$d\hat{R}_{LM}/dr^* = [V_{LM}(r)-\omega^2]\hat{K}_{LM}$. Fortunately, the problem has a solution (many, in fact, since it turns out that one of the functions f, g, h, k, or n may be chosen arbitrarily).

If we let $n(r)=1-2m/r$ where m is the mass of the Schwarzschild field we find that

$$f(r)=[\lambda(\lambda+1)r^2+3\lambda mr+6m^2]/r^2(\lambda r+3m), \quad g(r)=1,$$

$$h(r)=i[-\lambda r^2+3\lambda mr+3m^2]/(r-2m)(\lambda r+3m), \quad k(r)=-ir^2/(r-2m), \quad \lambda=\tfrac{1}{2}(L-1)(L+2), \tag{2}$$

and that

$$r^*=r+2m\ln(r/2m-1), \tag{3}$$

so that

$$d\hat{K}_{LM}/dr^*=\hat{R}_{LM}, \quad d\hat{R}_{LM}/dr^*=[V_L(r^*)-\omega^2]\hat{K}_{LM}, \tag{4}$$

where

$$V_L(r)=\left(\frac{1-2m}{r}\right)\frac{2\lambda^2(\lambda+1)r^3+6\lambda^2mr^2+18\lambda m^2r+18m^3}{r^3(\lambda r+3m)^2}. \tag{5}$$

Note that the system (4) written as a single second-order equation assumes formally the Schrödinger form

$$d^2\hat{K}_{LM}/dr^{*2}+[\omega^2-V_L(r)]\hat{K}_{LM}=0.$$

Thanks are due to John A. Wheeler for conversations in which he suggested the approach to use in finding an effective-potential form for the even-parity equations.

*Work supported by the National Science Foundation.

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ac-JOSEPHSON-EFFECT DETERMINATION OF e/h WITH SUB-PART-PER-MILLION ACCURACY*

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An ac-Josephson-effect determination of e/h has been made using a new method. The accuracy of the result referred to the local voltage standard is 0.13 ppm. The accuracy referred to the National Bureau of Standards as-maintained volt is 0.46 ppm. The final value is $2e/h = 483.593\,65 \pm 0.000\,22$ MHz/$\mu V_{69\,\mathrm{NBS}}$ (0.46 ppm).

The work of Parker, Taylor, and Langenberg[1,2] has established the ac Josephson effect in systems of weakly coupled superconductors as an important factor contributing to our knowledge of the fundamental physical constants[3] and as a potential dc voltage standard.[4] Both of these applications are based upon the fact that in a Josephson-junction device biased at a dc potential difference V, there exists a supercurrent oscillating at a frequency $\nu = 2eV/h$; the frequency-voltage ratio is thus simply the fundamental physical constant $2e/h$. High-accuracy determinations of the Josephson frequency-voltage ratio have been reported by Parker et al.[2] and by Petley and Morris.[5] The former has recently been reassessed and revised by Denenstein et al.[6] Both now have quoted one-standard-deviation uncertainties of 2.2 ppm and are in excellent agreement. A significant increase in the accuracy with which the frequency-voltage ratio could be determined would have important implications for our knowledge of the fundamental physical constants and for the future of the Josephson junction as a voltage standard. We report here the initial results of a program aimed at achieving an order of magnitude increase in accuracy.

The accuracy-limiting operation in any accurate determination of the Josephson frequency-voltage ratio is the comparison of the Josephson-junction voltage with a reference standard-cell voltage. In previous determinations, junction voltages of the order of 1 mV were measured using modifications of conventional potentiometers incorporating adjustable resistance elements equivalent to a slidewire. An appreciable increase in accuracy has been achieved in the present experiments through two major improvements. The first is based on the recognition that a Josephson junction is a variable voltage source; microwave-induced steps[7] can be adjusted to a desired voltage simply by varying the frequency of the input radiation. This fact makes it possible to carry out the voltage comparison with the reference standard cell using a fixed-ratio resistance network. The adjustable resistance element of the conventional potentiometer can be eliminated, together with its accompanying calibration uncertainties. The second improvement is an increase in the Josephson-junction voltage by a factor of 10, to about 10 mV. This has several advantages: (1) The random scatter of the data due to noise in the potentiometer and null detector and to fluctuations in thermoelectric emf's in the measuring circuit is scaled down in relative importance by a factor of 10 to about 0.1 ppm. (2) Uncertainties due to lead-resistance corrections are significantly reduced because the impedance level of the voltage-measuring system can be increased. (3) The fixed resistance ratio required for comparison of the junction voltage with the standard-cell voltage can be reduced from 1000:1 to 100:1, thus reducing the ratio-calibration uncertainty.

In the present experiments, we have used a double series-parallel exchange comparator (SPC) to establish an accurate 100:1 voltage ratio. A simplified circuit diagram is shown in Fig. 1. Its operation depends on the fact that if n nominally equal resistors are connected first in series and then in parallel, the ratio of the resistances of the two combinations is n^2 with an error reduced to second order in the degree of resistance matching.[8] The use of tetrahedral junctions between resistors[9,10] and compensating ("fan") resistors[11] (labeled R in Fig. 1) for paralleling the series-parallel network permit achievement of high accuracy in the series-to-parallel transfer despite relatively high lead and connection resistances. Two sets of ten matched resistors, one in series and one in parallel, thus provide a highly accurate 100:1 resistance ratio. The resistance network is fed by a high-stability (0.5 ppm/h current drift) power supply regulated by a mercury battery under essentially no load. For a current of 1 mA, 1 V appears across the series network and 10 mV appears across the parallel network. Thus a junction and a standard cell voltage can be balanced simultaneously. By reversing the SPC current, the standard cell, and the junction bias current, using the switches indicated in Fig. 1, the effect of thermoelectric voltages in the circuit is eliminated. A second pair of balances with the resistor sets "exchanged" reduces the effect of inequality of resistance between the two sets to second order, if the results from the two pairs of balances are averaged. While in principle a simultaneous balance is quite feasible and is indicated in Fig. 1 for simplicity, in practice a single null-detection system was used. This system consisted of a

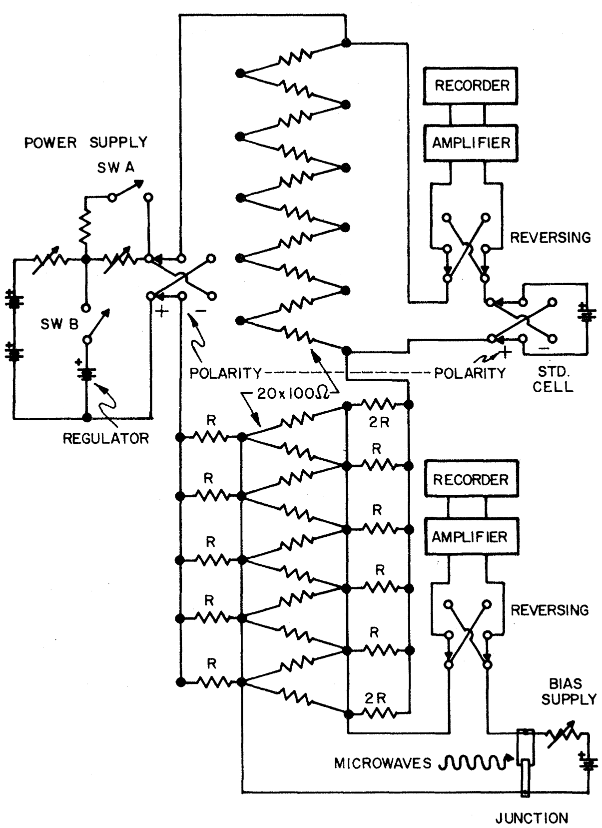

FIG. 1. Simplified circuit diagram of the voltage-measurement apparatus.

photocell-galvanometer amplifier with a specially added isolated output which fed a strip chart recorder. Each balance was recorded for both positions of the reversing switch and with an accurately known calibrating signal (using Sw. A in Fig. 1) of about 1 ppm. Usually the junction balance was made first, and then the null detector was immediately switched to the standard cell circuit for that balance. The effect of noise in the measuring circuit was reduced by visual averaging of the recorded null-detector outputs. The main 100-Ω resistors of the SPC were mounted in an oil-filled $\frac{1}{2}$-in.-thick aluminum case, and the entire instrument was housed in thermally insulated, thermistor-bridge temperature-regulated enclosure. Lack of space prohibits complete discussion of this instrument here, but a detailed discussion of its design and performance will be presented elsewhere.[12]

The 10-mV Josephson device voltage required by our present method could in principle be achieved by operating many (~10) Josephson junctions in series, each contributing ~1 mV. In practice, this would require careful matching of the characteristics of all the junctions and independent biasing of each junction, with a considerable resulting increase in the complexity of the device geometry and the bias circuitry. We have found that voltages >10 mV can reliably be achieved with a single junction or with at most two or three in series. Among the factors which make this possible are the following: (1) Development of techniques for fabricating near-ideal lead–lead-oxide–lead tunnel junctions with tightly controlled and matched characteristics. (2) Use of the "linear" or "in-line" junction geometry indicated in Fig. 1. This is crucial to the observation of satisfactory steps at high voltages; the self magnetic fields associated with the biasing current tend to "quench" the steps at voltages greater than $2\Delta/e$ in nonoptimal geometries like the conventional cross-type junction. We have observed steps in single junctions at voltages >10 mV ($n > 500$, $V_n > 7\Delta/e$) with amplitudes ~20 μA. (3) Overcoating of junctions for protection against the environment. This permits long-term storage (at liquid-nitrogen temperatures) and repeated reuse of exceptionally good junctions. One of the junctions used in the present experiments was stored for six months and reused several times.

The experiments were carried out at 1.2 K and in zero applied magnetic field, using steps induced by an X-band klystron phase-locked to a quartz-crystal oscillator. Ten runs were made using two different junction devices, one consisting of a single junction, the other of three junctions in series. The results are plotted in Fig. 2. The standard deviation of the weighted mean of the ten measurements is 0.05 ppm. It includes contributions from fluctuations in thermoelectric voltages and noise in the measuring circuitry, day-to-day temperature fluctuations of the local voltage standard, and any randomly fluctuating components of the possible systematic error sources discussed below. We note that the random uncertainty associated with maintenance of our local voltage standard via cell intercomparisons is completely negligible (≲0.005 ppm, or ± 5 nV).

The uncertainties associated with possible systematic errors in the voltage measurement are of two types. The first type consists of uncer-

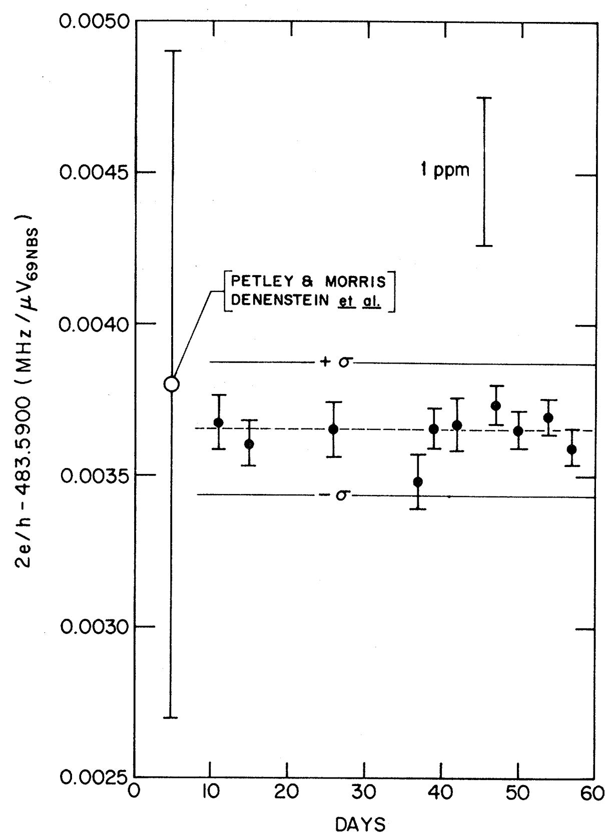

FIG. 2. Experimental values of $2e/h$ as a function of time. The measurements span the period December 1969 through February 1970. The error bars on the experimental points are the standard deviations of the data of the corresponding runs. The final value and its standard-deviation uncertainty are indicated by dashed and solid lines. The value quoted by previous workers is shown for comparison.

tainties in the comparison (via the SPC) of the Josephson-device voltage with our local voltage standard, a commercial set of six saturated standard cells mounted in a temperature-regulated air bath. The second type arises from the necessity for intercomparison of our local voltage standard with the National Bureau of Standards (NBS) as-maintained volt to permit comparison with other experiments. The possible sources of systematic error in the first category and the corresponding one-standard-deviation uncertainties are (in ppm) (1) mismatch of the series-parallel network, <0.005; (2) changes in self-heating of the series-parallel network under exchange, $\lesssim 0.05$; (3) transfer resistance of the tetrahedral junctions, <0.005; (4) mismatch of the fan resistors, <0.01; (5) temperature stability of the SPC enclosure, $\lesssim 0.01$; (6) stability of the SPC operating current, ~0.05; (7) accuracy of the calibrating signal, ~0.05; (8) leakage resistances, <0.02; and (9) aging or drift of the local voltage standard, ~0.05. Items (1), (4), (5), and (6) were frequently checked. Item (9) is the uncertainty in our knowledge of the drift of the local voltage standard over the period of the present measurements, based on $3\frac{1}{2}$ years experience with the standard, during which time four transfer intercomparisons at NBS were made. (All voltage-measurement equipment was powered with batteries in a special shielded room to preclude any spurious shifts in the standard-cell emf's from ac.[13]) The root-sum-square total of these uncertainties, together with the random uncertainty of the data cited above and the frequency measurement uncertainty (0.05 ppm), is 0.13 ppm. This is the "local accuracy" of the present experiments, a measure of the total experimental uncertainty associated with the comparison of our Josephson-device voltage with our local voltage standard.

The accuracy of the present experiments when referred to the NBS as-maintained volt is determined almost entirely by the uncertainty associated with intercomparison of the local voltage standard with the NBS volt. The standard NBS Report of Calibration form [form NBS-532a(11-68)] quotes a one-standard-deviation uncertainty of 0.2 ppm for the intercomparison at NBS. To this must be added an estimate of the uncertainty which should be associated with the effects of physical transport of the local standard to and from NBS. On the basis of NBS experience with such transfers,[14] we have assigned 0.4 ppm to this source. The root-sum-square total uncertainty is therefore 0.46 ppm. This is the measure of the overall accuracy of the present experiments.

Our final value of $2e/h$ with all corrections included is 483.59365 ± 0.00022 MHz/$\mu V_{69\,\mathrm{NBS}}$ (0.46 ppm). This is 0.3 ppm higher than the value reported by Parker et al.[2] and 0.3 ppm lower than the (coincident) values of Petley and Morris[5] and Denenstein et al.[6] The agreement is clearly excellent. The quoted accuracy of our value exceeds those of the earlier measurements by a factor of 5. The full implications of this value for our knowledge of the fundamental constants must await its inclusion in a new least-squares adjustment of the constants. It is, however, of interest to calculate the value of the fine-structure constant implied by our result. Using Eq. (92) of Ref. 3,

$$\alpha^{-1} = \left[\frac{1}{4R_\infty}\frac{c\,\Omega_{\mathrm{abs}}}{\Omega_{\mathrm{NBS}}}\frac{\mu_p'}{\mu_B}\frac{2e/h}{\gamma_p'}\right]^{1/2},$$

values of auxiliary constants listed in Table XI of Ref. 3, and a value of the proton gyromagnetic ratio γ_p' given by the weighted mean of the experimental values given in Eqs. (51) and (54) of Ref. 3 (converted to NBS 1969 units), we find $\alpha^{-1} = 137.036\,10(22)$ (1.6 ppm). This is 0.15 ppm higher than the value deduced "without quantum electrodynamic theory" (WQED) by Taylor, Parker, and Langenberg,[3] $\alpha_{\mathrm{WQED}}^{-1} = 137.036\,08(26)$ (1.9 ppm). We note that if α^{-1} could be determined experimentally with an uncertainty of 1 ppm or better, the above equation could be used to determine an indirect value of γ_p' with an accuracy greater than that of current direct experimental determinations. It should also be noted that our present experimental value of e/h is sufficiently accurate to qualify it as an "auxiliary constant" in the sense used by Taylor et al. and other workers; i.e., it now contributes no significant uncertainty to our knowledge of the fundamental constants.

With more measurements and added experience with the instrumentation, it should be possible to increase the local accuracy of the determination of e/h to better than 0.1 ppm. Our present local accuracy is already better than the accuracy quoted by NBS for routine standard-cell intercomparisons. The technology required to qualify the Josephson junction as a fundamental voltage standard is thus clearly already in hand. Furthermore, it should be possible to increase the overall accuracy of the method in terms of NBS units to the 0.1-ppm level or better by reducing the transportation and intercomparison uncertainties. The first may be accomplished by more frequent and more carefully studied voltage transfers or by performing the experiments at the site of the NBS voltage standard. The latter, however, does not solve the problem of disseminating the volt to other experimenters, in particular to other national laboratories such as the National Physical Laboratory (Great Britain). With regard to the second uncertainty, we note that the uncertainty associated with our own intercomparisons on our local voltage standard is ≤ 0.005 ppm. This suggests that significant improvement on the 0.2 ppm quoted by NBS for cell intercomparisons is possible. In any case, the present experiments indicate that the ultimate limit on the absolute accuracy of the ac-Josephson-effect determination of e/h is the accuracy with which the primary voltage standard can be established, maintained, and referred to.

We thank B. N. Taylor for his encouragement, RCA for supplying evaporation masks, and W. J. Hamer for arranging the calibration of the standard cells. One of us (A.D.) thanks the National Research Council for a NRC-NAS-NAE Postdoctoral Research Associateship which he held at NBS during an intermediate stage in the planning of the present experiments.

*Supported by the National Science Foundation and the Advanced Research Projects Agency.

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MEASUREMENT OF THE ISOTROPIC BACKGROUND RADIATION IN THE FAR INFRARED*

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(Received 23 January 1970)

Measurements made at an altitude of 40 km with a balloon-borne radiometer that has three separate spectral responses in the spectral region below 20 cm^{-1} indicate that the spectrum of the isotropic background radiation may not be thermal.

The discovery by Penzias and Wilson[1] of an isotropic background radiation and its subsequent interpretation by Dicke et al.[2] as the red-shifted remnant of the thermal radiation of a primordial cosmic fireball has prompted many measurements to determine the spectrum and isotropy of this radiation. Ground-based radiometer measurements[3] in the region from 0.1 to 3 cm^{-1} indicate that this part of the spectrum is consistent with a blackbody distribution of ~2.7°K. Furthermore, Bortolot, Clauser, and Thaddeus[4] have measured a temperature of 2.8°K at 3.8 cm^{-1} and inferred upper limits of 4.7, 5.4, 8.1°K at 7.6, 18, and 28 cm^{-1} from the population ratios of rotational states of interstellar molecules. The only direct measurements in the spectral region around the blackbody peak were made by Shivanandan, Houck, and Harwit[5] and by Houck and Harwit[6] with a rocket-borne far-infrared telescope. Their measured flux corresponds to a temperature of 8.3°K, if the source is a black body.

We report preliminary findings with a balloon-borne far-infrared radiometer with three spectral responses in the region below 20 cm^{-1}. The