erator staff for their aid in performing the experiment, and to Professor C. Baltay for permission to use an unpublished datum. Mr. J. Barney and M. E. Behr constructed much of the apparatus. One of us (L.P.) wishes to thank Professor T. D. Lee, Professor N. Christ, and Professor L. Durand, III, for enlightening discussions on radiative corrections.

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HIGH-ENERGY *ep* INELASTIC SCATTERING IN A RENORMALIZABLE THEORY*

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We have studied the ep deep inelastic scattering in a neutral pseudoscalar-meson theory without imposing a cutoff on the transverse momentum, by summing an infinite set of diagrams. The main results are that (1) the final particles fall naturally into two jets, (2) the Bjorken scaling law breaks down, (3) multiplicity of pions increases as $\log q^2$, and (4) a longitudinal impact parameter space is realized.

Recently there has been growing interest in the analysis of high-energy hadron electromagnetic processes through the parton model[1,2] originally suggested by Feynman. Although the original parton model is a physical picture of "bits" of the hadron scattering independently, Drell, Levy, and Yan[3] showed that analogous results can be obtained from a neutral pseudoscalar (ps) meson field-theory model. Their results are conditional on the existence of certain infinite-momentum limits; these conditions are satisfied in their model by imposing a transverse-momentum cutoff on each pion produced. They found that Bjorken's scaling law,[4] that $\nu W_2(q^2,\nu)$ is a function of the ratio $q^2/2m\nu$, is satisfied in their model. They also predict other features of ep inelastic scattering and of the $e\bar{e}$ annihilation process.

By introducing a cutoff on the transverse momentum of the pions, the ps meson theory becomes a super-renormalizable theory rather than a renormalizable theory. In this paper we shall study the form factor W_2 in deep inelastic ep scattering in a neutral ps meson theory without cutoff.[5] The purpose of this study is to find the characteristic behavior of high-energy ep inelastic scattering in a renormalizable theory. In view of the success of quantum electrodynamics, which is renormalizable, such a study is certainly desirable. As we shall see, our results are different in many ways from those obtained by Drell, Levy, and Yan.

The set of diagrams we considered are shown

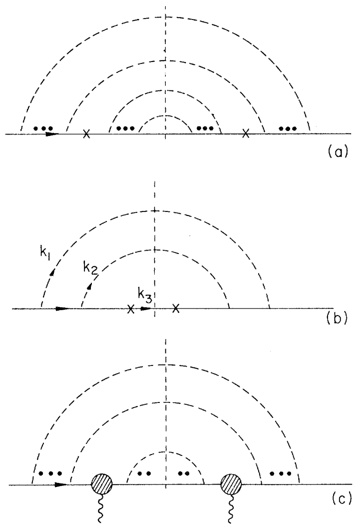

FIG. 1. (a) Set of straight ladder unitary diagrams considered. (b) Same diagram with two rungs. (c) Same as (a) with form factor included.

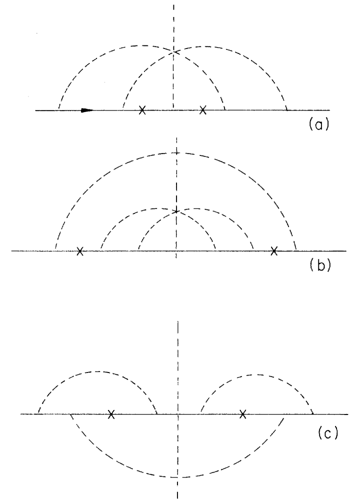

FIG. 2. (a), (b) Some lower-order ladder diagrams with crossed rungs. (c) Diagram with pions interacting in different sides of the currents. These diagrams are not asymptotically important compared with those of Fig. 1.

in Fig. 1. This is, in a sense, a set of ladder diagrams. As a first attempt at this problem, the possible nucleon-antinucleon pair creations and pion vertex corrections are ignored. It is not clear to us whether the inclusion of pion vertex corrections would change the qualitative results of this paper. However, see (f) below.

To each order in the pion-nucleon coupling constant g, we keep only the leading contribution in q^2 in our calculation of $W_2(q^2, \nu)$. $W_{1,2}(q^2, \nu)$ are, of course, the form factors of ep inelastic scattering; $-q^2$ and ν are the invariant momentum transfer squared and the energy loss of the electron, respectively, in the lab frame. In the forward, deep inelastic regions,

$$m^2 \ll q^2, \quad 2m\nu \ll s,$$

where s is the square of the c.m. energy and m is the proton mass. Here only W_2 contributes.

In order to justify our choice of the diagrams of Fig. 1, we have looked explicitly in lower order at other diagrams. In particular, we found that diagrams with crossed rungs [Figs. 2(a) and 2(b)] or with pions interacting between nucleons on different sides of the currents [Fig. 2(c)] are at least order $\log(q^2/m^2)$ smaller than the leading contribution of Fig. 1. This indicates it may not be a bad approximation to consider the straight ladder diagrams.

The results of our calculation can be summarized as follows:

(a) The final pions and proton fall naturally into two groups (jets) in which particles in a given group move close to each other. The first group contains all pions emitted before the proton interacts with the current (the outer "rainbow" of Fig. 1); while the second group contains the final proton and pions emitted after the proton interacts with the current (the inner "rainbow" of Fig. 1). The longitudinal momentum of the proton at the time of interaction, measured as a fraction x of the total longitudinal momentum, is still governed by the same $\delta(q^2/x-2m\nu)$ as in the parton model. This x measures the fraction of the longitudinal momentum left over by all the pions emitted before the proton interacts with the currents.

(b) The scaling law $\nu W_2 = \nu W_2(q^2/2m\nu)$ is violated in an interesting manner. For a process with n final pions, the partial W_2 contains a q-dependent factor $[\log(q^2/m^2)]^n$. Other than in $\delta(q^2/x-2m\nu)$, the partial W_2 for n pions emitted after the current insertion contains no x dependence; pions emitted before the current insertion introduce further x dependence.

(c) The total form factor W_2 is formed by summing over all pion rainbows. As mentioned above, W_2 factors into two parts, each associated with one group of particles:

$$\lim_{\substack{q^2,\nu\to\infty \\ q^2/\nu \text{ fixed}}} W_2(q^2,\nu) = \int_0^1 dx\, 2mA_1(q^2,x)A_2(q^2) \times \delta\left(\frac{q^2}{x}-2m\nu\right),$$

where

$$\int_0^1 A_1(q^2,x)x^{\lambda-1}dx = \exp\left(\frac{g^2}{16\pi^2}\frac{\log(q^2/m^2)}{\lambda(\lambda+1)}\right)-1,$$

$$A_2(q^2) = \left(\exp\frac{g^2}{32\pi^2}\log\frac{q^2}{m^2}\right).$$

The explicit structure of this result is presumably quite model dependent. However, the facts that A_1 has a simple exponential structure in the Mellin transform space and that A_1 and A_2 have explicit q^2 dependence may be the general properties of any renormalizable field theory. In analog to the eikonal form in the impact parameter space, the Mellin transform space for the longitudinal momentum has a profound physical meaning of its own. We can make the analog more precise by considering the two sets of variables[6]

$$(b_1, b_2, \lambda) = (E_1/x, E_2/x, K_3)$$

and

$$(p_1, p_2, \log x).$$

K_3 is the three-direction boost and $E_{1,2}$, the commuting generators of the infinite-momentum $E(2)$ subgroup, are combinations of boosts and rotations given by Chang and O'Raifeartaigh.[7] These variables form a commuting set of conjugate variables. Now the impact-parameter representation, which reflects the significance of a realization in $E_{1,2}$, comes from taking a two-dimensional Fourier transform over p_1 and p_2. Our λ space similarly comes from a Laplace transform over $\log x$, and reflects a similar realization in K_3. λ is thus a "longitudinal impact parameter."

(d) The number distribution for the pions emitted after the current insertion is a Poisson distribution. Because of the extra x dependence the distribution for the pions emitted before the current insertion is not Poisson; however, it is Poisson in the Mellin transform space. (The experimentally observed distribution is, of course, in the x space.) For the pions emitted after the current, the average number is

$$\bar{n}_2 = \frac{g^2}{32\pi^2}\log\frac{q^2}{m^2}.$$

For the pions emitted before the current, the average number is only simple for small x, when

$$\bar{n}_1 = \left(\frac{g^2}{16\pi^2}\log\frac{q^2}{m^2}\log\frac{1}{x}\right)^{1/2}.$$

In each case, $\bar{n}$ depends only logarithmically on q^2. There is additional peaking in $\bar{n}$, as x becomes small, i.e., as the first group of pions becomes soft.

(e) The longitudinal momentum distribution of the pions does not obey the simple dx/x rule, as suggested by phase space alone, because the integrand picks up extra x factors from the amplitude. In more complicated field theories, it may be hard to predict the x dependence of the pion momenta.

(f) We have seen that the largest momentum transfer takes place at the photon vertices rather than the individual pion vertices. This would indicate that inclusion of pure vertex corrections for the pions might have only small effect, but that corrections to the photon vertex alone may be important. Such a correction would just convert the γ_+ of the photon vertex to

$$F_1(q^2)\gamma_+ + (i/2m)F_2(q^2)\sigma_{+i}q_i;$$

$F_{1,2}(q^2)$ are the nucleon electromagnetic form factors. This is indicated in Fig. 1(c). Inclusion of this factor simply multiplies W_2 given above by $[F(q^2)]^2$.[8] Physically it is clear that the possible momentum transfer in the process must be damped by the nucleon form factor.

(g) Inclusion of multiphoton exchange in the production process, rather than one-photon exchange, can be made by using the infinite-momentum technique. Instead of the single-photon exchange amplitude, one has an eikonal form whose driving term comes from one-photon exchange.[9] The form of W_2 will be unchanged.

(h) Finally, we should note that accommodation of a factor like $\exp[(g^2/32\pi^2)\log(q^2/m^2)]$ would not be a severe strain on the data. For q^2 running 1 to 5, this factor varies from 1 to ~2.6, which is consistent with the present data.

To compute the ep inelastic form factors, we choose a particular frame such that the initial proton has the four-momentum $p_+ = p^0 + p^3 = 1$, $p_\perp = 0$, $p_- = p^0 - p^3 = m^2$, and the momentum transfer to the system is $q_+ = O(1/s)$, $\vec{q}$, $q_- = 2m\nu$. For an ep inelastic process of fixed q^2 and ν, but with very high energy s, we can ignore the term $O(1/s)$. Then the hadronic amplitude becomes s independent. The leading contribution for the process comes from the plus component of the current, which leads simply to $W_2(q,\nu)$.

We now give a very rough demonstration of our method of calculation. The example chosen is two pions emitted before the current, Fig. 1(b). We label the momenta of the final particles by

$$k_i^\mu = (x_i, \vec{k}_i, [\vec{k}_i^2 + \mu^2(m^2)]/x_i)$$

where μ (m) is the mass of pion (nucleon). The x_i's are the fractions of the longitudinal momentum taken by the ith particle in the ep c.m. frame. In terms of x and $\vec{k}$, the three-particle phase-space factor reduces to

$$D=8\pi^2\frac{dx_1}{4\pi x_1}\frac{dx_2}{4\pi x_2}\frac{dx_3}{4\pi x_3}\frac{d^2k_1}{(2\pi)^2}\frac{d^2k_2}{(2\pi)^2}\frac{d^2k_3}{(2\pi)^2}\delta(x_1+x_2+x_3-1)(2\pi)^2\delta^2(\vec{k}_1+\vec{k}_2+\vec{k}_3-q)$$
$$\times\delta\left(\frac{\vec{k}_1^{\,2}+\mu^2}{x_1}+\frac{\vec{k}_2^{\,2}+\mu^2}{x_2}+\frac{\vec{k}_3^{\,2}+m^2}{x_3}-2m\nu-m^2\right).$$

The last delta function is conservation of the minus component of the total momentum, and eventually supplies the $\delta(q^2/x-2m\nu)$ of the parton model.

The amplitude for this process is

$$M=ig^2\frac{\bar{u}(k_3)\gamma_+(\not{p}-\not{k}_1-\not{k}_2+m)\gamma_2(\not{p}-\not{k}_1+m)\gamma_5u(p)}{[(p-k_1-k_2)^2-m^2+i\epsilon][(p-k_1)^2-m^2+i\epsilon]}.$$

$|M^2|$, summed and averaged over final and initial spins, is to be integrated over the phase space to give W_2. Since $\int d^2k_1d^2k_2$ gives the $(\log q^2/m^2)^2$ term below, only the leading terms in $\vec{k}_1$ and $\vec{k}_2$ are kept in $\sum|M|^2$. Performing the d^2k_i, the contribution of Fig. 1(b) is then

$$\int_0^1 dx_2\,2m\,\delta\left(\frac{q^2}{x_3}-2m\nu\right)\int_0^1\frac{x_1dx_1}{(1-x_1)^2}x_2dx_2\delta(x_1+x_2+x_3-1)\frac{1}{2!}\left(\frac{g^2}{16\pi^2}\log\frac{q^2}{m^2}\right)^2.$$

In the Mellin transform space of x_3, the coefficient of $2m\delta(q^2/x_3-2m\nu)$ is

$$\frac{1}{N!}\left(\frac{g^2}{16\pi^2}\frac{\log q^2/m^2}{\lambda(\lambda+1)}\right)^N$$

for $N=2$. This result generalizes for N pions emitted before the current. Summing over N gives $A_1(q^2,x)$. The method is similar for pions emitted after the current; one finds no further x dependence in this case, leading to $A_2(q^2)$.

The details of this calculation and its applications to other electromagnetic processes, such as e^-e^+ annihilation into hadrons, will be published elsewhere. The authors wish to thank Professor R. Dashen for a very helpful discussion.

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[1]See R. Feynman, Phys. Rev. Letters 23, 1415 (1969).

[2]J. D. Bjorken and E. A. Paschos, Phys. Rev. (to be published).

[3]S. Drell, D. Levy, and T.-M. Yan, Phys. Rev. Letters 22, 744 (1969), and Phys. Rev. (to be published).

[4]J. Bjorken, Phys. Rev. 179, 1547 (1969).

[5]We have also studied this process in detail for a different super-renormalizable theory, namely $\lambda\Phi^3$. This is a useful theory because previous work on the infinite-momentum limit has told us precisely what diagrams to ignore. This theory also gives the scaling law. More details will be published elsewhere.

[6]We are indebted to Professor R. Dashen for pointing out this analogy to us. See also N. P. Chang, Phys. Rev. 172, 1796 (1968).

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[8]More precisely, the factor multiplied should be

$$F_1(q^2)^2+\frac{q^2}{4m^2}F_2(q^2)^2=\frac{G_E^{\,2}+(q^2/4m^2)G_M^{\,2}}{1+(q^2/4m^2)}.$$

However, it reduces to the universal form factor $F(q^2)^2$ if $G_E=G_M=F(q^2)^2$.

[9]For an elementary treatment of multiphoton processes, see, e.g., S. J. Chang and S. Ma, Phys. Rev. Letters 22, 1334 (1969).