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CONSEQUENCE OF HIGH-ENERGY NEUTRINO EXPERIMENT ON THE LEPTON NUMBER CONSERVATION LAW*

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Using the quoted μ^+-to-μ^- ratio obtained from the Berne-CERN-Fribourg neutrino experiment, the upper limit of C_M was estimated to be of the order of $5C_V$, where C_V is the universal vector-coupling constant of β decay, and C_M characterizes the coupling strength of a four-fermion weak interaction vertex only allowed by a multiplicative lepton-number conservation law.

As a result of the recent CERN high-energy neutrino experiment,[1] additional remarks can be made on the form of the lepton-number conservation law using the quoted μ^+-to-μ^- ratio. All present experimental evidence[2] is consistent with the existence of either (A) an additive conservation law of muonic and electronic lepton numbers, in which the sums $\sum L_\mu$ and $\sum L_e$ are separately conserved, or (B) a multiplicative conservation law[3,4] in which only the sum $\sum(L_\mu+L_e)$ and the sign $(-1)^{\Sigma L_\mu}$ are separately conserved. If the μ-e conservation law should take the multiplicative form, one would expect an overall production of two positive muons in the CERN experiment[1] via the double process

$$\pi^+ \text{ or } K^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu + {}_ZA \to {}_ZA + \mu^+ + e^- + \nu_e, \tag{1}$$

where coherent scattering of the virtual charged leptons from the Coulomb field of a target nucleus, ${}_ZA$, dominates Reaction (1).

Process (1) can be described in the lowest approximation of the weak and electromagnetic interactions by the sum of the two diagrams (a) and (b) of Fig. 1. For a zero-spin target, the total cross section is given by[5]

$$d\sigma(\mu^+e^-) = \frac{F(q^2)}{q^4} Z^2e^2 \frac{T_{\mu\nu}(p+p')_\mu(p+p')_\nu}{[(kp)^2-k^2p^2]^{1/2}} \cdot \frac{d^3p'}{(2\pi)^3 2p_0'}, \tag{2}$$

where $F(q^2) \simeq F(0)$ is the nuclear form factor of the nucleus (with $q=p-p'$), and $T_{\mu\nu}$ is a tensor describing the upper vertices of Fig. 1. From the gauge-invariance requirements we obtain[5]

$$T_{\mu\nu} = a[(kq)\delta_{\mu\nu} + q^2k_\mu k_\nu/(kq) - k_\mu q_\nu - k_\nu q_\mu] + b[q^2\delta_{\mu\nu} - q_\mu q_\nu], \tag{3}$$

where a and b are scalar functions.

It can be readily verified that as $q^2 \to 0$ and $(kq) \ll (kp)$ the quantity a reduces to $\sigma_{\text{ph}}/(kq)$ and the b de-

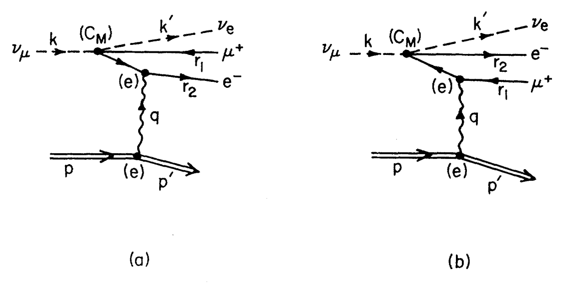

FIG. 1. The relevant Feynman diagrams of process (1). k, k', r_1, etc. are the four-momenta of the corresponding particles. (C_M) and (e) indicate the coupling strength of the corresponding vertex.

pendence in (3) can be ignored: It is therefore possible to simplify (2) into[5,6]

$$d\sigma(\mu^+e^-) = \frac{Z^2\alpha}{\pi}\,\sigma_{\rm ph}\left(1-\frac{x_2^{\,2}}{k_0^{\,2}x_1}\right)\frac{dx_1dx_2}{x_1x_2}, \tag{4}$$

where $\alpha = 1/137$, $\sigma_{\rm ph}$ = total cross section of the photoproduction process $\gamma+\nu_\mu\to\mu^++e^-+\nu_e$ with an unpolarized photon, and x_1, x_2 are the positive invariant quantities

$$x_1=-q^2, \quad x_2=kq.$$

The matrix element of the photoproduction process, based on the $V-A$ coupling, has the form

$$M=\frac{C_Me}{\sqrt{2}}\bar{u}(r_2)\left[\not{\epsilon}\frac{1}{\not{r}_2-\not{q}-m}\gamma_\sigma(1+\gamma_5)+\gamma_\sigma(1+\gamma_5)\frac{1}{-\not{r}_1+\not{q}-\mu}\not{\epsilon}\right]v(r_1)\bar{u}(k')\gamma^\sigma(1+\gamma^5)u(k), \tag{5}$$

where C_M is the weak-coupling constant only allowed by the multiplicative conservation law, e is the electric charge, u, v are spinors, and $\not{\epsilon}=\gamma_\mu\epsilon_\mu$, $\not{q}=\gamma_\mu q_\mu$, etc. The calculations lead to

$$d\sigma_{\rm ph}=\frac{32\alpha C_M^{\,2}}{(2\pi)^4}\cdot\frac{1}{(kq)}\cdot\left\{\frac{1}{(qr_2)}(qk')(kr_1)+\frac{1}{(qr_1)}(qk)(k'r_2)+\frac{1}{(qr_2)(qr_1)}[(2r_1r_2-qr_1-qr_2)(k'r_2)(kr_1)\right.$$
$$\left.+(qr_2)(k'r_1)(kr_1)+(qr_1)(k'r_2)(kr_2)-(r_1r_2)(k'q)(kr_1)-(r_1r_2)(kq)(k'r_2)]\right\}$$
$$\times\frac{d^3r_1}{2r_{10}}\cdot\frac{d^3r_2}{2r_{20}}\cdot\frac{d^3k'}{2k_0'}\delta^4(k+q-r_1-r_2-k'), \tag{6}$$

where terms proportional to m^2 [(electron mass)^2] and μ^2 [(muon mass)^2] have been left out. Introducing the invariant variables $x_3=(qr_1)$, $x_4=(qr_2)$, $x_5=(kr_1)$, $x_6=(kr_2)$, and substituting (6) into (4) we obtain[7]

$$\sigma(\mu^+e^-)=\frac{Z^2\alpha^2C_M^{\,2}}{2\pi^3}\int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2^{\,2}}\left(1-\frac{x_2^{\,2}}{k_0^{\,2}x_1}\right)\int dx_3\int dx_4\int\frac{dx_5}{[(x_2-x_3)^2-2x_1x_5+x_1]^{1/2}}$$
$$\times\left\{I_1(x_1\cdots x_5)+I_2(x_1\cdots x_5)\int x_6\frac{d\varphi}{2\pi}\right\}, \tag{7}$$

where $I_1(x_1\cdots x_5)$ and $I_2(x_1\cdots x_5)$ are polynomials in $x_1\cdots x_5$, and φ is the azimuth angle of $\vec{r}_2$ introduced by $x_6=(\vec{k}\cdot\vec{r}_2)$ in the $\vec{k}+\vec{q}-\vec{r}_2=0$ system, with the kinematically allowed integration limits, ignoring the electron mass, and expressing all quantities in unit of muon mass, we have integrated (7) with a Univac 1108 computer. The values of $\sigma(\mu^+e^-)(C_V/C_M)^2$ are shown in Fig. 2, where C_V is the universal vector-coupling constant of β decay.

After weighting the values of $\sigma(\mu^+e^-)(C_V/C_M)^2$ by the CERN ν_μ-energy spectrum (Fig. 2),[8] we find, for a $_{82}\mathrm{Pb}^{208}$ target, the average

$$\sigma(\mu^+e^-)=0.21\times10^{-40}(C_M/C_V)^2\ \mathrm{cm}^2. \tag{8}$$

If one uses the elastic cross section $\sigma_{\rm el}(\mu^-)$ for $\nu_\mu+n\to\mu^-+p$ at the same energy, and[7]

$$\sigma_{\rm el}(\mu^-)=0.63\times10^{-38}\ \mathrm{cm}^2/\mathrm{neutron},$$

then

$$\frac{\sigma(\mu^+e^-)}{\sigma_{\rm el}(\mu^-)}=\frac{0.2\times10^{-40}(C_M/C_V)^2}{0.63\times10^{-38}\times126}=0.25\times10^{-4}\left(\frac{C_M}{C_V}\right)^2.$$

Using the μ^+ to μ^- ratio quoted in the CERN paper,[1] we find that

$$\frac{\sigma(\mu^+e^-)}{\sigma_{\rm el}(\mu^-)}\leq\frac{\sigma(\mu^+)}{\sigma_{\rm el}(\mu^-)}<3.8\times10^{-3}\ \text{(90\% confidence level)}.$$

Therefore

$$C_M\leq12.3C_V. \tag{9}$$

Since none of the nine μ^+ events has been identified as a μ^+e^- event by the CERN experiment,[1] one

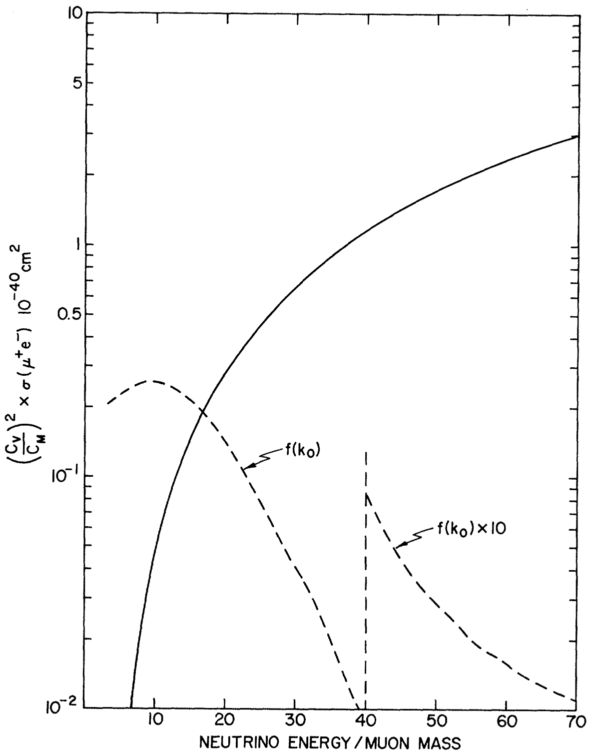

FIG. 2. The solid curve indicates the calculated values of $(C_V/C_M)^2\sigma(\mu^+e^-)$ as a function of the incident ν_μ energy. The dashed curve shows the weighting factor of $\sigma(\mu^+e^-)$ as a function of the ν_μ energy. To obtain the dashed curve we have normalized the area underneath the CERN ν_μ-energy spectrum to 1.

may also assume that

$$\frac{\sigma(\mu^+e^-)}{\sigma_{el}(\mu^-)} \leq 4.2\times 10^{-4},$$

which yields

$$C_M \leq 4.1C_V. \qquad (10)$$

Assuming the multiplicative conservation law (B), and characterizing the coupling strength of any local four-fermion weak-interaction vertex only allowed by (B) with C_M, one may compare the present result given by (9) or (10) with the previous limits. These limits, $C_M \leq 5800C_V$ and $C_M \leq 610C_V$, come from a muonium-antimuonium conversion ($\mu^+e^- \rightarrow \mu^-e^+$) experiment,[9] and a colliding beam ($e^-e^- \rightarrow \mu^-\mu^-$) experiment,[10] respectively. If we assume that the neutral leptons and charged leptons do behave identically in weak interactions, then Eq. (10) indicates that the upper limit of C_M should be of the order of $5C_V$. On the other hand, if they are different, then the absence of Reaction (1) does not necessarily imply the absence of $\mu^+e^- \rightarrow \mu^-e^+$ or $e^-e^- \rightarrow \mu^-\mu^-$.

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