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SIMPLE MODEL FOR PROTON-PROTON SCATTERING*

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A semiclassical model for inelastic and elastic proton-proton scattering is presented. A fit to recent π production counter data is made. The resulting proton-proton elastic scattering and proton electromagnetic form factor are discussed. Reasonable agreement with the data is obtained.

Sometimes very simple models are useful tools in gaining insight into limited aspects of complicated processes. Here we present a model which explains some aspects of elastic and inelastic proton-proton scattering. The model is closely related in philosophy to one originally suggested by Lewis, Oppenheimer, and Wouthuysen[1,2] and goes as follows. Since boson—in particular, pion—production dominates the cross section, one assumes that the bosons are produced in states corresponding to a field coupled to a classical source. The structure of the source is related to the structure of the colliding protons. The model's phenomenological input is the source function.

The analysis of a field coupled to a classical source[3] shows that the average number of particles produced per unit volume in momentum space is given by

$$\bar{n}(\vec{k})=(2\pi)^{-3}(2\omega)^{-1}|f(\vec{k})|^2, \tag{1}$$

where

$$f(\vec{k})=\int d^4x\,\rho(x)\,e^{-ik\cdot x}, \tag{2}$$

$$k\cdot x=\vec{k}\cdot\vec{x}-\omega t,\quad \omega=(\vec{k}^2+m^2)^{1/2},$$

and m is the pion mass. In a collision of two protons the source, $\rho(x)$, implicitly contains a dependence on the impact parameter, $b_\perp$, of the protons. The average of $\bar{n}$ over $b_\perp$ is directly measured in counter experiments which determine the flux of outgoing particle in a given range of momentum and solid angle, all other variables being ignored. The classical source model also corresponds to particles which are produced in uncorrelated states with a Poisson number distribution. Thus, the probability p_0 that a collision will take place with no particle production is given by

$$p_0=e^{-\bar{n}(b_\perp)},$$

where

$$\bar{n}(\vec{b}_\perp)=\int d^3k\,\bar{n}(\vec{k},\vec{b}_\perp).$$

This immediately gives the absorption disk for proton-proton elastic scattering. The elastic differential cross section is given by

$$\left.\frac{d\sigma}{dt}\right|_{\text{elastic}}=\pi\left|\int_0^\infty db_\perp\,b_\perp(e^{-\bar{n}(b_\perp)/2}-1)J_0(b_\perp\sqrt{-t})\right|^2. \tag{3}$$

This is the usual partial-wave expansion with the Legendre polynomials approximated by their large-l asymptotic form.

Now let us turn to the physical input, i.e., specification of the source function. It is natural to relate the source, ρ, to the overlap of the proton structures during their collision. To be specific, ρ will be taken to be the product of two Lorentz-contracted Gaussian blobs moving uniformly in opposite directions along the z axis with an impact parameter $b_\perp$. Thus, in the proton-proton center of mass system ρ is given by

$$\rho(x)=g(u)\psi^+(x)\psi^-(x), \tag{4}$$

where

$$\psi^\pm(x)=\exp\{-M^2[(\vec{x}_\perp \pm \tfrac{1}{2}\vec{b}_\perp)^2+(u_0 z \pm ut)^2]\}, \quad u=(\text{proton momentum})/(\text{proton mass}), \tag{5}$$

and

$$u_0=\gamma=(u^2+1)^{1/2}.$$

The constant M appearing in Eq. (5) has the dimensions of a mass and determines the size of the proton's structure. The coupling constant, its energy dependence, and M are left as free parameters to be determined by the experimental data. The Gaussian form for $\psi^\pm$ is very convenient for calculations and probably not too unreasonable physically.

The important quantity, $\bar{n}(\vec{k},\vec{b}_\perp)$, is easily determined from Eqs. (1), (2), (4), and (5). The result is

$$\bar{n}(\vec{k},\vec{b}_\perp)=G^2(g)e^{-M^2b_\perp^2}(1/\omega)\exp\{-(1/4M^2)[(1+u^{-2})k_\perp^2+(u_0^{-2}+u^{-2})k_z^2]\},$$

where

$$G^2(g)=g^2(\pi/4)(2M^2)^{-4}.$$

From this expression the two major results of this model immediately follow.

(I) Integrating $\bar{n}(\vec{k},\vec{b}_\perp)$ over $\vec{b}_\perp$ gives the production cross section relevant to the type of counter experiments mentioned above:

$$\frac{d^2\sigma}{d\Omega dk}=G^2\frac{\pi}{M^2}\frac{k^2}{\omega}\exp\{-(1/4M^2)[(1+u^{-2})k_\perp^2+(u_0^{-2}+u^{-2})k_z^2]\}. \tag{6}$$

(II) Integrating $\bar{n}(\vec{k},\vec{b}_\perp)$ over $\vec{k}$ gives one quarter of the imaginary part of the phase shift. For $M \gg \frac{1}{2}m$ the integral can be evaluated giving

$$\bar{n}(b_\perp)\simeq 2A\,e^{-M^2b_\perp^2},$$

where

$$A=2\pi M^2G^2\ln(2u^2+1).$$

This expression for $\bar{n}(b_\perp)$ can now be used to calculate the elastic differential cross section through Eq. (3). The result is

$$\left.\frac{d\sigma}{dt}\right|_{p\text{-}p\text{ elastic}}=\pi|F(t)|^2, \tag{7}$$

where

$$F(t)=\frac{1}{2M^2}\sum_{n=1}^{\infty}\frac{(-A)^n}{nn!}\exp\left(\frac{t}{4M^2n}\right).$$

Figures 1 and 2 show a comparison of Eq. (6) with the recent data of Day et al.[4] for π^+ production in proton-proton scattering at 12.2 GeV/c. For this comparison, the parameter M^2 was taken to be 0.0625 GeV^2 ($c=1$) and g^2 was chosen

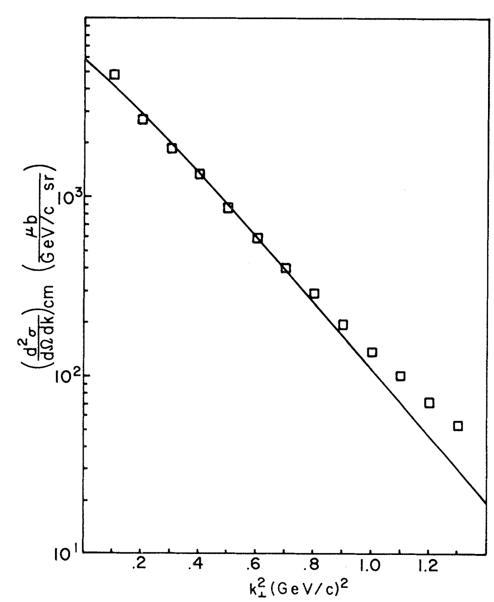

FIG. 1. A comparison of the π^+ data of Day et al., represented by open squares, with the present model for k_z fixed at 0.6 GeV/c.

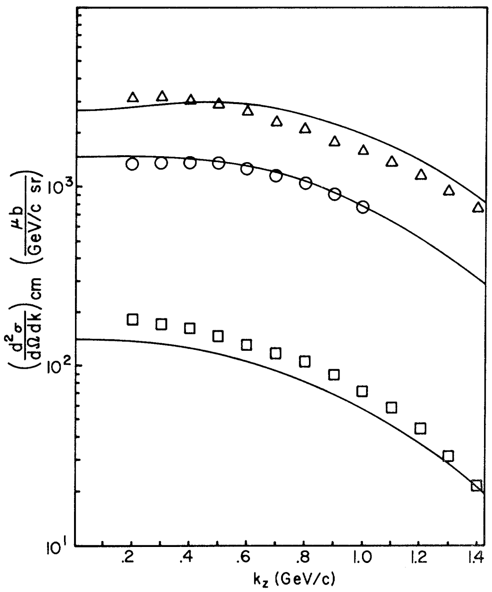

FIG. 2. A comparison of the π^+ data of Day et al. with the present model for $p_\perp^2$ fixed at 0.21 (open triangles), 0.41 (open circles), and 1.01 (open squares).

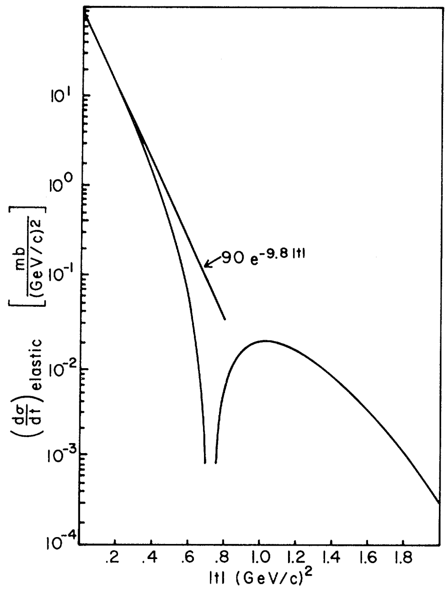

FIG. 3. A plot of p-p elastic scattering derived from the present model.

such that $G^2 = 0.78$ at the value of u corresponding to a beam momentum of 12.2 GeV/c. Figure 3 shows a plot of the proton-proton elastic differential cross section as calculated from Eq. (7). In calculating the elastic cross section, it has been assumed that π^- and π^0 production can be approximately accounted for by multiplying the absorption due to π^+ production by a factor of 2, and that all other production processes have negligible cross sections in comparison; i.e.,

$$\bar{n}(b_\perp)_{tot} = 2\bar{n}(b_\perp)_{\pi^+}.$$

A comparison can be made with the proton electromagnetic form factor if one identifies the form factor as the Fourier transform of $|\psi^+|^2$ in the proton rest system. The calculation gives an electromagnetic form factor proportional to $\exp(-2|t|)$. Thus, with this identification, the model approximately satisfies the Wu-Yang[5] conjecture and approximately fits the data.

The important results of this model are the quantitative relations between the transverse and longitudinal distributions of pions observed in proton-proton inelastic scattering, the proton-proton elastic cross section, and the electromagnetic form factor.

A word on the limitations of the model is called for at this point. No detailed information on the protons after an inelastic collision is contained in the model since the protons are assumed to move uniformly during the collision.

Isobar production is not specifically included in this model, but some of the production cross section may be approximating isobar production. One might conjecture that the isobars are the result of a final-state interaction which, on the average, may not significantly change a cross section of the type measured by Day et al.

Since the model is semiclassical, it represents an approximation to all processes simultaneously and the conclusions only apply to the most likely processes.

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