# PROPERTIES OF THE 5.256-MeV STATE IN $Si^{29}$: EVIDENCE FOR A ROTATIONAL BAND WITH $K^{\pi} = \frac{7}{2}^{-}$ *

T. T. Bardin, J. A. Becker, T. R. Fisher, and A. D. W. Jones
Lockheed Palo Alto Research Laboratory, Palo Alto, California 94304
(Received 18 February 1970)

The spin of the 5.256-MeV level in $Si^{29}$ has been determined to be $J=\frac{9}{2}$; its mean lifetime has been measured to be $\tau_m = (0.95 \pm 0.15)\times 10^{-13}$ sec. The electromagnetic decay of this state is >90 % via cascade radiation through the $J^{\pi}=\frac{7}{2}^{-}$ state of $Si^{29}$ at 3.624 MeV, with $E_{\gamma} = 1631.3 \pm 0.5$ keV; the transition has quadrupole/dipole admixture $\delta = -0.49 \pm 0.07$. This multipole mixing together with the measured lifetime fix parity $\pi = -1$ for this state.

The nucleus $Si^{29}$ is of particular interest since it is in a mass region where the nuclear deformation is not well defined and is changing from prolate to oblate.[1] A good account of the static and dynamic properties of the low-lying positive-parity levels has been obtained in terms of the Nilsson model[2] by assuming two mixed rotational bands and an oblate nuclear deformation, characterized by $\delta = -0.15$.[1,3] Negative-parity levels have been previously located at 3.624 MeV ($J^{\pi} = \frac{7}{2}^{-}$) and 4.935 MeV ($J^{\pi} = \frac{3}{2}^{-}$). The large reduced widths for these states observed in the reaction[4] $Si^{28}(d,p)Si^{29}$ suggests that they have most of the $f_{7/2}$ and $p_{3/2}$ single-particle strength. We have been engaged in a study of the $Si^{29}$ levels in the energy interval $4 < E_x(\text{MeV}) < 5.3$, observing $\gamma$ radiation produced in the reaction $Mg^{26}(\alpha,n)Si^{29*}$ ($Q_0 = 0.033$ MeV). For the level with $E_x = 5.256$ MeV, we have made an assignment $J^{\pi} = \frac{9}{2}^{-}$ and measured a partial width $\Gamma_{\gamma}(E2) = 27 \pm 7$ Weisskopf units for the $5.256 \to 3.624$ transition. We propose, in the language of the Nilsson model, that this level be described as the second member of a $K^{\pi} = \frac{7}{2}^{-}$ band based on a neutron in orbit 10, with the 3.624-MeV level ($J^{\pi} = \frac{7}{2}^{-}$) being the band head. This is the first such band located in the $s$-$d$ shell.

The 5.256-MeV level in $Si^{29}$ is well established through analysis of charged particle spectra following the reactions $Si^{29}(p,p')Si^{29}$, $Si^{28}(d,p)Si^{29}$, $Si^{30}(d,t)Si^{29}$, and $Al^{27}(He^3,p)Si^{29}$.[4] However, no spectroscopic information on this state has been extracted from any of these experiments. We have determined the $\gamma$-ray decay mode (hitherto unreported) of this level by analysis of $\gamma$-ray spectra produced by bombarding a 600-$\mu$g/cm$^2$ $Mg^{26}$ foil target with $\alpha$ particles. The $\gamma$ radiation was detected in a 37-cm$^3$ right cylindrical Ge(Li) diode, 40 mm diam $\times$ 32 mm long, which could be rotated about a vertical axis through the reaction site. Pulse-height distributions were collected using a 4096-channel analog-to-digital converter and computer-based data acquisition system. As the incident $\alpha$-particle energy is raised above threshold for production of the 5.256-MeV level, a $\gamma$ ray with energy $E_{\gamma} = 1631$ keV appears in the spectrum. We show in Fig. 1 portions of the $\gamma$-ray spectrum obtained in this manner, measured both above ($E_{\alpha} = 6.4$ MeV) and below ($E_{\alpha} = 6.0$ MeV) the threshold energy of 6.03 MeV. On the basis of its behavior near threshold and its measured energy $E_{\gamma} = 1631.3 \pm 0.5$ keV, we assign the $\gamma$ ray to the $5.256 \to 3.624$ transition. From energy determinations of $1596.5 \pm 0.5$ and $2027.7 \pm 0.5$ keV for the $\gamma$ rays from the decay of the 3.624-MeV level, we arrive at an excitation energy of $E_x = 5255.5 \pm 1.0$ keV for the 5.256-MeV level, slightly higher than the energy of $5250 \pm 7$ keV deduced from the charged particle work of Hinds and Middleton quoted by White.[5] The spectra show no evidence for any decay mode for the 5.256-MeV level other than via transitions to the 3.624-MeV level; we estimate the branching of this decay mode as >90%.

The angular distribution of the 1.631-MeV $\gamma$ ray was measured at three bombarding energies above threshold, $E_{\alpha} = 6.2$, 6.4, and 6.6 MeV. These distributions were parametrized by a Legendre polynomial expansion $W(\theta) = 1 + A_2Q_2P_2(\cos\theta) + A_4Q_4P_4(\cos\theta)$, where $Q_2$ and $Q_4$ represent the solid angle attenuation factors for our geometry. The results are summarized in Table I. Near threshold, the reaction is expected to be dominated by outgoing $s$-wave neutrons, and thus the recoiling $Si^{29}(5.256)$ nuclei are aligned with predominant populations of magnetic substates with quantum numbers $m = \pm\frac{1}{2}$. Angular distributions may be analyzed in terms of level spins and the $\gamma$-ray multipole mixing ratio in the formulation described by Litherland and Ferguson for a collinear geometry.[6] With the $J = \frac{7}{2}$ assignment for the 3.624-MeV level (see Ref. 4) an unambiguous spin assignment $J = \frac{9}{2}$ results for the 5.256-MeV level. The 1.631-MeV $\gamma$ ray has an $(L+1)/L$ multipole

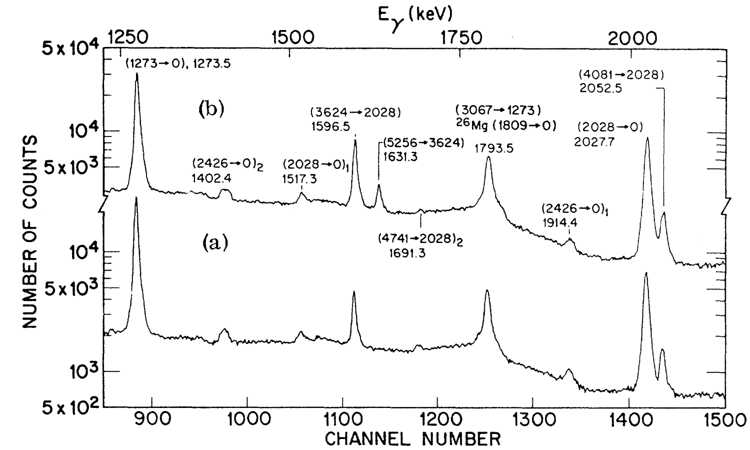


FIG. 1. Portions of the $\gamma$-ray spectra produced in the reaction $Mg^{26}$ ($\alpha$, $n$) $Si^{29}$ measured (a) at $E_\alpha = 6.0$ MeV, below threshold for the 5.256-MeV level, and (b) at $E_\alpha = 6.4$ MeV, 370 keV above threshold. The Ge(Li) $\gamma$-ray counter described in the text was located at $\theta_\gamma = 90°$, 15 cm from the reaction site in these measurements. The peaks in the spectrum are labeled in energy units of keV, as well as by the transition in $Si^{29}$ to which they correspond. Note the peak labeled 1631 keV is present in spectrum (b), but not in spectrum (a).

mixing $\delta(1.631) = -0.49 \pm 0.07$, which together with the partial quadrupole width $\Gamma(L=2) = 1.35 \pm 0.37$ meV requires negative parity for the 5.256-MeV level, since the 3.624-MeV level has negative parity. [The positive-parity assignment results in an $M2$ width $\Gamma(M2) = 838 \pm 232$ Weisskopf units.] Figure 2 displays the angular distribution of the 1.631-MeV radiation measured at $E_\alpha = 6.4$ MeV, together with the results of a least-squares fit to the angular correlation formulas in terms of the goodness-of-fit parameter, $\chi^2$ and $\tan^{-1}\delta(1.631)$. The preceding analysis might be expected to fail if the cross section near threshold were dominated by a resonance for the emission of $p$-wave neutrons. However, Fig. 2 shows that even the assumption of a 33% population for the $m = \pm\frac{3}{2}$ substates does not change the final results significantly.

The lifetime of the 5.256-MeV level was determined with a variant of the Doppler-shift attenuation method.[7] The Doppler shifts of the 1.631-MeV $\gamma$ ray were observed at $\theta_\gamma = 30°$ and $\theta_\gamma = 135°$ using two targets thick to the $Si^{29*}$ recoils and having different stopping characteristics: an isotopically pure $Mg^{26}$ foil and an alloy of 10% $Mg^{26}$ in Au. The beam energy, $E_\alpha = 7$ MeV, was chosen close to threshold to limit the recoiling nuclei to a well-defined forward cone. From a comparison of the centroid shifts observed with these two targets, the value $\tau_m(5.256) = (0.95 \pm 0.15) \times 10^{-13}$ sec ($\Gamma_\gamma = 6.9 \pm 1.1$ meV) was deduced. Nuclear stopping and scattering were included in the analysis, and the latter effect was calculated according to Blaugrund.[8]

Table I. Angular distribution coefficients for the 1.631-MeV $\gamma$ ray.

| $E_\alpha$ | $A_2Q_2$ | $A_4Q_4$ | $A_2$[a] | $A_4$[a] |
|---|---|---|---|---|
| 6.2 | 0.44 ± 0.08 | 0.06 ± 0.08 | 0.44 ± 0.08 | 0.06 ± 0.07 |
| 6.4 | 0.55 ± 0.02 | 0.13 ± 0.02 | 0.56 ± 0.02 | 0.14 ± 0.02 |
| 6.6 | 0.53 ± 0.03 | 0.11 ± 0.03 | 0.53 ± 0.03 | 0.11 ± 0.03 |

[a]Computed with $Q_2 = 0.99$ and $Q_4 = 0.96$.

The spectroscopy of the positive-parity levels in $Si^{29}$ with $E_x \leq 3.07$ MeV has been interpreted in the framework of the collective model by Bromley, Gove, and Litherland,[1] who concluded that a deformation $\delta = -0.15$ was in accord with the experimental data. In this model, the ground state and the levels at 2.43 and 2.03 MeV are, respectively, the $J^\pi = \frac{1}{2}^+$, $\frac{3}{2}^+$, and $\frac{5}{2}^+$ members of the $K^\pi = \frac{1}{2}^+$ ground-state band, while the 1.27- and 3.07-MeV levels are the $J^\pi = \frac{3}{2}^+$ and $\frac{5}{2}^+$ members of a $K = \frac{3}{2}^+$ band. Hirko has recently extended this interpretation and done a detailed band-mixing calculation.[3] While the inclusion of the Coriolis force to mix the $K^\pi = \frac{1}{2}^+$ and $K^\pi = \frac{3}{2}^+$ bands (based on neutrons in orbits 9 and 8, respectively) destroys the simple concept of a state belonging to one particular band, such a mixing accounts very well for the properties of the positive-parity states with $E_x \leq 4.07$ MeV, again with oblate deformation for the nucleus. The properties of the 3.624- and 5.256-MeV levels suggest that they might be described as members of a $K^\pi = \frac{7}{2}^-$ band based on Nilsson orbit 10 $[Nn_z\Lambda = 303]$. The symmetric rotor model with oblate deformation ac-

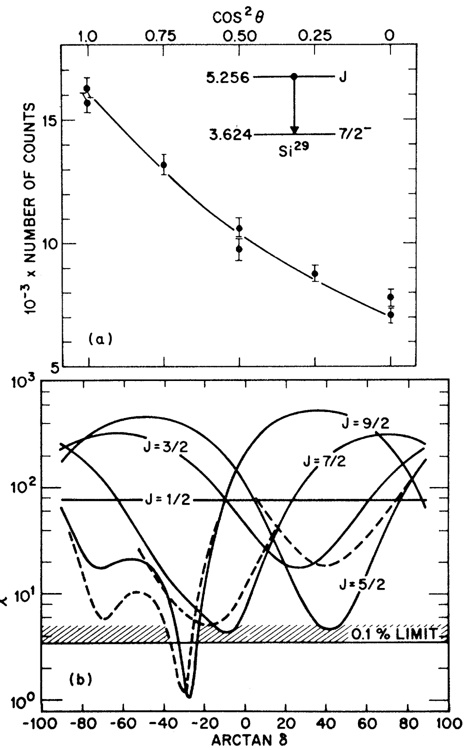


FIG. 2. Angular distribution of the 1631-keV $\gamma$ ray produced in the reaction $Mg^{26}$ ($\alpha$, $n$) $Si^{29}$ at $E_\alpha = 6.4$ MeV and assigned to the $Si^{29}$ 5.256 → 3.624 transition (upper half). The Ge(Li) counter was 15 cm from the reaction site. Lower half: The usual $\chi^2$ vs $\tan^{-1}\delta(1631)$ plots resulting from a fit of this distribution with the angular correlation formalism for the trial spins $J$ of the 5.256-MeV level. A relative population of 90 % and 10 % for the $|m| = \frac{1}{2}$ and $|m| = \frac{3}{2}$ substates was assumed, based on the relative transmission coefficients for $s$- and $p$-wave neutrons. The dashed curves show the results if a relative population of 67 % and 33 % is assumed.

counts for the low excitation energy of the $\frac{7}{2}^-$ state. Also, such a band is not expected to mix with states in the nearby positive-parity bands.[2]

We give some detailed consequences of this description. From the observed energy spacing between these two levels, we obtain $\hbar^2/2I = 180$ keV for the moment of inertia parameter, consistent with this picture. Extraband transitions to members of the $K = \frac{1}{2}$ or $\frac{3}{2}$ band are forbidden by $K$ selection rules; the 5.256-MeV level is observed to decay only to the 3.624-MeV level, while the lifetime of the 3.624-MeV state is long ($\tau_m = 4 \times 10^{-12}$ sec). We next examine the partial widths $\Gamma(M1)$ and $\Gamma(E2)$ of the 1.631 (5.256 → 3.624) transition, assuming this is a 100% branch. From the measured width of the 5.256-MeV level and multipole mixing of the 1.631-$\text{MeV}_\gamma$ ray we deduce the partial widths $\Gamma(M1) = 5.59 \pm 0.93$ and $\Gamma(E2) = 1.35 \pm 0.37$ meV, corresponding to $0.06 \pm 0.01$ and $27 \pm 7$ Weisskopf units, respectively. An $E2$ enhancement of this order is generally accepted as a sign of collective motion. Finally, we calculate the Nilsson-model estimate of $\Gamma(M1)$ and the predicted sign of the $E2/M1$ mixing ratio, and extract the quadrupole moment $|Q_0|$ from the measured $E2$ rate. The reduced $M1$ transition-matrix element $B(M1)$ for an intraband transition is given in terms of $g_K$ and $g_R$, the intrinsic and collective gyromagnetic ratios, and is

$$B(M1; J_iK \to J_fK) = \frac{3}{4\pi}\left(\frac{e\hbar}{2Mc}\right)^2 \langle J_iK10|J_fK\rangle^2 K^2 (g_K - g_R)^2. \qquad (1)$$

Here, $\langle J_iK10|J_fK\rangle$ is a vector coupling coefficient. For orbit 10, $g_K = -0.546$ and we estimate $g_R$ as $Z/A$. We find $\Gamma(M1, \text{Nilsson}) = 27.6$ meV, which overestimates the measured $\Gamma(M1)$ by a factor of 5.

The expression for $B(E2)$, the reduced $E2$ transition probability, in terms of the quadrupole moment $Q_0$, is

$$B(E2; J_iK \to J_fK) = (5/16\pi)e^2\langle J_iJ20|J_fK\rangle^2 Q_0^{\,2}. \qquad (2)$$

The measured transition speed $1.35 \pm 0.37$ meV results in $|Q_0| = 66 \pm 9$ F$^2$. For a comparison the Thomas-Fermi value estimated with deformation $\delta = -0.15$ is $|Q_0| = 20$ F$^2$.

The model predicts the sign of the $E2/M1$ mixing ratio and the sign of the deformation to be the same in this band.[9] Since $\delta(1.631) = -0.49$, the deformation is predicted to be oblate, in agreement with the conclusions of Bromley, Gove, and Litherland[1] and Hirko.[3] Altogether then, the model gives a reasonable account of the observed properties of the 3.624- and 5.526-MeV levels. We conclude, however, with the note that the observed properties of these two negative-parity states may be in agreement with weak-coupling model predictions if we attribute the 3.624-MeV level to the $[(Si^{28})_{0^+} \times \nu_{7/2^-}]_{7/2^-}$ configuration, and the 5.256-MeV level as the $J^\pi = \frac{9}{2}^-$ member of the multiplet due to the $[(Si^{28})_{2^+} \times \nu_{7/2^-}]_J$ configuration. Since our work suggests no other negative parity levels which could be reasonably assigned to the $[(Si^{28})_{2^+} \times \nu_{7/2^-}]$ configuration below an excitation energy 5.284 MeV, and the next known state is at 5.649 MeV,[4] we favor the strong-coupling description at this time. We are extending our study of $Si^{29}$ to include levels above 5.3-MeV excitation energy with the aim of establishing a clear choice between these descriptions.

*Research supported by the Lockheed Independent Research Fund. Work performed at the Stanford University tandem Van de Graaff Laboratory, supported by the National Science Foundation, and at the Lockheed Nuclear Physics Laboratory.

[1]D. A. Bromley, H. E. Gove, and A. E. Litherland, Can. J. Phys. 35, 1057 (1957).

[2]S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 29, No. 1 (1955).

[3]R. G. Hirko, thesis, Yale University, 1969 (unpublished).

[4]P. M. Endt and C. van der Leun, Nucl. Phys. A105, 1 (1967).

[5]R. E. White, Phys. Rev. 119, 757 (1960).

[6]A. E. Litherland and A. J. Ferguson, Can. J. Phys. 39, 788 (1961).

[7]E. K. Warburton, D. E. Alburger, and D. H. Wilkinson, Phys. Rev. 129, 2180 (1963).

[8]A. E. Blaugrund, Nucl. Phys. 88, 501 (1966).

[9]E. K. Warburton, A. R. Poletti, and J. W. Olness, Phys. Rev. 168, 1232 (1968).

---

# ANISOTROPIC SUPERFLUIDITY IN NEUTRON STAR MATTER*

M. Hoffberg
Vassar College, Poughkeepsie, New York

and

A. E. Glassgold and R. W. Richardson
New York University, New York, New York 10012

and

M. Ruderman
Columbia University, New York, New York 10027
(Received 16 February 1970)

The magnitude of the expected superfluid transition temperature and the form of the gap are obtained for regimes of neutron density expected in neutron-star cores. For neutron densities exceeding $1.5\times10^{14}$ g cm$^{-3}$ the gap is anisotropic but nodeless, leading to thermodynamic properties of a conventional BCS-type superfluid.

In the density regime $5\times10^{13}$ g cm$^{-3}$ $<\rho<6\times10^{14}$ g cm$^{-3}$, characteristic of neutron-star cores, the neutron density varies from about $\frac{1}{3}$ to near 4 times the density of neutrons in normal nuclei. The attraction between appropriately paired neutrons at the top of their Fermi sea causes a BCS-type superfluid behavior.[1-3] The magnitude and form of the gap in the single-particle excitation spectrum depends sensitively upon density. This is because the appropriate Fermi energy varies as $\rho^{2/3}$ and the known neutron-neutron phase shifts depend strongly on energy. When $\rho\lesssim1.5\times10^{14}$ g cm$^{-3}$, $^1S_0$ attraction gives conventional Cooper pairing; when $\rho\gtrsim1.5\times10^{14}$ g cm$^{-3}$, the significant attraction is in the $^3P_2$ state. All other $S$ and $P$ phase shifts are much smaller or repulsive in this latter regime, and the resulting gap is nonisotropic. Anisotropic superfluidity has been considered earlier for liquid He$^3$ with $P$- and $D$-wave pairing.[4,5]

We assume that for neutrons near the top of their Fermi sea, all interactions vanish except $^1S_0$ and $^3P_2$, and that these can be adequately described by a separable-type potential chosen to fit the scattering data.[6] Inside normal, symmetrical nuclear matter the effective phase shift $\delta^*$ and mass $m^*$ differ from their values appropriate to free space. In an almost pure neutron environment, these differences are expected to be much smaller because the $n$-$p$ interactions, which give the dominant contribution to them in normal nuclear matter, are not present. We assume $\delta^*\approx\delta$ and $m^*\approx m$, which is also supported by explicit numerical calculations at densities near those of neutrons in nuclei.[7]

The superfluid transition temperature $T_c$ obtained numerically from the usual BCS linear equation[8] with this interaction is given in Fig. 1. A qualitative fit[3] to these results and to others with slightly different $m^*$ is

$$\ln\frac{kT_c}{2E_\mathrm{F}}\approx-\frac{\pi}{2}\frac{m}{m^*}\cot\delta^*, \tag{1}$$

where $\delta^*$ is either the $^1S_0$ or $^3P_2$ phase shift, whichever is the dominant attractive one at the