would be $L = 2\xi = 4R$. The value of κ depends[2] on the mean free path δ in the film. We can estimate δ for the films of GD from the value $\lambda(T) = 2800$ Å at $T_c/\Delta T = 28.5$, calculated from (2) using the measured depression[1] of the flux quantum in one of their cylinders.

Using standard values for the London penetration depth and coherence length in pure tin and the well-known formulas for dirty materials,[2] we can estimate $\delta \approx 430$ Å for their material. This value is rather low compared with values obtained by others[6] in evaporated tin, so we take it as a lower limit and calculate a maximum value $\kappa = 0.62$. Taking $R = 3\times10^{-3}$ cm and $t = 7\times10^{-5}$ cm we obtain $\Delta G_k/k_BT_c = 14\Delta H/H_0$. Thus we might expect nucleation in two different states over a range $\Delta H \sim 0.1\Phi_0/2\pi R^2$. However, this should only be taken as a very loose upper limit on the range over which fluctuations could be important. If the cylinder supercools before making the transition, ΔG_k could be much larger; and the nucleation region could well be longer than $4R$ by the time it finally closes a loop around the cylinder.

For two reasons we do not believe that the domain structure observed by GD is nucleated by the above thermodynamic fluctuation mechanism. First, GD observe domains over a range ΔH which seems excessively large in view of the above arguments. Second and more important, they observe that in ranges of H where domains are present, particular places on the cylinder are biased in favor of the high state or the low state for each succeeding n. Furthermore, GD obtain highly reproducible domain structures upon repeated cooling in constant H. We believe instead that nucleation of the domains observed by GD is governed by a local variation of ΔH along the cylinder which is probably due to ferromagnetic impurities in the glass tube which encases their cylinder. Thus for a range of H the local equilibrium state of the cylinder actually varies along the length between n and $n+1$. This means also that there is little pressure to move the vortices until the applied field is turned off, and we have shown that the vortices are strongly pinned well below the transition.

We can conclude then that while domains may be nucleated in a uniform applied field by thermodynamic fluctuations, the range of field over which this effect is important is certainly smaller than the range over which GD observe domains.

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THERMAL FLUCTUATIONS IN SUPERCONDUCTING WEAK LINKS*

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The current-voltage characteristics of superconducting weak links are measured near their transition temperature. The effect of thermal fluctuations on these characteristics is found to be in excellent agreement with a calculation of Ambegaokar and Halperin.

The effects of thermal fluctuations on the current-voltage characteristics of Josephson junctions have been the subject of several recent theoretical and experimental papers.[1-4] The most reliable experimental results have been reported by Anderson and Goldman[4] who measured the I-V characteristics of an essentially ideal Josephson tunnel junction at temperatures within a few millidegrees of the superconducting transition temperature. Their results are in fair qualitative agreement with the calculation of Ambegaokar and Halperin.[2] Quantitative comparison of the experimental and theoretical results was difficult, first because one of the theoretical assumptions was only marginally satisfied in the experiment, and second because there were indications that some external noise was present in the apparatus. In this Letter we report measurements of the current-voltage characteristics of superconducting weak links which avoid both of these difficulties and which are in very good quantitative agreement with the predictions of Ambegaokar and Halperin.

The calculation of Ambegaokar and Halperin is valid only in the limit that the product of the time constant of the junction and the Josephson plasma frequency approaches zero;

$$(RC)(2eI_1/\hbar C)^{1/2} \equiv \Omega \ll 1, \tag{1}$$

where R is the normal resistance of the junction near the critical temperature, C is the capacitance of the junction, and I_1 is the maximum value of the Josephson supercurrent. The problem with using the usual oxide-barrier tunnel junction to test the theoretical predictions is that this geometry results in such a large capacitance that expression (1) becomes difficult to satisfy experimentally. In order to avoid this difficulty, we have investigated the I-V characteristics of thin superconducting films containing a constriction at one point in the film constraining the current to pass through the narrow neck or bridge (such a device is usually referred to as a Dayem bridge[5] or as a weak link). The inequality in expression (1) is more easily satisfied experimentally because the geometry of these weak links greatly reduces the capacitance of the junction.

The calculations of Ambegaokar and Halperin, although derived for Josephson tunnel junctions, should remain valid for weak links since the only property of the tunnel junction essential to their results is a current-phase relation of the form $I = I_1 \sin\varphi$, where φ is the quantum phase difference across the junction. It is well known that the Josephson effect is a general phenomenon which appears whenever two superconductors are coupled together weakly by any mechanism and that, in general, the current-phase relation may be any odd, periodic function of the quantum phase difference across the junction.[6-8] It is only in the limit of I_1 going to zero that the current-phase relation necessarily takes the form assumed by Ambegaokar and Halperin ($I = I_1 \sin\varphi$), independent of the mechanism which couples the superconductors.[8] Fortunately, the final results of the experiments reported here are rather insensitive to the exact shape of this function, especially at temperatures where noise effects are large. This fact is discussed in more detail below.

The weak links are fabricated by first evaporating a thin film of Sn approximately 300 Å thick onto a glass substrate. The film is then scratched using an electroetched tungsten point so that it contains a small bridge of approximately 2 μ width (measured perpendicular to the current flow) and a length of 6 μ. In an attempt to minimize any external noise incident on the weak link, the cryostat was surrounded by a Mumetal shield, reducing ambient magnetic fields to less than 1×10^{-3} Oe. In addition, the weak link was mounted within an aluminum block which provided electromagnetic shielding. Low-pass filters, located in an adjacent compartment of the aluminum block, were inserted in series with all signal leads, reducing room-temperature thermal noise and rf interference propagating down the leads. Many small holes were drilled in the aluminum block allowing the weak links to be in direct thermal contact with the liquid-helium bath. The temperature of this bath was controlled to within $\pm2\times10^{-4}$ K by vapor-pressure regulation, and relative temperatures were determined with an accuracy of $\pm1\times10^{-3}$ K from the helium vapor pressure. Instead of directly measuring the I-V curve of the weak link, which would have been difficult because of the small voltages involved, the differential resistance dV/dI as a function of the current I was measured using a standard phase-sensitive detection technique at 200 Hz.[9] Numerical integration of the dV/dI versus I curves yielded the current-voltage characteristics of the weak links.

Ambegaokar and Halperin's final result for the time-average voltage $\overline{V}$ appearing across a junction, in terms of the reduced variables $x \equiv I/I_1$, $v = \overline{V}/I_1R$, and $\gamma = I_1\hbar/ekT_N$, is

$$v = \frac{4\pi}{\gamma}\left\{(e^{\pi\gamma x}-1)^{-1}\left[\int_0^{2\pi} d\varphi f(\varphi)\right]\left[\int_0^{2\pi} d\varphi' \frac{1}{f(\varphi')}\right] + \int_0^{2\pi} d\varphi \int_\varphi^{2\pi} d\varphi' \frac{f(\varphi)}{f(\varphi')}\right\}^{-1}, \tag{2}$$

where

$$f(\varphi) \equiv \exp[\tfrac{1}{2}\gamma(x\varphi + \cos\varphi)].$$

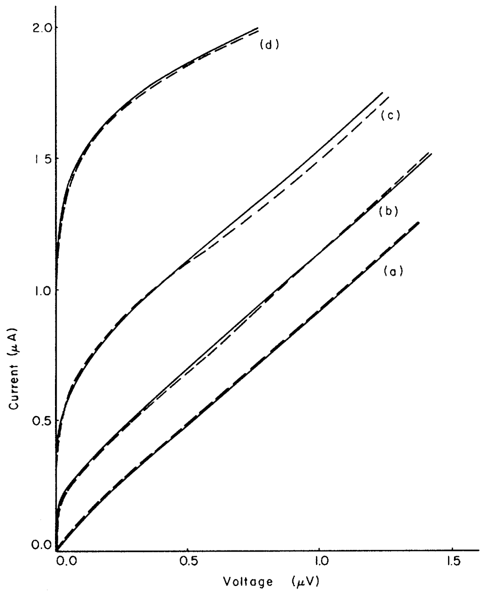

FIG. 1. Current-voltage characteristics of a weak link at several temperatures near T_c. The solid lines are experimental curves and the dashed curves are a one-parameter fit with the theory of Ambegaokar and Halperin. The relevant parameters are (*a*) $T = 3.873°K$, $\gamma = 2.0$, $I_1 = 0.16\ \mu A$; (*b*) $T = 3.869°K$, $\gamma = 6.2$, $I_1 = 0.50\ \mu A$; (*c*) $T = 3.866°K$, $\gamma = 13$, $I_1 = 1.05\ \mu A$; and (*d*) $T = 3.862°K$, $\gamma = 28$, $I_1 = 2.3\ \mu A$.

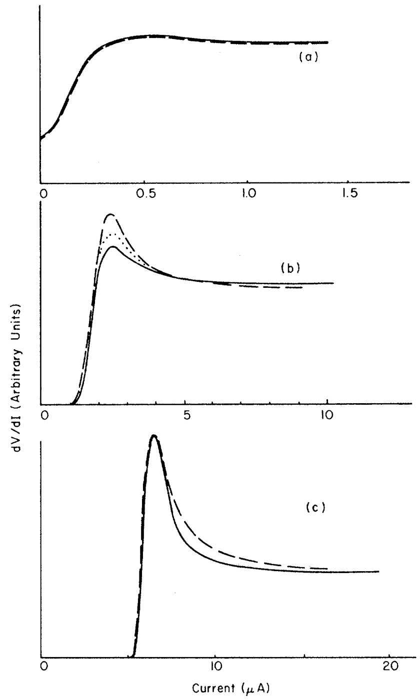

FIG. 2. Differential resistance-current characteristics of a weak link at several temperatures near T_c. The solid lines are experimental curves and the dashed curves are a one-parameter fit with the theory of Ambegaokar and Halperin. The dotted curve in (b) is a one-parameter fit using an alternative current-phase relation to describe the weak link (see text). The relevant parameters are (a) $T = 3.871°K$, $\gamma = 3.1$, $I_1 = 0.25\ \mu A$; (b) $T = 3.862°K$, $\gamma = 28$, $I_1 = 2.3\ \mu A$; and (c) $T = 3.854°K$, $\gamma = 80$, $I_1 = 6.50\ \mu A$.

Note that γ expresses the ratio of the coupling energy of the superconductors to the thermal energy. The concept of an effective noise temperature is explicitly introduced by writing the temperature as T_N. In order to make a comparison with the experimental dV/dI versus I curves, we implicitly differentiated Eq. (2) with respect to x prior to performing the necessary numerical integrations.

Figure 1 shows I-V curves for several values of T near T_c. The dashed lines represent a one-parameter fit by Eq. (2). The actual temperature of the bath was used as the effective noise temperature T_N, and the critical current I_1 was adjusted for the best fit over the range $0 < I < 2.5I_1$. The agreement between the experimental and theoretical curves is excellent. A more sensitive comparison can be make by considering the derivative of the I-V curves. Figure 2 shows representative dV/dI versus I curves for several values of T near T_c. The dashed lines represent the one-parameter fit by the derivative of Eq. (2). Again, we have used the actual temperature of the bath as the effective noise temperature T_N and adjusted I_1 for the best fit over the range $0 < I < 2.5I_1$. From the comparison in Fig. 2, and from other data not presented, several conclusions can be drawn. (1) For values of $\gamma \leq 10$, the agreement between theory and experiment is essentially perfect. Since this is the region where noise effects dominate the junction characteristics, we conclude that the noise contribution to the I-V curves of Josephson junctions in the limit of $\Omega \ll 1$ is completely described by the model of Ambegaokar and Halperin. (2) For intermediate values of γ ($15 < \gamma < 50$), the peak in the curves of dV/dI versus I consistently falls below the theoretical values. (3) For $\gamma > 80$, the calculations do not agree exactly with the data for currents $I_1 < I < 2I_1$. In particular, the value of dV/dI falls to its limiting value faster than the calculations indicate it should.

In an attempt to improve the fit between the experimental and theoretical results, we have numerically investigated the effect of assuming a current-phase relation other than $I = I_1 \sin\varphi$. One form which is particularly easy to treat is that of a sawtooth of either positive or negative slope. These current-phase relations can be integrated to obtain the free energy as a function of the phase which can then be inserted into the calculations of Ambegaokar and Halperin in the appropriate manner.[10] The results of numerical integration lead to the following conclusions: (1) For values of $\gamma < 10$, (i.e., for those temperatures where noise has the greatest effect on the I-V curves) the exact form of the current-phase relation produces little detectable change in either the calculated I-V or dV/dI-I curves. (2) For greater values of γ $(15 < \gamma < 50)$, both sawtooth current-phase relations yield dV/dI-I curves which have a smaller peak than the results using $I = I_1 \sin\varphi$, but still greater than observed experimentally. These numerical results are illustrated in Fig. 2(b) by the dotted curve. For the largest values of γ $(60 < \gamma < 100)$, the new dV/dI-I curves again become essentially identical in shape to those computed using $I = I_1 \sin\varphi$. We conclude that alternative current-phase relations do improve the agreement between the theoretical and experimental results, but some discrepancies still remain.

As mentioned earlier, the theoretical expression [Eq. (2)] is exactly valid for $\Omega \ll 1$. This criterion is satisfied for the weak links used in these experiments. By observing the hysteresis in the current-voltage characteristics at temperatures well below T_c and comparing the results with the calculation of McCumber,[11] we are able to estimate the capacitance of the weak links to be of order 0.3 pF. With this value of the capacitance, and the measured values of R and I_1, the largest value of Ω corresponding to the data in Fig. 2 is less than 0.1.

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