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FREQUENCY OF PULSAR STARQUAKES*

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A relationship between the frequency of the occurrence of starquakes on pulsars and the shear strength of the solid crust of these objects is derived assuming various mechanisms of damping of their rates of rotation. Tentative conclusions concerning the shear strength, the age, and the composition of the Crab and the Vela pulsars are made.

Several models[1-4] have been proposed to explain the sudden increases in 1969 in frequency[5-8] of pulsars NP0532 (Crab) and PSR0833 (Vela). In this note only Ruderman's model, in which it is assumed that the spinup is a result of a sudden adjustment of the shape of the solid crust of the star, i.e., a starquake, will be discussed. These periodic adjustments relieve strains which build up in the crust as the pulsar's angular velocity slowly decreases. There are two (approximate) equations[2] which govern these phenomena:

$$\varphi \simeq (7R^3/8GM)(\omega_i^2-\omega^2)\sin^2\theta, \tag{1}$$

$$\frac{\Delta R}{R} \simeq \frac{95\mu\varphi_m R}{7GM\rho}\left[1-\left(\frac{R_i}{R}\right)^7\right](1-3\cos^2\theta), \tag{2}$$

in which R and M are the radius and mass of the pulsar, R_i is the inner radius of the crust, G is gravitational constant, φ is the angular shearing strain on the surface of the crust produced when the initial angular velocity ω_i gradually drops to ω, μ is the shear modulus, φ_m is the maximum elastic shear angle, and θ is the latitude measured from the equator. For the Crab and Vela pulsars, the best estimated values for R are 14.6 and 17.4 km, those for M are $0.30M_\odot$ and $0.21M_\odot$ where $M_\odot$ is solar mass; the central densities are $(3.2 \text{ and } 2.45)\times 10^{14}$ g cm^{-3}, and the present angular velocities are 190 and 70 sec^{-1}, respectively. These values are obtained from a variety of considerations[9,10] and the Cameron-Tsuruta[11] equation of state. The inner radius for the Vela[9] pulsar is $R_i = 8.4$ km; for the Crab pulsar this radius is unknown but one expects it to be much smaller. The shear modulus of a Coulomb lattice, i.e., nuclei of charge Z embedded in a relativistic degenerate electron Fermi sea, can be obtained[12] from the approximate relationship $\mu \sim (Ze)^2 m^{4/3}$, where m is the numerical density of nuclei, giving[2] (for Z between 30 and 50) $\mu = 10^{30}$ dyn cm^{-2}. As the angular velocity of the pulsar drops, the surface strains build up, according to Eq. (1), until they reach φ_m. Then, according to Eq. (2), a sudden change of shape occurs which leads to an increase in angular velocity given by $\Delta\omega/\omega = 2\Delta R/R$ for $\theta = \frac{1}{2}\pi$. The observed $\Delta\omega/\omega$ indicate that only a very small fraction of the total surface strain φ_m is removed suddenly during the quake, which suggests that the rest of the strain is relieved gradually through plastic deformation of the solid crust. Such deformation is known to account very well for the continental drift on Earth which is of the order of a few cm per year. This gradual relief of strain will affect the $\omega(t)$ curve[13] and superimpose on the effect of the various damping mechanisms between the core and the crust.[14] It will be assumed here that the gradual relief of the total long-range strain does not affect the interval τ between quakes.

Clearly, the least certain quantity in the above equations is φ_m. Simple theoretical calculations[15] based on chemically and crystallographically perfect crystals of "terrestrial" metals lead to values between 10^{-1} and 10^{-2}. Unfortunately, φ_m is a "structurally sensitive" quantity and, in contrast to the shear modulus μ, its value is radically changed by going from an ideal crystal to a real polycrystal which has chemical impurities, lattice defects, etc. One would expect the pulsar crust to be "impure" in the sense that there may be a whole range of Z values rather than just one, as a result of partial burning and incomplete mixing[16,17], and one would also expect it to contain a host of defects such as grain boundaries, dislocations, etc., produced during solidification and during prior deformations. Since at present there is no reliable theory which would permit calculating φ_m of real polycrystals, it is best to use an analogon of a Coulombic lattice such as alkali metals in which the ionic cores are far apart (unlike, for instance, in Cu) and in which the wave functions of the "free" electrons are close to plane waves. For these metals φ_m is[18,19] typically 10^{-4}. In more imperfect metals it may go down to 10^{-5} or so. An estimate of φ_m for the pulsar crust can also be made using Eq. (2) and putting the observed $\Delta\omega/\omega$ equal to 10^{-8} for the Crab and 2×10^{-6} for the Vela pulsar. The result is (1 to 3)$\times 10^{-4}$. Inserting $\varphi = \varphi_m$ in Eq. (1), and assuming $\omega_i \gg \omega$, gives minimum values of ω_i which are required to produce a quake. Table I lists these minimum angular velocities for various values of φ_m. It is usually assumed that the gradual drop of ω can be described by

$$d\omega/dt = K\omega^n, \tag{3}$$

where $n=1$ for an exponential decay (for which no theoretical model exists), $n=3$ for damping due to magnetic radiation, and $n=5$ for damping due to gravitational radiation.[20] An extrapolation of the present ω_0 of the Crab pulsar, using Eq. (3) with $n=3$, to the instant of its formation over 900 yr ago gives the initial angular velocity[20] of about 370 as compared with the present $\omega_0 = 200$. Thus φ_m has to be smaller than 10^{-2}. Since there were probably other quakes of the Crab pulsar besides that observed in 1969, one concludes that $\varphi_m < 5\times 10^{-3}$. All one can say about the Vela pulsar, with its present $\omega_0 = 70$, is that $\varphi_m = 10^{-3}$ would probably account for the quake observed in 1969 but would exclude any subsequent quakes. Thus an upper limit of $\varphi_m \sim 10^{-4}$ seems indicated in both cases.

The model here discussed permits calculating the probable time τ until the next quake by substituting in Eq. (1) various values of $\varphi = \varphi_m$ (for $\theta = \frac{1}{2}\pi$), putting ω_i equal to the present angular velocity ω_0, and using Eq. (3) for the rate of decay of the angular velocity:

$$\tau = \gamma_1 \ln(1-\varphi_m/c\omega_0^2)^{-1} \quad \text{for } n=1,$$
$$\tau = \gamma_3 \varphi_m \omega_0^{-2}(c\omega_0^2 - \varphi_m)^{-1} \quad \text{for } n=3,$$
$$\tau \simeq \gamma_5 \varphi_m / c\omega_0^6 \quad \text{for } n=5, \tag{4}$$

where $\gamma_n = \omega_0^n(2d\omega/dt)^{-1}$, ω_0 is the present angular velocity, and $c = 7R^3/8GM$. [In the notation of Ref. 19, $\gamma_3 = \alpha^{-1}$ and $\gamma_5 = \beta^{-1}$.] It is interesting to note that for the present ω_0 and for small φ_m, i.e., in the range considered in this note,

$$\tau \simeq \varphi_m[2c\omega_0 d\omega/dt]^{-1}, \tag{5}$$

which is independent of n and thus independent of the nature of the proposed mechanism of the slowing down of the pulsar. Eventually when ω drops to very low values Eq. (5) becomes invalid.

The coefficient of φ_m in Eq. (5) is equal to 1.5×10^7 yr for the Vela pulsar and 5.5×10^5 yr for the Crab pulsar. It follows that, if φ_m of both crusts is the same, then quakes on the Vela pulsar are 30 times rarer than on the Crab pulsar and that a coincidence such as observed in 1969 is a most exceptional occurrence (the probability of both quakes occurring within one year is $1.34\times 10^{-13}\times \varphi_m^{-2}$). An alternate possibility is that φ_m of the Vela pulsar crust is, say, an order of magnitude lower than for the Crab pulsar crust. This agrees with other arguments suggesting that the Vela pulsar is a much older object than the Crab pulsar and that it has undergone many more quakes which produced a high concentration of lattice defects. Clearly, this implies also that φ_m decreases with time and that quakes will occur with an increasing probability. From the ratio of the values of $\Delta\omega/\omega$ and of φ_m for the two pulsars one can, in principle, estimate the ratio of the shear moduli μ and thus of the mean values of Z. If φ_m for the Crab pulsar is indeed higher than for the Vela pulsar then it is likely that Z is lower in the former. This may have interesting consequences concerning the origin and history of these objects.

Although only one quake on each of the two pulsars was observed, it is tempting to speculate that they occur not more frequently than once every few years and not less frequently than, say, once every 100 yr. Assuming these two limits and assuming that the numerical values of R, M, etc. are correct, one concludes that the limits for φ_m for the Vela pulsar are

Table I. Minimum angular velocities (in $\sec^{-1}$) ω_i for the occurrence of pulsar quakes as a function of the maximum elastic shear φ_m.

φ_m	Crab	Vela
10^{-2}	385	250
10^{-3}	122	78
10^{-4}	38.5	25
10^{-5}	12.2	7.8
10^{-6}	3.9	2.5

10^{-7} to 10^{-5}, and for the Crab pulsar 10^{-6} to 10^{-4}. Clearly, a change in the value of M and especially of R would affect strongly these limits. It should be added that the calculated oblateness of pulsars is very small so that the sudden adjustments in the shape are of the order of a centimeter for the Vela and tenths of a millimeter for the Crab pulsar. Thus it is hard to believe that the strength of the crust or the frequency of its quakes is seriously affected by details of its external and internal shapes.

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EVIDENCE AGAINST A_1 PRODUCTION IN HIGH-ENERGY K^+p INTERACTIONS*

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The production of the A_1 has been reported at 12.8 GeV/c in the reaction $K^+p \to Kp(3\pi)$. It has also been reported at 9.0 GeV/c in the reactions $K^+p \to Kp(3\pi)$ and $K^+p \to Kp(4\pi)$. At 12.0 GeV/c, with five times the data of the 12.8-GeV/c experiment and three times the data of the 9.0-GeV/c experiment, we see no evidence for A_1 production in any of these reactions.

The A_1 enhancement[1] has been seen mainly in the reaction $\pi^\pm p \to A_1^\pm p$, $A_1^\pm \to \rho^0\pi^\pm$, $\rho^0 \to \pi^+\pi^-$, where its interpretation as a resonance has been questioned because Deck or other diffraction processes may be present.[2] In K^+p interactions, where A_1 simulating effects of this type are probably not so prominent, the observation of the A_1 would greatly favor the resonance interpretation independently of the concept of duality.[2]

Recently, observations of the A_1 have been reported in the reactions

$$K^+p \to K^+p\pi^+\pi^-(\pi^0) \tag{1}$$

(Berlingieri et al.[3]),

$$K^+p \to K^0p\pi^+\pi^+\pi^- \tag{2a}$$

(Ref. 3 and Alexander, Firestone, and Goldhaber[4]),

$$K^+p \to (K^0)p\pi^+\pi^+\pi^- \tag{2b}$$

(Ref. 4), and

$$K^+p \to K^0p\pi^+\pi^+\pi^-(\pi^0) \tag{3}$$

(Ref. 4), in bubble-chamber experiments at 12.8[3] and 9.0[4] GeV/c (the brackets indicate a particle not detected). In our experiment at 12.0 GeV/c, we have studied these reactions with a path length corresponding to about 35 events/μb—at least three times as great as in either of the other experiments.[5]

All our events were measured on a spiral reader, which, in addition to the coordinate measure-