EFFECT OF CORE POLARIZATION IN THE REACTION ^{209}Bi(p,p')^{209}Bi(1.61 MeV) AT 39.5 MeV*

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The cross section for the $1h_{9/2}$-$1i_{13/2}$ single-proton transition in the reaction ^{209}Bi(p, p')^{209}Bi* at 39.5 MeV has been measured. It is shown that most of the observed cross section is due to admixtures of the state formed by coupling the valence proton of ^{209}Bi to the highly collective 3^- state of ^{208}Pb at 2.614 MeV. An estimate of core polarization, based on the work of Kuo and Brown, grossly underestimates the $L=3$ component of the cross section.

In ^{209}Bi there are two low-lying $\frac{13}{2}^+$ states of very different character. The 1.609-MeV level lies near the expected position of a single proton outside a closed ^{208}Pb core and shows up strongly in ^{208}Pb(^{3}He,d)^{209}Bi.[1] Some (^{3}He,d) strength is observed in the $\frac{13}{2}^+$ state at 2.602 MeV. Both of these states decay to the ground state by strong $E3$ transitions.[2,3] In the simplest picture the ground state is a $1h_{9/2}$ proton outside a closed ^{208}Pb core, and the 1.609-MeV state is a $1i_{13/2}$ proton outside the same core, whereas the 2.602-MeV state is a member of a weak-coupling septet formed by coupling the $1h_{9/2}$ single particle to the strongly collective 3^- state of ^{208}Pb at 2.614 MeV. A perturbation calculation by Mottelson[4] has shown that the two $\frac{13}{2}^+$ states are mixed. The admixture of the 2.602-MeV state into the 1.609-MeV state is $\epsilon^2=4.8\times10^{-2}$. In this calculation the coupling matrix element was estimated from the γ decay of the 3^- state of ^{208}Pb. The mixing of the states accounts for the observed (^{3}He,d) strengths.

In this Letter new experimental data on the 1.609-MeV, $\frac{13}{2}^+$ state in ^{209}Bi are presented. The differential cross section for the excitation of this level in the (p,p') reaction was obtained using 39.5-MeV protons from the Michigan State University cyclotron and a self-supporting Bi foil. The scattered particles were detected in a lithium-drifted germanium detector in a side-entry configuration. The energy resolution was typically about 50 keV overall, and the peak-to-valley ratio was over 2000 to 1. Figure 1, a spectrum obtained at 35°, illustrates the difficulties of seeing the relatively weak single-particle states, which are shown underlined.

Recent progress has been made in understanding the (p,p') reaction in terms of "realistic forces." The main features of the differential cross sections for several transitions in the reactions ^{12}C(p,p') and ^{40}Ca(p,p') at incident energies from 25 to 55 MeV can be reproduced in local distorted wave calculations using Kallio-Kolltveit (K-K) force[5] as the projectile-target interaction with an approximate treatment of antisymmetrization.[6,7] The K-K force is a good approximation to the central part of the shell-model reaction matrix. This same approach has been successful in describing the inelastic proton scattering from low-lying states in ^{50}Ti, ^{89}Y, and ^{90}Zr.[7,8] Core polarization, which is important in these transitions, has been estimated by including 2p-1h (two-particle, one-hole) or 3p-1h components, whichever is appropriate, in the target wave functions. These components are calculated using first-order perturbation theory, and only those states formed by coupling the valence nucleon to particle-hole excitations of the core with energies up to $2\hbar\omega$ are considered. This is essentially the approach first used by Horie and Arima to calculate effective charges.[9] The same picture is used by Kuo and Brown in their work on the bound state problem.[10]

Kuo has suggested[11] that the particle-hole treatment of core polarization may not be adequate when there is the possibility of contributions from highly collective phonons of the core. This appears to be the case for this transition. Because of this we calculate the cross section in two ways: (1) including only 2p-1h components

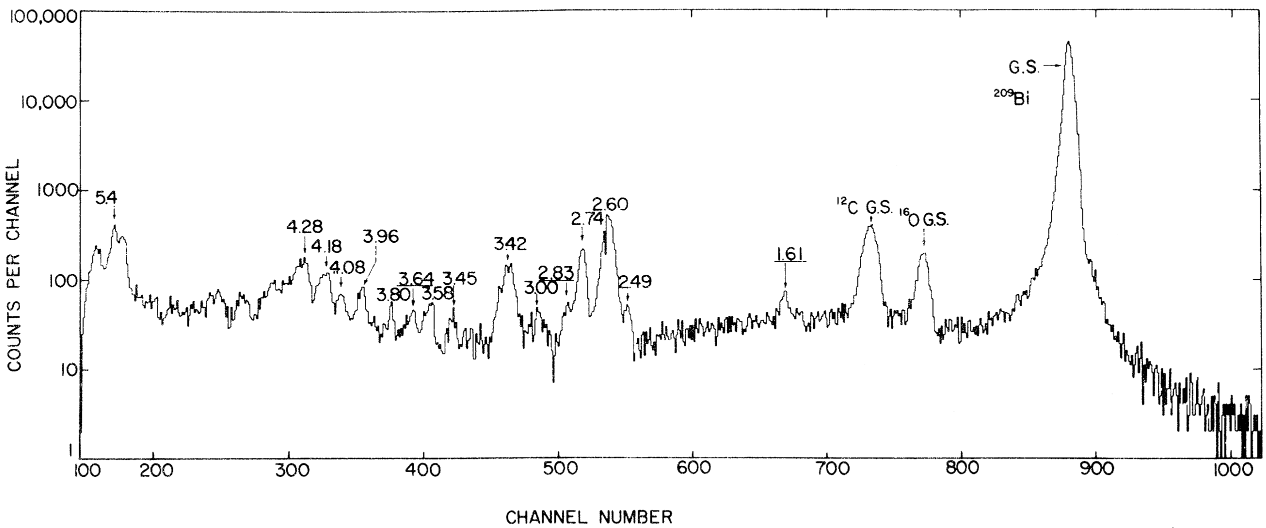

FIG. 1. Spectrum of protons from the reaction ^{209}Bi$(p,p')^{209}$Bi taken at 35° in the lab. The resolution is about 45 keV. The peaks shown underlined are single-particle states in ^{209}Bi.

in the wave functions, (2) replacing the components with p-h coupled to angular momentum $J_c = 3$ by components which contain the 3^- core state of ^{208}Pb. In the latter calculation we use the macroscopic vibrational model to describe the core. The procedure is essentially that described by Mottelson in Ref. 4. The calculation could have been kept completely microscopic by using the random-phase approximation vector of Gillet to describe the 3^- state in ^{208}Pb.[12]

The wave functions which contain only the 2p-1h admixtures will be designated Set I. They are defined as follows:

$$|jm\rangle' = |jm\rangle + \sum A_j(j'(\mathrm{ph})J_c)|[j'(\mathrm{ph})J_c]jm\rangle, \quad (1)$$

where the sum is over j' and $\mathrm{ph}J_c$ and

$$A_j(j'(\mathrm{ph})J_c) = -(\Delta E)^{-1}\langle[j'(\mathrm{ph})J_c]j|G|j\rangle. \quad (2)$$

Here $|jm\rangle$ specifies the state of the valence proton in the presence of a closed core. The quantity $(\mathrm{ph})J_c$ refers to a particle-hole state of the core with angular momentum J_c and excitation energy ΔE. This is coupled to a valence proton in the state j' to give j. G denotes the coupling interaction which is taken to be the K-K force. Particle-hole pairs are formed from the shells shown in Table I. Harmonic-oscillator wave functions have been used, and the energy denominators were taken in part from experiment[13] and in part from the Nilsson scheme at zero deformation. The size parameter $\hbar\omega$ is 6.8 MeV.

The wave functions used in the second calculation will be designated Set II. They are the same as Set I except for the replacement

$$\sum_{\mathrm{ph}} A_j(j'(\mathrm{ph})J_c = 3)\,|[j'(\mathrm{ph})J_c = 3]jm\rangle \to B_j(j')\,|j'\otimes 3^-; jm\rangle, \quad (3)$$

$$B_j(j') = (E_j - E_{j'} - \hbar\omega_3)^{-1}\langle j'\otimes 3^-; j|V|j\rangle, \quad (4)$$

$$\langle j'\otimes 3^-; j|V|j\rangle \sim \langle k\rangle(\hbar\omega_3/2C_3)^{1/2}\langle j'\|Y_c\|j\rangle, \quad (5)$$

$$V = -k(r)\sum_{LM}\alpha_{LM}Y_{LM}(\hat{r}). \quad (6)$$

Equations (3)-(6) are the usual expressions encountered when the macroscopic vibrational model is used in the treatment of particle-vibrational coupling. The quantity $k(r) = RdU(r)/dr$, where $U(r)$ is the single-particle potential seen by the extra-core proton, R specifies the nuclear radius, $\langle k\rangle$ denotes a radial integral, $\hbar\omega_3$ is the ex-

Table I. Particle and hole orbitals used in microscopic calculation. The absence of total angular-momentum subscript indicates that both $j = l \pm \frac{1}{2}$ orbits are included.

Particles		Holes	
Protons	Neutrons	Protons	Neutrons
$1h_{9/2}$	$1i_{11/2}$	$1d$	$1f$
$2f$	$2g$	$2s$	$2p$
$3p$	$3d$	$1f$	$1g$
$1i$	$4s$	$2p$	$2d$
$2g$	$1j$	$1g$	$3s$
$3d$	$2h$	$2d$	$1h$
$4s$	$3f_{7/2}$	$3s$	$2f$
$1j_{15/2}$		$1h_{11/2}$	$3p$
$2h_{11/2}$			$1i_{13/2}$

citation energy of the 3^- phonon of ^{208}Pb, and C_3 gives a measure of the core stiffness to this lowest octupole vibration. Reference 4 gives $\langle k \rangle = 60$ MeV and $C_3 = 649$ MeV. Analyses of the reaction $^{208}\mathrm{Pb}(p, p')^{208}\mathrm{Pb}$ give $\beta_3 \sim 0.13$ for this state[14] which is the only state with a large value of β in ^{208}Pb. The relation $\beta_3 = 7^{1/2}(\hbar\omega_3/2C_3)^{1/2}$ implies $C_3 = 543$ MeV which is smaller than the value from Ref. 4 and corresponds to an admixture $\epsilon^2 = 5.5\times10^{-2}$ of the 2.602-MeV, $\frac{13}{2}^+$ state in the 1.609-MeV, $\frac{13}{2}^+$ state. The smaller value of C_3 was used in this work.

The cross section for this transition has 20 components each designated by (LSJ) referring to orbital, spin, and total angular momentum transfer. Details for calculating the cross section from the wave functions being considered in this work are given in Ref. 8. In this work, as a matter of convenience, we have used a pseudopotential for the projectile-target interaction which is known to give results consistent with those obtained using the K-K force and treating antisymmetrization approximately. The 2p-1h components of the cross section have been included only in the $S=0$ terms in the cross section because it is only in these components that they add coherently. In using wave function Set II the components of the wave functions defined by Eqs. (3)-(6) contribute only to the $(LSJ) = (303)$ component of the cross section. The remaining 19 components are the same in Sets I and II.

Figure 2 shows the total differential cross sections obtained with wave function Set I and Set II. The (303) components are also shown for both cases. The differential cross section (II) gives a good fit to the experimental data. The (303) (II) component is dominant at forward angles. The enhancement due to core polarization, i.e., ratio of integrated cross sections with and without core polarization, of (303) (II) is about 200. Because of this large enhancement the valence contribution to (303) (II) is small. Considering only this component and neglecting the valence contribution, the data place an upper limit on $\epsilon^2 = 10^{-1}$. Wave function Set II gives $B(E3) = 2.4\times10^{-2}e^2$ b^3 which is slightly larger than the experimental values $(1.3\text{-}2.0)\times10^{-2}e^2$ b^3,[2,3] obtained from Coulomb excitation measurements.

The particle-hole model fails to reproduce the effect of the 3^- phonon of ^{208}Pb. The enhancement of (303) (I) is about 13 which is an order of magnitude smaller than the value obtained for (303) (II). This model predicts that many components make important contributions to the total differential cross section. In particular, (303) (I) is comparable in magnitude with (112) which involves the lowest allowed L and J transfers. As the lowest J transfer is highly favored in γ transitions, the particle-hole model predicts that the 1.609-MeV, $\frac{13}{2}^+$ state will decay to the ground state predominantly by an $M2$ transition which is in contradiction to experiment. In a previous analysis of the reaction $^{89}\mathrm{Y}(p, p')^{89}\mathrm{Y}$(0.908 MeV),[8] which involves a single proton going from the $2p_{1/2}$ to the $1g_{9/2}$ level, the particle-hole model gave a good fit to the experimental angular distribution and predicted appreciable contributions from both the (314) and (505) components. Here the levels are known to be connected by an $M4$ γ transition.[15] In this case there are no strongly collective core states contributing because ^{88}Sr has no strong low-lying 5^- state.

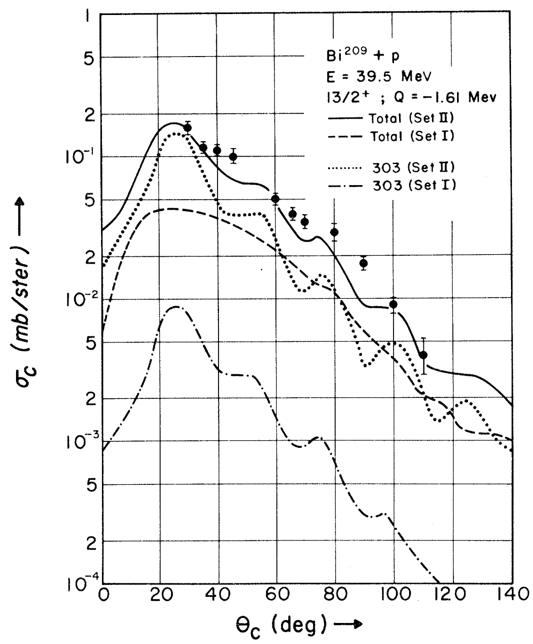

FIG. 2. The experimental data compared with the theoretical results obtained with both sets of wave functions. The total differential cross sections and the (303) component are shown for both cases.

It is concluded that highly collective core phonons can play an extremely important part in the core polarization process, and that care must be exercised in applying the uncorrelated particle-hole model for core polarization.

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EFFECT OF PAIRING CORRELATIONS ON ^{22}Ne SPECTRUM

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The effects of pairing correlations on the ground-state band of ^{22}Ne are investigated. It is shown that the computed low-lying excited-state spectrum is significantly affected, whereas the $E2$ transition probabilities are almost unaffected by the pairing correlations.

It is generally found[1] that the small energy gap between the last occupied and the first unoccupied single-particle Hartree-Fock (HF) orbital in the even-even nucleus indicates the importance of pairing correlations in the nucleons. As a probable consequence of this, the energy spectrum projected from such an intrinsic HF state of the nucleus is quite compressed.[2] It should be mentioned in this connection that the fault is not with the projection method, the accuracy of which is found[3] to be quite good. The compressed nature of the projected energy spectra then suggests that the HF state, due to a small HF gap at the Fermi surface, cannot be trusted as a good intrinsic state of the nucleus. In view of this fact, the next step is to construct a better intrinsic state of the nucleus by including the pairing correlations between nucleons. Though the pairing correlations can be self-consistently included in the framework of Hartree-Fock-Bogoliubov (HFB) theory,[4] the projection formalism in conjunction with the intrinsic HFB state of the nucleus becomes complicated[5] because of number and angular momentum projection. It is nevertheless essential to investigate the effect of pairing correlations on the projected nuclear spectra. This is done for the first time and the results of such calculations in ^{22}Ne are presented in this note. It should be mentioned here that the effect of pairing correlations on the ground-state bands of sd-shell nuclei has been investigated recently[6] by performing HFB calculations in a model space of single sd shell by treating ^{16}O as an inert core. The HFB calculations[1,6] for ^{22}Ne show that the intrinsic mass quadrupole moment is only slightly changed, whereas the energy gap at the Fermi surface of neutrons is substantially increased by the onset of pairing correlations. These HFB calculations[1,6] only demonstrate the effect of pairing correlations on the gross properties such as the intrinsic deformation and the single-particle structure of the variational state of the nucleus. The motivation behind the present work is to investigate the effect of pairing correlations on