WHAT IS THE POMERANCHUK TRAJECTORY?*

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We calculate the high-energy amplitude in quantum electrodynamics and scalar electrodynamics, and find that the Pomeranchon is a moving Regge pole near $J=\frac{3}{2}$ when t is equal to or near the two-particle threshold, and is a fixed branch point at $J=1$ for $t\leq 0$. This is due to the promotion from 1 to $\frac{3}{2}$ for the power of s of the tower-diagram amplitudes at threshold. The Gribov paradox is thereby automatically resolved.

Over the past two years, we have carried through a field-theoretic investigation[1-3] on the high-energy behavior of scattering amplitudes near the forward direction, i.e., in the region $s\to\infty$ with t fixed at a nonpositive value, where s is the square of the c.m. energy and t is the negative of the momentum-transfer squared. In this Letter, we shall consider these amplitudes in the region $s\to\infty$ with t fixed at a positive value.

The contents of this Letter are first summarized as follows: (i) At $t=4\lambda^2$, where $\lambda>0$ is the mass of the exchanged vector meson ("photon"), there is a promotion for the power of s from 1 to $\frac{3}{2}$. For example, the scattering amplitude corresponding to the diagrams in Fig. 1(a) is proportional to s for $t\neq 4\lambda^2$, but is proportional to $s^{3/2}$ at $t=4\lambda^2$. This promotion also occurs for other diagrams with two-photon cuts in the t channel. Some additional examples are shown in Figs. 1(b) and 1(c). These diagrams will be called tower diagrams. (ii) The impact-diagram rules[3] can be generalized to the region $t\sim 4\lambda^2$. Applying these rules to the tower diagrams of Figs. 1(a)-1(c), we obtain the corresponding amplitude explicitly. One particular result is that the amplitude for the sum of N loop diagrams in Fig. 1(c) is of the order of $s^{3/2}(\ln s)^N$ at $t=4\lambda^2$. [This is to be compared with $s(\ln s)^N$ for $t\neq 4\lambda^2$.] Similar considerations apply to other processes $a+b\to a'+b'$. (iii) Because of the phenomenon of promotion, other diagrams, such as those illustrated on Fig. 1(d), can no longer cancel the diagrams in Fig. 1(c) as they do in the region $t\leq 0$ to preserve unitarity. Put in another way, the scattering amplitude does not have to satisfy the Froissart bound[4] when $t>0$, and the tower diagrams in Figs. 1(a)-1(c) dominate over those in Fig. 1(d) when t is at or near $4\lambda^2$. (iv) Summing up the high-energy amplitudes for the tower diagrams in the region t at or near $4\lambda^2$, we find that the scattering amplitude for the process $a+b\to a'+b'$ is of the form

$$\tfrac{1}{8}s^{3/2+\kappa}(1+i)\lambda^{-3}I^{aa'}I^{bb'}, \qquad (1)$$

where $I^{aa'}$ ($I^{bb'}$) is a function of a and a' (b and b') only and κ is proportional to e^4. This means that the leading singularity in the J plane is a moving Regge pole located to the right of $J=\frac{3}{2}$ as t is near $4\lambda^2$. (v) The Gribov paradox,[5] which forbids the scattering amplitude from taking the form $sf(t)$, is automatically resolved.

Having summarized the results, let us begin the discussion by observing that an example of promotion had already been found in potential scattering.[6] Consider, for example, the Schrödinger equation with a Yukawa potential $-Ge^{-r}/$

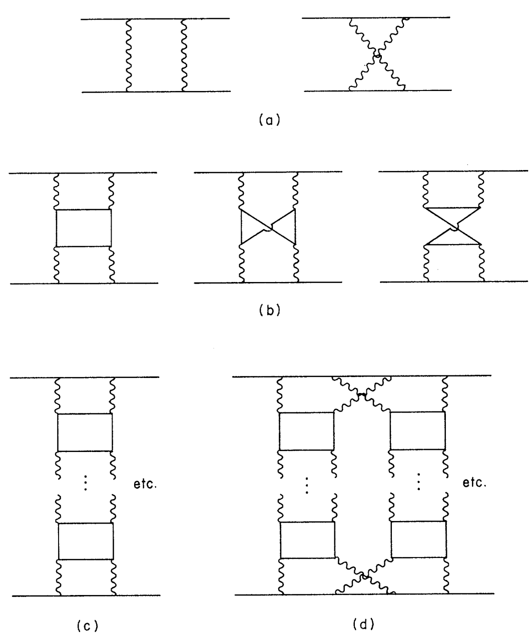

FIG. 1. Some relevant diagrams for electron-electron scattering. The s channel is from left to right, and the t channel is from top to bottom.

r, $1 \gg G > 0$. In this case the leading Regge pole is given by the perturbation series[7]

$$\alpha(k_t) \sim -1 + iG/(2k_t), \tag{2}$$

where k_t is the momentum of the particle. Thus the leading Regge pole is located near -1 in the l plane, if the potential is weak. The perturbation series (2) breaks down at the threshold $k_t = 0$, and it has been found that at $k_t = 0$, there is a Regge pole located at[6]

$$\alpha(0) \sim -\tfrac{1}{2} + \tfrac{1}{2}G. \tag{3}$$

We have further shown[8] that (2) and (3) correspond to the same Regge pole; therefore there is a promotion for the leading pole from the neighborhood of $l = -1$ to that of $l = -\frac{1}{2}$, if the coupling is weak. This promotion occurs when $|k_t|$ is within G. As G increases, the leading Regge pole moves further to the right.

This promotion also occurs for the ψ^3 theory. Consider the ladder diagrams in the limit $s \to \infty$, t fixed. Then promotion occurs for each individual diagram. Specifically, it is well known that the diagram of $N+1$ rungs gives the amplitude[9]

$$\mathfrak{M}_N \sim -g^2 s^{-1} (\ln s)^N [I(t)]^N / N!, \tag{4}$$

where

$$I(t) = -(g/4\pi)^2 \int_0^1 dx [tx(1-x) - 1 + i\epsilon]^{-1}. \tag{5}$$

In the above, g is the coupling constant, and the mass of the scalar mesons is chosen to be unity. At $t = 4$, the right side of (5) is a divergent integral. This does not mean that $\mathfrak{M}_N$ does not exist at $t = 4$. It simply means that the asymptotic form (4) does not hold at $t = 4$. To obtain the asymptotic form of $\mathfrak{M}_N$ at $t = 4$, we must start again from the exact expression for $\mathfrak{M}_N$, set $t = 4$, and take the limit $s \to \infty$. We get, for $t = 4$,

$$\mathfrak{M}_N \sim i\pi g^{2(N+1)} 2^{-5N} \pi^{-N} [(N-1)!]^{-I} s^{-1/2} \times (\ln s)^{N-1}. \tag{6}$$

Comparing (4) with (6), we see that the power of s for the scattering amplitude of an individual diagram is promoted from -1 for $t \neq 4$ to $-\frac{1}{2}$ for $t = 4$. Summing over all N, we get, for $t = 4$, the amplitude

$$ig^4 (32)^{-1} s^{-1/2 + g^2 (32\pi)^{-1}}. \tag{7}$$

Thus, similar to the potential-scattering case, the leading Regge pole is promoted from the neighborhood of $l = -\frac{1}{2}$ to that of $l = -1$, if the coupling constant is small.

This promotion, while of academic interest in potential scattering and ψ^3 theory, is very significant in quantum electrodynamics. This is because it throws light on the nature of the Pomeranchon. We recall that in quantum electrodynamics, there are Feynman diagrams which give amplitudes of the order of $s(\ln s)^N$, $N = 1, 2, 3, \cdots$ in the limit $s \to \infty$, t fixed. The lowest order Feynman diagrams which give an amplitude proportional to $s(\ln s)^N$ are the N electron-loop diagrams illustrated in Fig. 1(c). We have obtained the leading term for corresponding scattering amplitude in the limit $s \to \infty$, t fixed.[10] If we sum over these leading terms, we obtain, at the forward direction $t = 0$, a result of the order of $is^{1+a}/\ln s$, where

$$a = 11\alpha^2\pi/32, \tag{8}$$

in massive quantum electrodynamics, and

$$a = 5\alpha^2\pi/32, \tag{9}$$

in massive scalar electrodynamics. This result violates s-channel unitarity and is therefore physically meaningless. Since the perturbation series can always be made manifestly unitary, it is possible to "unitarize" the scattering amplitude by including more Feynman diagrams. In particular, to unitarize the amplitude from the tower diagrams of Figs. 1(a)-1(c), we must include the diagrams like those illustrated in Fig. 1(d). The amplitudes from the diagrams of Fig. 1(d) cancel those of Figs. 1(a)-1(c) and unitarity is preserved. This point has been discussed previously.[11] This is no assurance, however, that such a procedure yields the correct asymptotic amplitude, which remains one of the most difficult and challenging quantities to calculate in the field of high-energy physics.

Such complications disappear, however, when t is at or near $4\lambda^2$. This is because the amplitudes from the tower diagrams of Fig. 1(c) are promoted from s to $s^{3/2}$ (logarithmic factors unchanged), while those from Fig. 1(d) remain to be of the order of s. Thus the former amplitudes dominate over the latter, and cancellation cannot occur.

The amplitudes from the tower diagrams of Fig. 1(c) are difficult to treat with the conventional Feynman method. We have generalized the impact-diagram method to handle this case, and the amplitude for the sum of the N-loop tower diagrams for $a + b \to a' + b'$ is then found to be, at $t = 4\lambda^2$,

$$\tfrac{1}{8} s^{3/2} (1+i) \lambda^{-3} I^{aa'} I^{bb'} \kappa^N (\ln s)^N / N!. \tag{10}$$

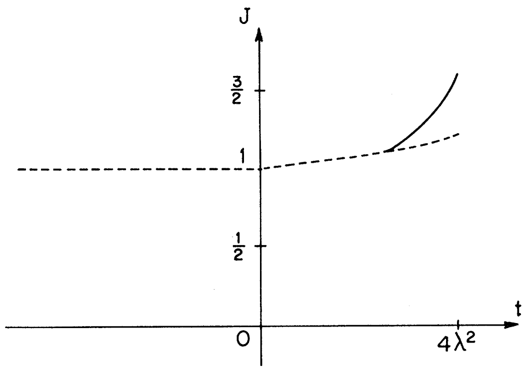

FIG. 2. Schematic plot for the Pomeranchon. The dotted line represents the position of the branch point, and the solid line represents the position of the p Regge pole.

Summing up all N, we obtain (1).

We discuss the significance of (1) and (10). Eq. (1) shows that when t is near $4\lambda^2$, the leading singularity in the J plane comes from the tower diagrams and is a moving Regge pole located to the right of $J=\frac{3}{2}$. In hadron physics, where the coupling constant is strong, this Regge pole should be further to the right, and makes a bound state if it passes $J=2$. Our previous calculations[10] indicate that for $t \leqslant 0$, the leading singularity in the J plane, coming from all the diagrams of Fig. 1, is a fixed branch point at $J=1$. This branch point starts to move when t is positive, and for some t between 0 and $4\lambda^2$ a Regge pole emerges from the second sheet through the branch point and moves ahead.[12] At $t=4\lambda^2$, this Regge pole is in the neighborhood of $J=\frac{3}{2}$ if the coupling is weak, and is further to the right for strong couplings. This is schematically plotted in Fig. 2.

Gribov argued that the scattering amplitude cannot be of the form $sf(t)$ when t is above the elastic threshold. Apparently, the promotion phenomenon guarantees the scattering amplitude to be at least of the order of $s^{3/2}$ at $t=4\lambda^2$. Thus Gribov's paradox is trivially resolved.

Promotion always occurs when t is at a two-body threshold. What we have found here is a diagrammatic way to study the promotion. For instance, our argument shows that the diagrams which generate the fermion Regge pole[13] are promoted from $s^{1/2}$ to s at $t=(M+\lambda)^2$, where M is the mass of the fermion. Thus the fermion pole is in the neighborhood of $J=1$ at $t=(M+\lambda)^2$. On the other hand, no promotion occurs on a three-particle threshold.

Finally, we mention that this complicated behavior for the Pomeranchon causes similar complications in all other Regge trajectories. This has obvious experimental implications which can be easily tested.

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