and the position of the $\rho^\pm$ bands [$0.67 \leq M(\pi^\pm\pi^0) \leq 0.86$ GeV] for $M(3\pi) = 1$ GeV. The arrows show the directions in which the respective ρ bands move for increasing $M(3\pi)$. (d) The RPD—the fraction of the periphery of the Dalitz plot ($\lambda \leq 0.006$) occupied by the $\rho^\pm$ bands as a function of $M(3\pi)$.

ρ in the final state is reasonably constant from 900 to 1400 MeV, then selecting events from the periphery of the Dalitz plot will cause an enhancement in the region $M(3\pi)$ when the ρ lies mainly in the periphery of the Dalitz plot, i.e., below ~1200 MeV. Similarly, when selecting events from the center of the Dalitz plot we should not see any enhancement below the point where there is appreciable ρ in this area of the plot, i.e., below ~1200 MeV.

The $\pi^+\pi^-\pi^0$ peak at the A_1 observed by UR in the periphery of the Dalitz plot was seen by them to be greatly enhanced when the events were restricted to those consistent with coming from a ($\rho^\pm\pi^\mp$) intermediate state. This was taken to be further evidence for A_1 production, although it was stated that the enhancement might be due in part to a kinematic effect. Figure 2(b) shows those events of Fig. 2(a) consistent with the decay of at least one charged ρ [$0.67 \leq M(\pi^\mp\pi^0) \leq 0.86$ GeV]. The enhancement at 1.0 GeV is striking and looks very similar to the one obtained by UR. We believe that this enhancement can be understood as being entirely due to kinematics and therefore no resonance interpretation is necessary.[15] To show this we define the rho probability distribution (RPD) as the fraction of the outer Dalitz plot ($\lambda \leq 0.006$) occupied by the $\rho^{\pm}$ bands [i.e., for $M(3\pi) = 1$ GeV the value of the RPD is the fraction of the outer Dalitz plot in Fig. 2(c) occupied by the hatched area]. Figure 2(d) shows the RPD as a function of $M(3\pi)$. A prominent enhancement is seen in the region around 1 GeV. Thus, even a flat three-pion spectrum always appears to have a $(\rho\pi)$ enhancement in the outer Dalitz plot at about 1 GeV. The result of folding the RPD into the events in Fig. 2(a) is shown by the dashed lines in Fig. 2(b). The shape of the experimental enhancement is reproduced closely; the absolute normalization is low by only about 15%. It should be kept in mind that the dashed lines are the result of assuming a uniform Dalitz-plot population (i.e., no ρ production), and so agreement to within 15% is remarkable.

We now consider the two-pion spectrum as a function of three-pion mass for events with $|t_{pp}| \leq 1$ GeV^2. Following UR, we define regions below, in, and above the A_1 as 840 to 940, 940 to 1140, and 1140 to 1240 MeV, respectively. Figures 3(a) and 3(b) show the $(\pi^+\pi^-)$ and $(\pi^{\pm}\pi^0)$ signals for events in Fig. 1(a) occurring in the lower region. No obvious evidence for the presence of the ρ is seen or expected because of lack of phase space. Figures 3(c)-3(f) show these distributions for events in and above the A_1 region. The existence of the ρ signal is apparent in the upper control region, and its strength is comparable with that in the A_1 region.[16] The ratio of charged to neutral ρ appears to be the same above the A_1 region as in the A_1 region, making it unnecessary to invoke the presence of the A_1 to explain the data.

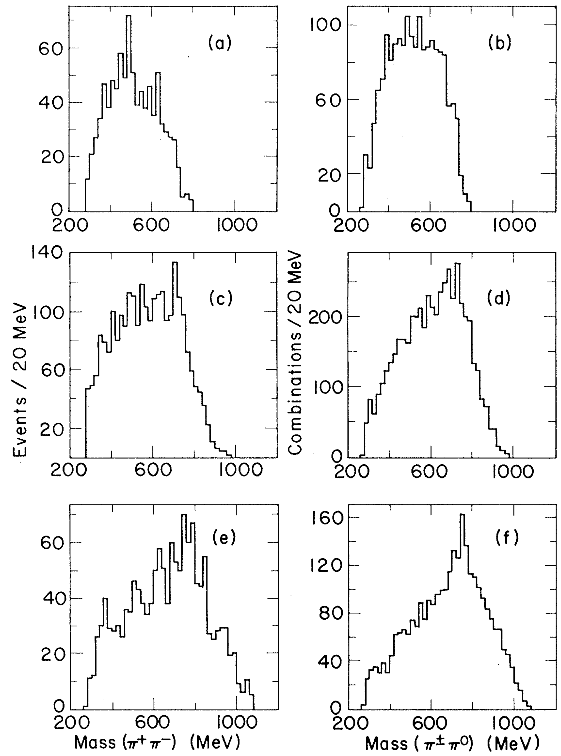

FIG. 3. (a), (c), (e) The $(\pi^+\pi^-)$ mass distribution for events of Fig. 1(a) below (900 events), in (2559 events), and above the A_1 region (1411 events), respectively. (b), (d), (f) The sum of the $(\pi^+\pi^0)$ and $(\pi^-\pi^0)$ mass distributions for events of Fig. 1(a) below, in, and above the A_1 region, respectively (two combinations per event).

Reactions (2a), (2b), and (3).—UR has 381 examples of Reaction (2a), and the 9.0-GeV/c K^+p experiment[4] has 456 events in Reaction (2a), 1475 in (2b), and 1141 in (3).

In our experiment, examples of Reactions (2b) and (3) were chosen from the four-pronged and V–four-pronged events, respectively, by criteria similar to those used in the study of Reaction (1). In addition, for Reaction (2b), we required that the missing mass squared be negative when the track fitted as a π^+, and with the greater momentum in the laboratory frame, was interpreted as a K^+. This removed background produced by the reaction $K^+p \to K^+p\pi^+\pi^-(n\pi^0)$, for $n > 1$, but did not change the shape of the distribution of interest, namely $M(\pi^+\pi^+\pi^-)$. Reaction (2a) comes from seven-constraint fits to the V–four-pronged events and is unambiguous.

Imposing the requirements that $|t_{pp}| \leq 1.0$ GeV^2 leaves us with 1454 examples of Reaction (2a), 5431 examples of Reaction (2b), and 2647 examples of Reaction (3). The relevant three-pion mass spectra are shown in Fig. 4. There is no evidence of A_1 production in these data, especially after we have removed from Reaction (3) events where one of the π^+ is consistent with coming from the decay $K^{*+}(890) \to K^0\pi^+$, a competing reaction (see hatched graphs in Fig. 4).

In summary, we conclude that there is evidence in our experiment against the production of the A_1 in Reaction (1) with a cross section greater than 5 μb, and that there is no convincing evidence for its production in the data presented by UR, where a production cross section of 40 μb was quoted. There is also no evidence in our experiment for A_1 production in Reactions (2a), (2b), or (3) with a cross section greater than 5 μb.

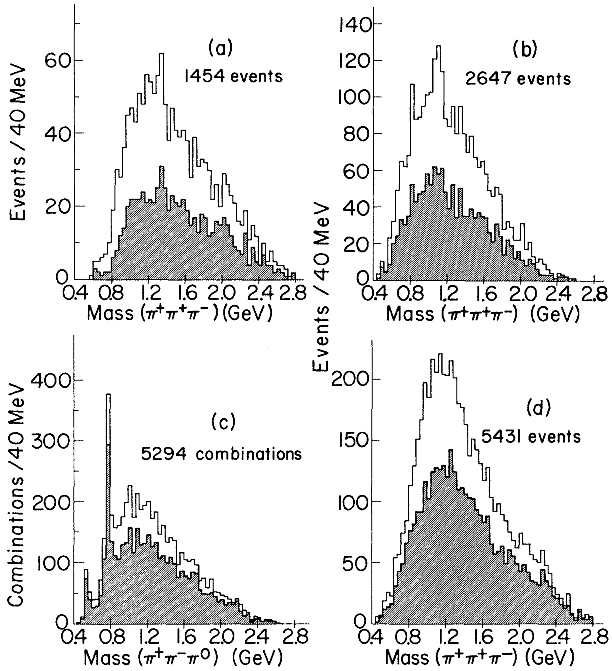

FIG. 4. Three-pion mass spectra for events with $|t_{pp}| \leq 1.0$ GeV^2. The hatched events are those with a competing $K^{*+}(890)$ removed. (a) $M(\pi^+\pi^+\pi^-)$ from Reaction (2a). (b) $M(\pi^+\pi^+\pi^-)$ from Reaction (3). (c) $M(\pi^+\pi^-\pi^0)$ from Reaction (3), two combinations per event. (d) $M(\pi^+\pi^+\pi^-)$ from Reaction (2b).

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†Visiting from Laboratoire Interuniversitaire Belge des Hautes-Energies, Brussels, Belgium.

[1]See, for example, B. French, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, September 1968* (CERN Scientific Information Service, Geneva, Switzerland, 1968).

[2]R. T. Deck, Phys. Rev. Letters 13, 169 (1964); E. L. Berger, Phys. Rev. 166, 1525 (1968). Recently, however, duality has been invoked to suggest that t-channel mechanisms such as these are just alternative descriptions of resonance states. See, for example, R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968); G. F. Chew and A. Pignotti, Phys. Rev. Letters 20, 1078 (1968).

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[5]600 000 pictures were taken in the Stanford Linear Accelerator Center 82-in. hydrogen bubble chamber exposed to an rf-separated, 12-GeV/c K^+ beam. For a description of the beam see S. Flatté, Lawrence Radiation Laboratory Group A Physics Note No. 746, 1966 (unpublished).

[6]The validity of this procedure was checked on the scanning table.

[7]Such events constitute only 6.5% of our sample. The general features of K^+p interactions at high energies make it very unlikely that this assignment will be incorrect.

[8]We have used the program PHONY [E. Burns and D. Drijard, Trilling-Goldhaber Technical Note No. 143, 1968 (unpublished)] to generate the events. The PHONY results were checked by comparing their predictions for the ω mass region with what was actually observed, and they were found to be good to a few per cent.

[9]The percentages increased from the lowest to the highest value given as a monotonic function of $M(3\pi)$. The total losses from misidentifications and measurement failures were estimated to be about 15% at the ω mass and about 23% at $M(3\pi) = 2$ GeV.

[10]By plotting the three-pion mass spectrum in finer bins, we are able to estimate our 3π mass resolution in the ω region as being about ±14 MeV.

[11]See, for instance H. Cramér, *Mathematical Methods of Statistics* (Princeton Univ., Princeton, N.J., 1946), p. 448, formula (30.6.2).

[12]We exclude the ω region because as a result of its narrow width small systematic differences or differences in resolution could cause large differences in the χ^2 value. We limit our comparison to the 3π mass region above 0.85 GeV.

[13]P. Slattery (University of Rochester), private communication. The $|t_{pp}| \leq 1.0$ GeV^2 cut mentioned by UR referred only to those events scanned for in an unbiased way. Two thirds of the data used for the sample of events of Reaction (1) came from the measurement of events selected by requiring that a dark proton track be identified on the scanning table. The scanning efficiency for these events was mearly 100% for $|t_{pp}| \leq 0.3$ GeV^2 and decreased rapidly with increasing $|t_{pp}|$.

[14]The inner and outer regions of the Dalitz plot are delimited by a contour which contains 50% of the events resulting from the decay of a $J^P = 1^-$ system (i.e., a decay matrix element of the form $|\vec{P}_+ \times \vec{P}_-|$, where $\vec{P}_+$ and $\vec{P}_-$ are the three-momenta of the π^+ and the π^-, respectively, in the 3π c.m.) We define this contour using the parameter $\lambda = |(\vec{P}_+ \times \vec{P}_-)/(Q^2 + 2Qm)|^2$, where Q is the total kinetic energy of the three pions in their c.m. and m is the sum of the masses of the three pions. This definition of λ is an energy-invariant generalization of the definition given in Ref. 3 and is also the one

used by UR (private communication from P. Slattery). The value of λ used is 0.006 (UR used 0.0065).

[15]The technique we use is similar to that used in studying the kinematical origin of the H meson. See, for example, A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. 40, 77 (1968); S.-Y. Fung, W. Jackson, R. T. Pu, D. Brown, and G. Gidal, Phys. Rev. Letters 21, 47 (1968).

[16]Adding the control regions together causes the ρ signal to be obscured. This explains the absence of any indication of the ρ signal in Figs. 3(b) and 3(d) of Ref. 3.

ERRATA

ac-FIELD–INDUCED GRANDJEAN PLANE TEXTURE IN MIXTURES OF ROOM-TEMPERATURE NEMATICS AND CHOLESTERICS. W. Haas, J. Adams, and J. B. Flannery [Phys. Rev. Letters 24, 577 (1970)].

Reference 6 should read, "W. Haas and J. Adams, J. Electrochem. Soc. 116, C297 (1969)."

DYNAMIC SCALING THEORY FOR ANISOTROPIC MAGNETIC SYSTEMS. E. Riedel and F. Wegner [Phys. Rev. Letters 24, 730 (1970)].

Part of a crucial sentence has been omitted in the course of printing. Following Eq. (11), insert "In the critical regime, $\tau \ll q^{1/\nu} \ll \Delta^{1/\varphi}$, on the ⋯."