clear choice between these descriptions.

*Research supported by the Lockheed Independent Research Fund. Work performed at the Stanford University tandem Van de Graaff Laboratory, supported by the National Science Foundation, and at the Lockheed Nuclear Physics Laboratory.

[1]D. A. Bromley, H. E. Gove, and A. E. Litherland, Can. J. Phys. 35, 1057 (1957).

[2]S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 29, No. 1 (1955).

[3]R. G. Hirko, thesis, Yale University, 1969 (unpublished).

[4]P. M. Endt and C. van der Leun, Nucl. Phys. A105, 1 (1967).

[5]R. E. White, Phys. Rev. 119, 757 (1960).

[6]A. E. Litherland and A. J. Ferguson, Can. J. Phys. 39, 788 (1961).

[7]E. K. Warburton, D. E. Alburger, and D. H. Wilkinson, Phys. Rev. 129, 2180 (1963).

[8]A. E. Blaugrund, Nucl. Phys. 88, 501 (1966).

[9]E. K. Warburton, A. R. Poletti, and J. W. Olness, Phys. Rev. 168, 1232 (1968).

ANISOTROPIC SUPERFLUIDITY IN NEUTRON STAR MATTER*

M. Hoffberg
Vassar College, Poughkeepsie, New York

and

A. E. Glassgold and R. W. Richardson
New York University, New York, New York 10012

and

M. Ruderman
Columbia University, New York, New York 10027

(Received 16 February 1970)

The magnitude of the expected superfluid transition temperature and the form of the gap are obtained for regimes of neutron density expected in neutron-star cores. For neutron densities exceeding 1.5×10^{14} g cm^{-3} the gap is anisotropic but nodeless, leading to thermodynamic properties of a conventional BCS-type superfluid.

In the density regime 5×10^{13} g cm^{-3} $<\rho<6\times10^{14}$ g cm^{-3}, characteristic of neutron-star cores, the neutron density varies from about $\frac{1}{3}$ to near 4 times the density of neutrons in normal nuclei. The attraction between appropriately paired neutrons at the top of their Fermi sea causes a BCS-type superfluid behavior.[1-3] The magnitude and form of the gap in the single-particle excitation spectrum depends sensitively upon density. This is because the appropriate Fermi energy varies as $\rho^{2/3}$ and the known neutron-neutron phase shifts depend strongly on energy. When $\rho\lesssim1.5\times10^{14}$ g cm^{-3}, $^1S_0$ attraction gives conventional Cooper pairing; when $\rho\gtrsim1.5\times10^{14}$ g cm^{-3}, the significant attraction is in the $^3P_2$ state. All other S and P phase shifts are much smaller or repulsive in this latter regime, and the resulting gap is nonisotropic. Anisotropic superfluidity has been considered earlier for liquid He^3 with P- and D-wave pairing.[4,5]

We assume that for neutrons near the top of their Fermi sea, all interactions vanish except $^1S_0$ and $^3P_2$, and that these can be adequately described by a separable-type potential chosen to fit the scattering data.[6] Inside normal, symmetrical nuclear matter the effective phase shift δ^* and mass m^* differ from their values appropriate to free space. In an almost pure neutron environment, these differences are expected to be much smaller because the n-p interactions, which give the dominant contribution to them in normal nuclear matter, are not present. We assume $\delta^*\approx\delta$ and $m^*\approx m$, which is also supported by explicit numerical calculations at densities near those of neutrons in nuclei.[7]

The superfluid transition temperature T_c obtained numerically from the usual BCS linear equation[8] with this interaction is given in Fig. 1. A qualitative fit[3] to these results and to others with slightly different m^* is

$$\ln\frac{kT_c}{2E_F}\approx-\frac{\pi}{2}\frac{m}{m^*}\cot\delta^*, \tag{1}$$

where δ^* is either the $^1S_0$ or $^3P_2$ phase shift, whichever is the dominant attractive one at the

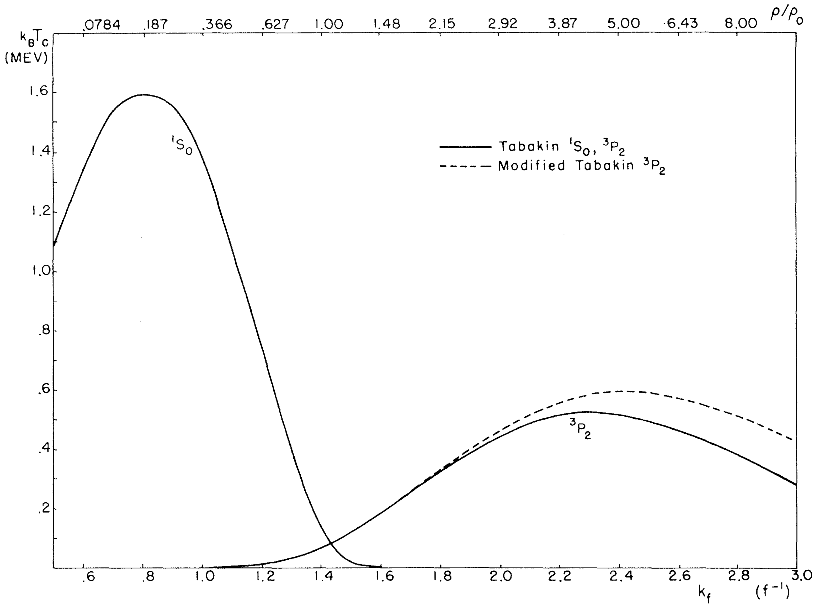

FIG. 1. Computed superfluid transition temperature for neutron matter as a function of Fermi wave number k_F (bottom scale) and of neutron density ρ (top scale) with $\delta^*=\delta$ and $m^*=m$. The neutron density ρ_0 in normal nuclei is 1.5×10^{14} g cm^{-3} and the density at the core-crust interface of a neutron star is about 5×10^{13} g cm^{-3}. The solid curves were obtained with Tabakin's potentials (see Ref. 6), and the dashed curve with a one-term modification of the Tabakin potential.

Fermi momentum appropriate to the given neutron density. The computed transition temperatures are well above temperatures expected for neutron-star interiors.[9] Coexisting with the neutrons is a degenerate proton fluid a few percent as dense. A very substantial superfluid transition temperature ($>10^9$ °K) seems likely also for the protons.

Although the transition-temperature calculations are essentially unchanged for pairing in any spin and angular-momentum state, the elucidation of the form of the gap below T_c requires an extension of conventional BCS theory. We use the generalization to triplet pairing of Balian and Werthamer[10] in which the gap is a matrix $\Delta_{m_1m_2}(\vec{k})$, where m_i is dichotomic and corresponds to the two values of the z component of the spin of either paired electron. Balian and Werthamer[10] applied their formalism to J-independent ^3P interactions and showed that, in this case, the spin and angular dependence of the gap matrix is that of a $^3P_0$ state. An important condition that is placed on the gap matrix in this formalism is

$$\sum_m \Delta_{mm_1}{}^*(\vec{k})\Delta_{mm_2}(k) = D^2(\vec{k})\delta_{m_1m_2}. \tag{2}$$

We have obtained this condition from the time-reversal invariance of the Hamiltonian and the assumption that, except for rotational degeneracy, the condensed state is unique. The quantity D^2 in (2) is the square of the energy gap in the quasiparticle spectrum. In the case treated by Balian and Werthamer this energy gap is isotropic. However, in the case of $^3P_2$ pairing, the gap matrix must have the form

$$\begin{pmatrix} \sqrt{2}\Delta_0Y_{1-1}+6^{1/2}\Delta_1Y_{10}+12^{1/2}\Delta_2Y_{11} & -\sqrt{3}\Delta_1{}^*Y_{1-1}+2\Delta_0Y_{10}+\sqrt{3}\Delta_1Y_{11} \\ -\sqrt{3}\Delta_1{}^*Y_{1-1}+2\Delta_0Y_{10}+\sqrt{3}\Delta_1Y_{11} & \sqrt{2}\Delta_0Y_{11}-6^{1/2}\Delta_1{}^*Y_{10}+12^{1/2}\Delta_2{}^*Y_{1-1} \end{pmatrix}, \tag{3}$$

and D^2 is a linear combination of spherical harmonics of orders 0 and 2. In a principal-axis system for $D^2(\vec{k})$, in which there is no longer rotational degeneracy, we have the additional restrictions

$$8^{1/2}\Delta_0\mathrm{Im}\Delta_2=\sqrt{3}\,\mathrm{Im}(\Delta_1)^2,$$
$$\Delta_0\Delta_1+6^{1/2}\Delta_1{}^*\Delta_2=0, \tag{4}$$

where Δ_0 is chosen real. We find that the solution of the gap equation with the lowest energy has $\Delta_1=\Delta_2=0$, i.e., the gap has the spin and angular dependence of a $^3P_2$ state with $M_J=0$. Then $D^2(\vec{k})$ has the form

$$D^2(\vec{k})=\Delta_0{}^2(k)(1+3\cos^2\theta), \tag{5}$$

and $\Delta_0(k_F) = 1.2kT_c$. The energy gap is nodeless; this is expected for the lowest energy state whenever a nodeless solution of the gap equation exists.[5] An important consequence of the nodeless gap is that the single-particle contribution to neutron-matter heat capacity contains the exponentially vanishing factor $e^{-T_c/T}$, just as for an isotropic superfluid.

The rotational degeneracy associated with the arbitrariness of the $\theta = 0$ direction is resolved in a neutron star by considerations of its stability. The lowest-energy configuration for the star is achieved when the gap polar axis ($\theta = 0$), along which the compressibility is greatest, orients itself in the radial direction in the star. There will, of course, exist some collective excitation modes[5] in the star in which the gap polar axis departs from its locally preferred direction.

---

*Supported in part by the National Science Foundation and the Office of Naval Research.

[1]V. Ginzburg and D. Kirzhnits, Zh. Eksperim. i Teor. Fiz. 47, 2007 (1964) [Soviet Phys. JETP 20, 1346 (1965)].

[2]N. N. Bogoliubov, Dokl. Akad. Nauk SSSR 119, 52 (1958) [Soviet Phys. Doklady 3, 279 (1958)]; L. N. Cooper, R. L. Mills, and A. M. Sessler, Phys. Rev. 114, 1377 (1959); K. Brueckner, T. Soda, P. W. Anderson, and P. Morel, Phys. Rev. 118, 1442 (1960).

[3]V. J. Emery and A. M. Sessler, Phys. Rev. 119, 248 (1960); R. C. Kennedy, L. Wilets, and E. Henley, Phys. Rev. 133, B1131 (1964); E. Jakeman and S. A. Mozkowski, Phys. Rev. 141, 933 (1966).

[4]V. J. Emery and A. M. Sessler, Phys. Rev. 119, 43 (1960); Brueckner, Soda, Anderson, and Morel, Ref. 2.

[5]P. W. Anderson and A. M. Morel, Phys. Rev. 123, 1911 (1961).

[6]F. Tabakin, Ann. Phys. (N.Y.) 30, 51 (1964).

[7]H. A. Bethe, Pulsar Symposium at the Aspen Institute of Physics, Aspen, Colorado, 1969 (to be published) and private communication.

[8]J. Bardeen, L. N. Cooper, J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

[9]S. Tsuruta and A. G. W. Cameron [Can. J. Phys. 44, 1863 (1966)] have estimated the temperature of the neutron-star core to be $<5\times10^8$ °K, for times greater than 10^3 yrs after formation.

[10]R. Balian and R. Werthamer, Phys. Rev. 131, 1553 (1963).

EVIDENCE FOR Ξ RESONANCES IN THE Ξ(1530)π SYSTEM*

S. Apsell, N. Barash-Schmidt, L. Kirsch, and P. Schmidt
Department of Physics, Brandeis University, Waltham, Massachusetts 02154

and

C. Y. Chang, R. J. Hemingway, B. V. Khoury, A. R. Stottlemyer, H. Whiteside,† and G. B. Yodh
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

and

S. Glickman, M. Goldberg, S. Jacobs, K. Jaeger, C. McCarthy, B. Meadows, and G. C. Moneti‡
Department of Physics, Syracuse University, Syracuse, New York 13210

and

J. Bartley,§ R. M. Dowd, J. Schneps, and G. Wolsky
Department of Physics, Tufts University, Medford, Massachusetts 02155

(Received 15 January 1970)

Evidence is presented for the decay of the Ξ(1820) and Ξ(1930) resonances to Ξ(1530)π, and for an $I=\frac{1}{2}$ assignment to these states. By combining data from the Ξπ and Ξ(1530)π decays of these resonances, we have determined their masses (1820 ± 7, 1963 ± 8) MeV, widths (64 ± 23, 89 ± 33) MeV, and branching ratios [Ξπ/Ξ(1530)π $= 1.5^{+0.6}_{-0.4}$, $2.8^{+0.7}_{-0.6}$].

The existence of Ξ resonant states in the Ξ(1530)π system has yet to be convincingly demonstrated.[1-4] In this paper, evidence is presented for the decay of Ξ(1820) and Ξ(1930) into Ξ(1530)π channels. The data have been analyzed to determine mass, width, and isospin values for these resonances. In addition, a comparison with our previously reported result[5] on resonant states in the Ξπ system yields branching-ratio information for these particles.

The data are based on the analysis of 10^6 pictures of K^-p interactions at 2.87 GeV/c momen-