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ASYMMETRY PARAMETER FOR $\Lambda^0 \to n\pi^0$ †*

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The asymmetry parameter α_0 for $\Lambda^0 \to n\pi^0$ has been measured relative to α_- by comparing the neutron distribution with the proton distribution from the decay $\Lambda^0 \to p\pi^-$ for polarized Λ^0 hyperons. A sample of 4760 neutron decay events and 8500 proton decay events gave $\alpha_0/\alpha_- = 1.000 \pm 0.068$ in good agreement with the $|\Delta\vec{I}| = \frac{1}{2}$ rule.

This Letter reports the result of a second experiment designed to test the validity of the $|\Delta\vec{I}| = \frac{1}{2}$ rule in the hadronic decays of hyperons. The first experiment was a measurement of proton polarization in $\Sigma^+ \to p\pi^0$.[1] Because the Λ^0 hyperon is an isospin singlet, the $|\Delta\vec{I}| = \frac{1}{2}$ rule leads to a very simple relation between the amplitudes for $\Lambda^0 \to p\pi^-$ and $\Lambda^0 \to n\pi^0$, namely, $S_- = -\sqrt{2}S_0$, $P_- = -\sqrt{2}P_0$; here S and P represent the amplitudes for orbital angular momentum 0 and 1, respectively. In terms of these amplitudes the decay rate is given by $\Gamma = |S|^2 + |P|^2$ and the asymmetry parameter by $\alpha = 2\,\mathrm{Re}S^*P/(|S|^2 + |P|^2)$. The spatial distribution of nucleons is of the form $N(\omega) = (1/4\pi)(1 + \alpha P_\Lambda \cos\omega)$, where ω is the angle between the Λ^0 spin and the proton momentum in the hyperon rest frame, and P_Λ is the average Λ^0 polarization. In the absence of radiative corrections the $|\Delta\vec{I}| = \frac{1}{2}$ rule predicts the branching ratio to be $\Gamma_0/\Gamma_- = 0.5$, and the asymmetry parameter ratio to be $\alpha_0/\alpha_- = 1$. The best experimental value for the branching ratio is $\Gamma_0/\Gamma_- = 0.550 \pm 0.019$.[2] An earlier measurement of the asymmetry parameter ratio yielded $\alpha_0/\alpha_- = 1.10 \pm 0.27$.[3] The branching ratio is sensitive primarily to $|\Delta\vec{I}| = \frac{3}{2}$ S-wave amplitudes because $|S|/|P| \approx 3$, while the asymmetry parameter ratio is equally sensitive to S- and P-wave $|\Delta\vec{I}| = \frac{3}{2}$ terms. [See Eq. (5) below]. Within experimental errors these data allow P-wave $|\Delta\vec{I}| = \frac{3}{2}$ amplitudes of ~20%.[4] Radiative corrections are somewhat uncertain, but should be at the 3% level.[5] The ratio α_0/α_- was measured directly in this experiment by comparing the spatial distributions $N(\omega) = (1/4\pi)(1 + \alpha P_\Lambda \times \cos\omega)$ for neutrons and protons following the decay of polarized Λ^0 hyperons.

The Princeton-Pennsylvania Accelerator furnished a secondary positive beam at 1.0 GeV/c. The π^+ intensity in the beam was 10^5/sec with a ±1% momentum spread; the spot size at the final focus was 3.5 cm × 4.5 cm. The polarized Λ^0 hyperons were produced in liquid deuterium by the reaction $\pi^+ n(p) \to K^+\Lambda^0(p)$, where (p) represents the spectator proton. The K^+ mesons were detected electronically to select associated production events. The nucleons from Λ^0 decay were detected in a spark chamber array with 11 scintillators and 33 polyethylene plates each 60 cm square. Each scintillator was 0.3 g/cm^2 thick, and each polyethylene plate was 0.9 g/cm^2 thick. Neutrons were detected by observing recoil proton tracks in the spark-chamber volume. A floor plan of the apparatus is shown in Fig. 1. The K^+ mesons satisfied $\beta < 0.75$, stopped in the large water tank, and decayed into μ^+ each of which registered in one of 12 wrap counters surrounding the water tank. To study $\Lambda^0 \to n\pi^0$, a K^+ in coincidence with $\bar{P}N_I$ was required, where P was a veto counter to suppress charged particles, and N_I was a count in any one of the eleven scintillators in the polyethylene array. This signature was designed to detect proton recoils from n-p collisions. To study $\Lambda^0 \to p\pi^-$ the P counter was placed in coincidence. The charged trigger rate was 1 picture/sec, about half caused by $\Lambda^0 \to p\pi^-$. The neutral trigger rate was $\frac{1}{4}$ picture/sec, about $\frac{1}{4}$ caused by $\Lambda^0 \to n\pi^0$. Most of the rejected neutron triggers had no visible recoil proton in the polyethylene chamber. The detection efficiency for the chamber was 12% for 50- to 250-MeV neutrons from Λ^0 decay.[6] For each event the K^+ direction was recorded in a foil spark chamber

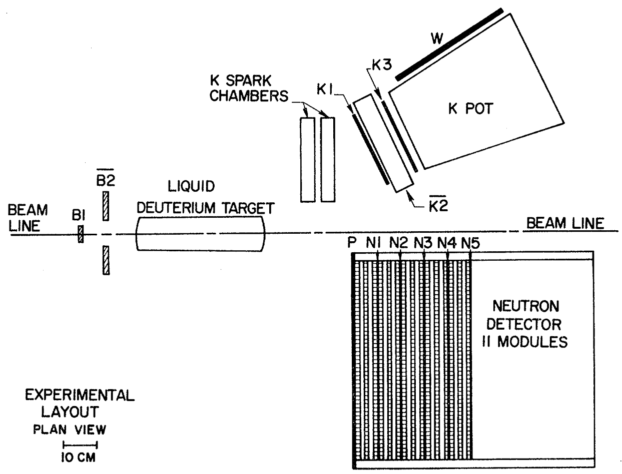

FIG. 1. Floor plan of the apparatus. The positive 1.0-GeV/c beam was about 75% protons and 25% pions. Pions were identified by time of flight between the internal target of the Princeton-Pennsylvania Accelerator and counters ($B1\times\overline{B2}$). The counter $K2$ was a water-filled Cherenkov counter and was used as a veto to reject fast pions from the target. The signature ($B1\times\overline{B2}$) + ($K1\times\overline{K2}\times K3$) + (KPOT $\times W$) identified a $\pi^+\to K^+$ reaction, and the decay modes were selected by $\overline{P}\times N_i$ for $\Lambda^0\to n\pi^0$ and ($P\times N_i$) for $\Lambda^0\to p\pi^-$, respectively, where N_i was any one of the eleven counters in the neutron detector assembly. The time between $K3$ and W was used to measure the time distribution between stopping K^+ and decay μ^+.

and the delay time between the stopping K^+ and the decay μ^+ was recorded on nixie tubes. Register lights were also flashed indicating which neutron counters N_i fired. A total data sample of 250 000 neutral triggers and 100 000 charged triggers was obtained.

Neutral and charged film were treated in the same way and analyzed with the same reconstruction programs to avoid differences in the nucleon asymmetry distributions caused by the analysis. Single K^+ and "recoil" proton[7] tracks were required for an acceptable event. The recoil proton track was restricted to a fiducial volume which eliminated a 2.5-cm border around the active area of the spark chamber plates. The last spark-chamber gap was a veto for all events, and the first two gaps were veto gaps for neutron events. A minimum proton range of three sparks was also required. Background in the neutron sample from conversion of γ rays in the polyethylene was reduced to a negligible level by requiring a straight single-track recoil. Most pairs from γ-ray conversion were sufficiently energetic to leave the chamber fiducial volume. The film was scanned by hand, and about 10 000 neutron events and 25 000 proton events were selected for measurement. The measurements were made with the Michigan Automatic Scanning System, a computer-controlled flying-spot scanner.

The reconstruction of an event would have been simple had the target neutron been at rest. Under this assumption the measured K^+ direction would predict a Λ^0 direction and energy, assuming the beam π^+ to be coincident with the beam center line. The hyperon information together with the measured point of origin of the recoil proton would give a family of possible neutron lines of flight, depending on the Λ^0 decay point. Then the recoil proton direction and range would fix the neutron energy for a particular neutron line of flight, assuming that the recoil proton originated in hydrogen rather than carbon (true for 70% of the recoils). Since the neutron energy and direction were not independent, a one-constraint fit could be performed to $\pi^+n\to\Lambda^0K^+$, $\Lambda^0\to n\pi^0$. The motion of the target neutron in deuterium made it impossible to perform a constrained fit in this manner. Instead a rather ad hoc technique was devised to extract the associated production events from the background. The target neutron was allowed five momentum values (0, ±25 MeV/c, and ±50 MeV/c along the incident π^+ direction) and the angle between the decay neutron and the Λ^0 in the laboratory predicted by each fit was compared with the measured angle. The first value of the target momentum which yielded a difference in angles $|\Delta\theta|<4°$ stopped the calculation, and the event was accepted. If none of the five target momenta satisfied this criterion, the event was rejected. In this manner 4760 neutron events and 8500 proton events were obtained.

Figure 2 shows the K^+-lifetime curves associated with these data; these curves were consistent with K^+ decay, so that the sample appeared to be pure associated production at a level of ~2%. Pions at 1.0 GeV/c were below threshold for Σ-hyperon production on a free nucleon target, but Σ^0 and Σ^+ could be produced in deuterium. Monte Carlo estimates of Σ^0 and Σ^+ production were 6 and 4% relative to Λ^0, using measured cross sections and a Hulthén momentum distribution of nucleons in the deuteron.[8] The detector efficiencies for Λ^0-decay neutrons and protons did not depend on whether the Λ^0 hyperons were produced directly or by the decay of Σ^0 hyperons. The small Σ^0 hyperon contamination, therefore, decreased slightly the polarization of the Λ^0 sample, but had a negligible effect on the asymmetry parameter ratio. The two Σ^+ decay

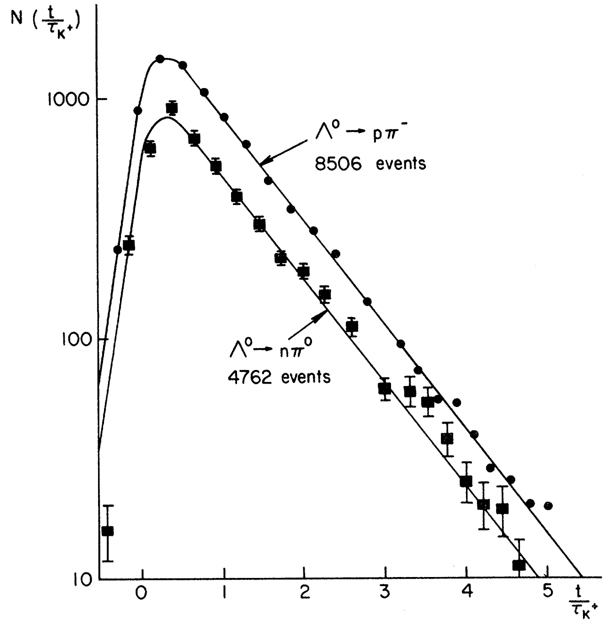

FIG. 2. Time distributions for the difference between the K^+ stop time ($K3$) and the apparent $K^+ \rightarrow \mu^+\nu$ decay time (W) for the charged and neutral Λ^0-decay data samples after the quasireconstruction described in the text. The abscissa is in units of the K^+ lifetime, taken to be 12.5 nsec. The solid curves were obtained by folding a Gaussian resolution function 2.5 nsec wide into a pure exponential, and normalizing to the number of events for $t > \tau_k 2$. The data are consistent with pure associated production for each of the two decay modes of the Λ^0.

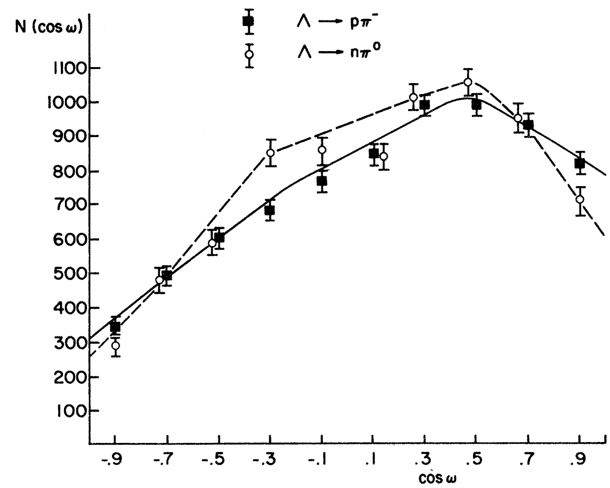

FIG. 3. Distributions in $\cos\omega = \hat{q} \times \hat{S}_\Lambda$ in the hyperon rest frame for the charged and neutral data. The neutral data have been normalized to the charged to facilitate comparison by eye. The measurement errors and geometrical effects which distort the curves are discussed in the text. The solid and dashed lines represent Monte Carlo calculations of the expected distributions, assuming $\alpha_0 P_\Lambda = \alpha_- P_\Lambda = 0.6$ for production on free neutrons. For comparison the individual ϵ's defined in the text are: data $\epsilon_0 = 0.47 \pm 0.03$, $\epsilon_- = 0.48 \pm 0.02$; Monte Carlo $\epsilon_0 = 0.44 \pm 0.01$, $\epsilon_- 0.44 \pm 0.01$.

modes $\Sigma^+ \rightarrow n\pi^+$ and $\Sigma^+ \rightarrow p\pi^0$ could have produced neutrons and protons with different spatial distributions, since $\alpha_+^{\Sigma} \approx 0$ and $\alpha_0^{\Sigma} \approx -1$. This background effect was also negligible because the Σ^+ tended to have low polarization, and the decay neutrons and protons were too energetic to reconstruct as Λ^0 decays in most cases.

For each event the production plane normal $\hat{n}$ was assumed to be parallel to $\vec{k}_\pi \times \vec{k}_K$. The component of the laboratory nucleon momentum vector along $\hat{n}$, $\vec{p} \cdot \hat{n}$, served to define the quantity $\cos\omega = \vec{p} \cdot \hat{n}/q$, where q is the nucleon momentum in the hyperon rest frame. The distributions in $\cos\omega$ for the complete samples of neutron and proton data are shown in Fig. 3. The neutron distribution has been normalized to the total number of proton events for comparison. The ratio $p/q \sim 6$ for a typical event, so that uncertainties in $\vec{p} \cdot \hat{n}$ were amplified in calculating $\cos\omega$. The ratio α_0/α_- was not obtained directly from these curves because different distributions in θ_K^*, the K^+-production angle in the πn center-of-mass system, could result in different values of P_Λ, the average hyperon polarization. Thus the $\cos\omega$ data were divided into seven different bins according to $\cos\theta_K^*$ for $-0.5 \leq \cos_K^* \leq +0.7$ and the asymmetry ratio was determined for each bin. The curves of Fig. 3 are nevertheless useful for discussing the distortions.

Although qualitatively similar, neither curve in Fig. 3 has the simple form $N(\omega) = (1/4\pi)(1 + \alpha P_\Lambda \times \cos\omega)$. Two types of effects distorted these curves: (a) errors in the $\pi^+ n \rightarrow \Lambda^0 K^+$ reconstruction, including target neutron motion and geometrical uncertainties which caused the true Λ^0 production plane to differ from the apparent $\pi^+ K^+$ plane; and (b) biases in the detection of the decay neutrons and protons. Effects of the first type washed out the apparent Λ^0 polarization, but contributed equal distortions to the neutron and proton distributions. Effects of the second type included the fact that the polyethylene spark chamber did not intercept all nucleons from Λ^0 decay, and the fact that the energy dependences of the detection probabilities were not the same for neutrons and protons. The neutron detector favored lower energies, whereas the corresponding protons tended to stop in the deuterium target. Care was taken to insure that the apparatus was up-down symmetric, so that energy-dependent distortions depended on $|\cos\omega|$, but not on the algebraic sign.

Asymmetries were extracted from data in the form shown in Fig. 3 by defining a quantity ϵ for each pair of bins with the same value of $|\cos\omega|$:

$$\epsilon = \frac{N(+|\cos\omega|)-N(-|\cos\omega|)}{N(+|\cos\omega|)+N(-|\cos\omega|)}\frac{1}{|\cos\omega|}. \qquad (1)$$

Each pair of bins was assigned an error

$$\delta\epsilon = 2(N_+N_-)^{1/2}(N_+ + N_-)^{-3/2}|\cos\omega|^{-1}. \qquad (2)$$

Then the weighted average of the five values of ϵ was calculated to obtain the best value for the entire curve. In the absence of distortions the quantity ϵ defined in Eq. (1) equals αP_Λ for each value of $|\cos\omega|$. The independence of the distortions on the sign of $\cos\omega$ was checked both with the real data and by using Monte Carlo techniques. The energy distribution of neutrons for $\cos\omega > 0$ was found to be identical to the energy distribution for $\cos\omega < 0$. The proton data also showed this symmetry. The parameters ϵ defined by Eq. (1) for charged and neutral data did not change outside of statistical fluctuation for the various values of $|\cos\omega|$. These considerations combined with the similarity between the proton and neutron points in Fig. 3 led to the conclusion that if $\alpha_0/\alpha_- \simeq 1$, then $\epsilon_0/\epsilon_- = \alpha_0/\alpha_-$ to within $\leq 2\%$. The Monte Carlo calculations verified this conclusion to a statistical accuracy of $\pm 3\%$.

The ratios α_0/α_- were calculated in this manner for each 0.2-wide range in $\cos\theta_K{}^*$ (fixed P_Λ) discussed earlier, and then averaged to obtain the final result $\alpha_0/\alpha_- = 1.000 \pm 0.062$. The error is a statistical standard deviation. It is interesting to note that if there were no distortions in the curves in Fig. 3, the expected error based on 4760 neutron events and 8500 proton events would be ± 0.045. If background had been present due to nonassociated production, the ratio would have depended on the observed K^+-decay time shown in Fig. 2. To check for an effect of this type, proton and neutron data were divided into early and late time bins at a decay time $t = \tau_{K^+}/2$. The results of this check were $(\alpha_0/\alpha_-)_{\rm early} = 1.001 \pm 0.091$, and $(\alpha_0/\alpha_-)_{\rm late} = 0.939 \pm 0.094$, consistent with no contamination. To obtain the final result a 2% error to account for uncertainty in the early time background and 2% error to account for possible distortions in the $\cos\omega$ distributions were folded in, resulting in the ratio

$$\alpha_0/\alpha_- = 1.000 \pm 0.068. \qquad (3)$$

This result was combined with the results of Ref. 3 to give a weighted average

$$\alpha_0/\alpha_- = 1.006 \pm 0.066. \qquad (4)$$

Let $\Delta\alpha$ be the measured value of α_0/α_- minus the predicted value, with a similar definition of $\Delta\Gamma$ for the branching ratio Γ_0/Γ_-. Then assuming $|S_0| > |P_0|$ [9] and $\alpha_- = 0.65 \pm 0.02$,[10] linear expressions for the $|\Delta\vec{I}| = \frac{3}{2}$ S- and P-wave amplitudes can be obtained:

$$\Delta\alpha = -1.56(S_3/S_1) + 1.64(P_3/P_1),$$

and

$$\Delta\Gamma = 1.85(S_3/S_1) + 0.24(P_3/P_1). \qquad (5)$$

Here the $|\Delta\vec{I}| = \frac{3}{2}$ amplitudes are expressed relative to the $|\Delta\vec{I}| = \frac{1}{2}$ amplitudes; the uncertainties in the coefficients are small compared with uncertainties in $\Delta\alpha$ and $\Delta\Gamma$. Final-state πN interactions have been included in these relations, but have a very small effect. Three different choices were made for the predicted values: (a) uncorrected $|\Delta\vec{I}| = \frac{1}{2}$ rule, $\alpha_0/\alpha_- = 1$, $\Gamma_0/\Gamma_- = 0.5$; (b) $|\Delta\vec{I}| = \frac{1}{2}$ with phase-space corrections only, $\alpha_0/\alpha_- = 1.025$, $\Gamma_0/\Gamma_- = 0.52$; and (c) $|\Delta\vec{I}| = \frac{1}{2}$ with complete radiative corrections given in Ref. (5), $\alpha_0/\alpha_- = 1.03$, $\Gamma_0/\Gamma_- = 0.485$. The results were as follows: (a) $S_3/S_1 = 0.024 \pm 0.010$, $P_3/P_1 = 0.026 \pm 0.037$; (b) $S_3/S_1 = 0.017 \pm 0.010$, $P_3/P_1 = 0.004 \pm 0.037$; and (c) $S_3/S_1 = 0.034 \pm 0.010$, $P_3/P_1 = 0.018 \pm 0.037$. The quoted errors were obtained from the experimental standard deviations; no uncertainty in the radiative corrections was included in group (c). These uncertainties are difficult to estimate, but could be as large as 3%.

It is well known that the hadronic $|\Delta\vec{I}| = \frac{1}{2}$ rule does not follow naturally from the current $\times$ current picture of the weak-interaction Lagrangian density. The rule has been conjectured to follow either from "octet" enhancement of the $|\Delta\vec{I}| = \frac{1}{2}$ terms, or from direct cancelation of these terms in the Lagrangian by the addition of neutral hadronic currents.[11] Results reported here and in Ref. 1 agree with the $|\Delta\vec{I}| = \frac{1}{2}$ rule at the 5% level, almost sufficient precision to make the octet-enhancement scheme seem unlikely. More accurate tests of rule for the Λ^0 hyperon, for example, may be able to settle this question, particularly if the radiative corrections can be done with confidence at the 1% level. The difficulties encountered in calculating the $|\Delta\vec{I}| = \frac{1}{2}$ forbidden transition $K^+ \to \pi^+\pi^0$ using radiative effects should perhaps not preclude the possibility of accurate radiative corrections to most hadronic weak decays where the $|\Delta\vec{I}| = \frac{1}{2}$ rule is well satisfied.

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*Experiment performed at the Princeton-Pennsylvania Accelerator.

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HIGH-ENERGY ep INELASTIC SCATTERING IN A RENORMALIZABLE THEORY*

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We have studied the ep deep inelastic scattering in a neutral pseudoscalar-meson theory without imposing a cutoff on the transverse momentum, by summing an infinite set of diagrams. The main results are that (1) the final particles fall naturally into two jets, (2) the Bjorken scaling law breaks down, (3) multiplicity of pions increases as $\log q^2$, and (4) a longitudinal impact parameter space is realized.

Recently there has been growing interest in the analysis of high-energy hadron electromagnetic processes through the parton model[1,2] originally suggested by Feynman. Although the original parton model is a physical picture of "bits" of the hadron scattering independently, Drell, Levy, and Yan[3] showed that analogous results can be obtained from a neutral pseudoscalar (ps) meson field-theory model. Their results are conditional on the existence of certain infinite-momentum limits; these conditions are satisfied in their model by imposing a transverse-momentum cutoff on each pion produced. They found that Bjorken's scaling law,[4] that $\nu W_2(q^2,\nu)$ is a function of the ratio $q^2/2m\nu$, is satisfied in their model. They also predict other features of ep inelastic scattering and of the $e\bar{e}$ annihilation process.

By introducing a cutoff on the transverse momentum of the pions, the ps meson theory becomes a super-renormalizable theory rather than a renormalizable theory. In this paper we shall study the form factor W_2 in deep inelastic ep scattering in a neutral ps meson theory without cutoff.[5] The purpose of this study is to find the characteristic behavior of high-energy ep inelastic scattering in a renormalizable theory. In view of the success of quantum electrodynamics, which is renormalizable, such a study is certainly desirable. As we shall see, our results are different in many ways from those obtained by Drell, Levy, and Yan.

The set of diagrams we considered are shown