

total discharge current and about 0.3% of the total ion number in the dense column. Now when the orbiting motion of the ions is also taken into account, the current due to these accelerated ions will be appreciably smaller than the above value.

The ion accelerations achieved with various current distributions will be discussed in a future publication along with more details on the neutron production.

\*Work supported by U. S. Air Force.

<sup>1</sup>O. A. Anderson, W. R. Baker, S. A. Colgate, J. Ise, Jr., and R. V. Pyle, *Phys. Rev.* **110**, 1375 (1958).

<sup>2</sup>A. M. Andrianov et al., in Proceedings of the Second International Conference on Peaceful Uses of Atomic Energy (United Nations, Geneva, Switzerland, 1958),

Vol. 31, p. 348.

<sup>3</sup>J. W. Mather and A. H. Williams, in Proceedings of the Second International Conference on Peaceful Uses of Atomic Energy (United Nations, Geneva, Switzerland, 1958), Vol. 32, p. 26.

<sup>4</sup>D. A. Meskan, H. L. L. van Paassen, and G. G. Comisar, in Proceedings of the Conference on Pulsed High Density Plasmas, Los Alamos Scientific Laboratory Report No. LA-3770 (unpublished).

<sup>5</sup>M. J. Bernstein, D. A. Meskan, and H. L. L. van Paassen, *Phys. Fluids* **12**, 2193 (1969).

<sup>6</sup>N. V. Filippov, T. I. Filippova, and V. P. Vinogradov, *Nucl. Fusion: Suppl. Pt. 2*, 277 (1962).

<sup>7</sup>P. J. Bottoms, J. P. Carpenter, J. W. Mather, K. D. Ware, and A. H. Williams, in Plasma Physics and Controlled Nuclear Fusion Research (International Atomic Energy Agency, Vienna, Austria, 1969), Vol. II, p. 67.

<sup>8</sup>M. J. Bernstien and F. Hai, to be published.

## TRANSPORT PROPERTIES OF CHROMIUM THROUGH THE NÉEL POINT

M. J. Laubitz and T. Matsumura

Division of Physics, National Research Council of Canada, Ottawa, Canada

(Received 16 January 1970)

We have measured precisely the transport properties of a pure polycrystalline specimen of chromium, with a residual resistivity ratio of 380, from 300 to 320 K. We have found no evidence whatsoever for the 5% peak in the thermal conductivity recently reported by Meaden, Rao, and Loo.

In a recent letter, Meaden, Rao, and Loo<sup>1</sup> (henceforth referred to as Meaden) have reported their measurements of the thermal conductivity of a specimen of chromium, with a residual resistivity ratio of 178, which showed a peak at 313.5 K of about 5%. Previous work, done by a group at the Oak Ridge National Laboratory<sup>2</sup> (ORNL), on two different specimens of chromium with residual resistivity ratios of 58 and 280 showed no such peaks, a failure which Meaden somewhat gratuitously ascribes to a lack of precision. In an attempt to resolve this discrepancy, we have borrowed the purer of the two specimens from ORNL. Its properties were fully characterized before<sup>2</sup>; since then, however, it has undergone extensive annealing (4 days at 1100 K, followed by 1 day at 1200 K), and its residual resistivity ratio increased from 280 to 380, which is more than twice as large as the value reported by Meaden for their specimen. Briefly, the results that we have obtained for this specimen in its annealed condition confirm none of the "peaky" features reported by the latter, and elsewhere, by Meaden and Sze.<sup>3</sup>

Two separate experiments were performed on

the specimen. In the first, we determined very precisely its electrical resistivity through the Néel point in an oil bath used for the calibration of platinum resistance thermometers. The temperature of the oil bath was uniform and stable to within 1 mK, and was measured with a primary standard resistance thermometer to  $\pm 1$  mK. The electrical resistivity was measured by a precise ac technique<sup>4</sup> at 7 Hz to eliminate any possible errors stemming from the high thermoelectric power of chromium, which could easily affect dc measurements.<sup>5</sup> The absolute accuracy of our measurement was  $\pm 0.2\%$  and the precision  $\pm 0.003\%$ . There was no discernible frequency effect within that precision.

The results of this experiment are shown by a solid curve in Fig. 1. This curve, taken on a heating cycle, was synthesized from 68 points, none of which departed from the smooth curve by more than the imprecision of the measurements.<sup>6</sup> The cooling curve coincided with the heating curve above the Néel point, but below it was somewhat lower, about 0.15% at 300 K.

The specimen was then mounted in a thermal conductivity apparatus described elsewhere in

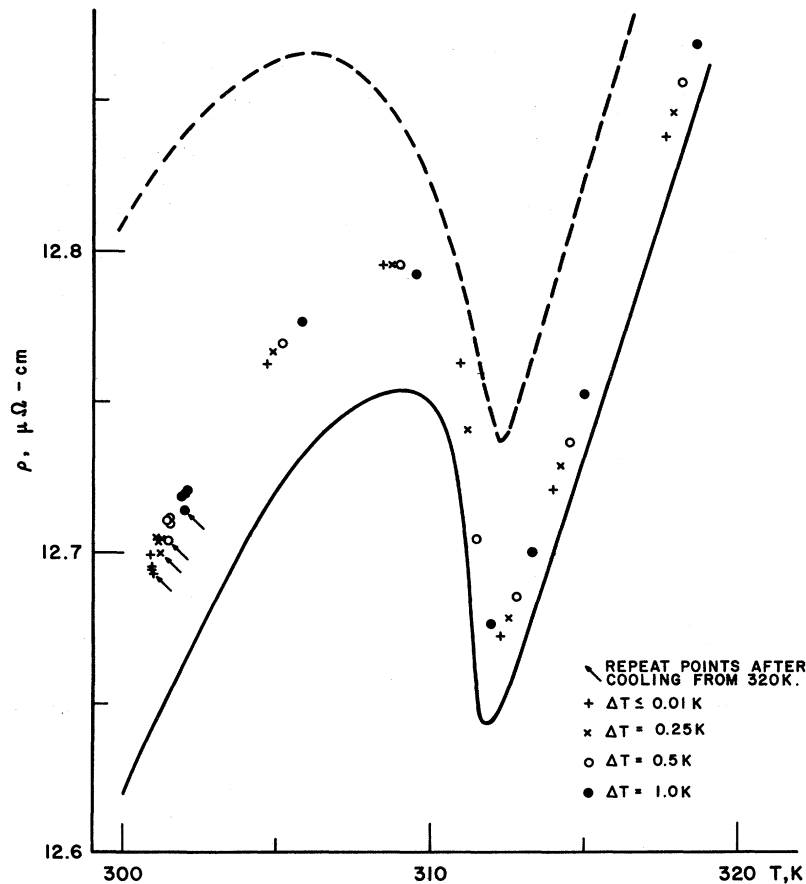


FIG. 1. The electrical resistivity of chromium. Dashed line: the results of Meaden, Rao, and Loo (Ref. 1). Solid line: our results in the oil bath. Dots, circles, and crosses: our results obtained in the thermal conductivity apparatus.

detail,<sup>7</sup> and its thermal and electrical conductivities were measured concurrently. At each temperature, the thermal conductivity was determined using three distinct temperature differences,  $\Delta T$ , between the two specimen thermocouples nominally 5 cm apart; these were 1.0, 0.5, and 0.25 K. The readings were taken in a sequence of increasing temperatures from 301 to 319 K, except for the last point which was a repeat measurement at 301 K. The electrical resistivity was measured at each temperature not only at each gradient employed, but also, as a reference, at no gradient, with  $\Delta T \leq 0.01$  K. The same ac technique was used as in the oil bath, except that now there was a small (0.05%) frequency effect, for which we applied a correction.

The possible systematic errors in the thermal conductivity are estimated to be less than  $\pm 0.7\%$ ; on top of that, there is a random scatter due to imprecision in the measurement of  $\Delta T$ , which we estimate to be  $\pm 0.1$ ,  $0.2$ , and  $0.4\%$  for  $\Delta T$

equal to 1.0, 0.5, and 0.25 K, respectively.

The electrical resistivity measured in the second experiment agreed with that obtained in the oil bath within the combined experimental error above the Néel point (Fig. 1); below it, there is a difference which appears to be significant. The important points to notice, however, are that our results show a much sharper dip at the Néel point than those of Meaden, indicating a better degree of annealing<sup>8</sup>; that the position of our minimum, at 311.7 K, agrees very well with the position of the maximum observed in the specific heat of chromium by Beaumont, Chihara, and Morrison<sup>9</sup>; and that there are no systematic effects due to averaging among the results obtained with the various  $\Delta T$ 's.

The thermal conductivity obtained showed no trace of the 5% peak observed by Meaden; in fact, as ORNL have observed, in the temperature range here investigated, the thermal conductivity  $\lambda$  is constant. Averaging the results for each of the groups of points with different

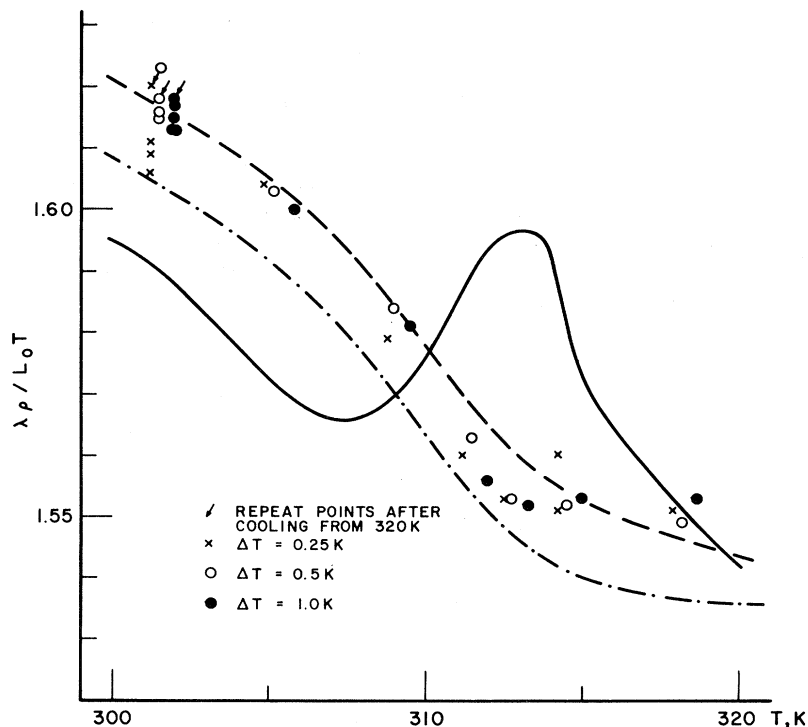


FIG 2. The Wiedemann-Franz ratio of chromium. Solid line: the results of Meaden, Rao, and Loo (Ref. 1). Dashed and dash-dotted lines: the results of the ORNL group (Ref. 2) for specimens with residual resistivity ratios of 280 and 58, respectively. Dots, circles, and crosses: our experimental points.

$\Delta T$ 's we have obtained the following values:  $\lambda = 0.9365 \pm 0.0015$  W/cm K for  $\Delta T = 1.0$  K;  $0.9365 \pm 0.0016$  for  $\Delta T = 0.5$  K; and  $0.9345 \pm 0.0033$  for  $\Delta T = 0.25$  K, where the errors indicate rms deviations which are in good agreement with the estimated imprecision of measurement. As the spread in the averages is only 0.21%, it is clear that there are no systematic differences among the three groups of points, contrary to the implication of Meaden that small temperature gradients are necessary to observe the peak in the conductivity. Our range of gradients, incidentally, spans that used by the latter.

Using the individual values of  $\lambda$  and  $\rho$  for each temperature  $T$  and each  $\Delta T$ , we have computed the (reduced) Wiedemann-Franz ratio,  $\lambda\rho/L_0T$ , where  $L_0$  is the standard Lorenz number. These individual points are shown in Fig. 2, together with the results of ORNL for their two specimens, and those of Meaden. Again, our results agree both qualitatively and quantitatively<sup>10</sup> with ORNL, and show no trace of the peak reported by Meaden. Our results have, in fact, the characteristic shape frequently found at magnetic transition points.<sup>11,12</sup> The close agreement among the results obtained at ORNL and in our own laboratory, on specimens of widely differ-

ent residual resistivity ratios and states of annealing, and determined by essentially quite different techniques, casts doubt on the relevance of the results reported by Meaden, as far as reproducible properties of pure chromium are concerned. It is perhaps conceivable that their "peaky" results, and those of Meaden and Sze, are peculiar to their particular specimen.

We are grateful to the Heat and Solid State Section of our Division for their permission to use their laboratory facilities, and to Dr. D. L. McElroy, of the Oak Ridge National Laboratory, for the loan of the chromium specimen.

<sup>1</sup>G. T. Meaden, K. V. Rao, and H. Y. Loo, Phys. Rev. Letters **23**, 475 (1969).

<sup>2</sup>J. P. Moore, R. K. Williams, and D. L. McElroy, in Thermal Conductivity, Proceedings of the Seventh Conference, edited by D. R. Flynn and B. A. Peavy, Natl. Bur. Stds. Special Publication No. 302 (U.S.G.P.O., Washington, D. C., 1968).

<sup>3</sup>G. T. Meaden and N. H. Sze, Phys. Letters **29A**, 162 (1969).

<sup>4</sup>L. Crovini and C. G. M. Kirby, "Resistance Comparisons at Nanovolt Levels Using an Isolating Current Ratio Generator," Rev. Sci. Instr. (to be published).

<sup>5</sup>See for instance, M. J. Laubitz, Can. J. Phys. **45**,

3677 (1967).

<sup>6</sup>Our results here are in sharp contrast to those of Meaden and Sze, Ref. 3, who have observed all kinds of bumps in  $\rho$  for a specimen which, as far as one can tell, is identical with, if not actually the same as, the one used by Meaden.

<sup>7</sup>T. Matsumura and M. J. Laubitz, to be published. In the temperature range of interest here, our technique yielded results for silver which differed by only 0.2% from those of M. J. Laubitz [Can. J. Phys. 47, 2633 (1969)], which were obtained using a different technique.

<sup>8</sup>T. M. Sabine and A. C. Svenson, Phys. Letters 28A, 443 (1968).

<sup>9</sup>R. H. Beaumont, H. Chihara, and J. A. Morrison, Phil. Mag. 5, 188 (1960).

<sup>10</sup>ORNL estimate that their error in  $\lambda$  is 1.2% and in  $\rho$ , 0.4%. The combined errors in the Wiedemann-Franz ratio are then of the order of 2%, whereas the observed values differ at most by 0.7%.

<sup>11</sup>W. Fulkerson, J. P. Moore, and D. L. McElroy, J. Appl. Phys. 37, 2639 (1966).

<sup>12</sup>D. W. Boys and S. Legvold, Phys. Rev. 174, 377 (1968).

## DYNAMIC SCALING THEORY FOR ANISOTROPIC MAGNETIC SYSTEMS

E. Riedel\*

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850

and

F. Wegner

Institut Max von Laue-Paul Langevin, (8046) Garching bei München, Germany

(Received 15 January 1970)

Dynamic scaling laws for anisotropic magnetic systems are derived where the anisotropy parameters are explicitly treated. The approach is applied to calculate the critical spin relaxation rates for Heisenberg ferromagnets and antiferromagnets with one, two, and three easy axes of magnetization at  $T \geq T_c$ .

In this Letter we outline a scaling theory for dynamic properties in anisotropic magnetic systems. Anisotropy acts to suppress critical fluctuations perpendicular to the easy axes of magnetization and breaks certain local conservation laws. This has a marked effect on the dynamic spin-spin correlation functions in the critical temperature region. Using the generalized scaling approach presented earlier,<sup>1</sup> we calculate the linewidths,  $\Gamma$ , of the spin-density fluctuations in those systems for  $T \geq T_c$ .

For isotropic systems Ferrell et al.<sup>2</sup> and Halperin and Hohenberg<sup>3</sup> first applied homogeneity arguments to dynamic critical phenomena. Basic to dynamic scaling is the existence of a unique correlation length,  $\xi \propto \epsilon^{-\nu}$ , and correlation time,  $1/\Gamma_q$ , which both diverge at the critical point. The time scale of the system in the critical temperature region,  $q\xi \gg 1$ , is calculated from the scaling assumption  $\Gamma_q = \epsilon^{\psi} f(q/\epsilon^{\nu})$ , where  $\psi$  is the dynamic scaling exponent.

To find the scaling properties of the linewidths,  $\Gamma$ , for anisotropic systems we introduce the anisotropy parameters as further "critical" variables. Then a universal exponent  $\psi$  exists, that determines the overall scaling properties of the dynamic correlation functions. The actual dependence of  $\Gamma$  on temperature, wave vector, and anisotropy, however, depends crucially on the dynamics of the spin components involved, and the relative values of the variables. For the anisotropic Heisenberg model we give explicit results for the linewidths of the different correlation functions (Table I). They can be tested by inelastic neutron-scattering experiments.

We start from a Heisenberg model with anisotropic spin exchange interaction

$$\mathcal{H} = -\frac{1}{2} \sum_{r,r'} I(r-r') \{ S_{r,z} S_{r',z} + (1-\Delta_1) S_{r,x} S_{r',x} + (1-\Delta_1-\Delta_2) S_{r,y} S_{r',y} \}. \quad (1)$$

The anisotropy parameters,  $\Delta_{1,2}$ , are assumed small and positive. We will mainly discuss systems with one ( $\Delta_1 = \Delta, \Delta_2 = 0$ ) or two ( $\Delta_1 = 0, \Delta_2 = \Delta$ ) easy axes of magnetization in the paramagnetic temperature range. The linewidth  $\Gamma_i$  of the  $i$ th component of the dynamic spin-spin correlation function is defined microscopically by<sup>4</sup>

$$\Gamma_i(\tau, q, \Delta) = \frac{1}{\langle S_{q,i}^\dagger S_{q,i} \rangle} \operatorname{Re} \int_0^\infty dt \langle \dot{S}_{q,i}^\dagger(0) \dot{S}_{q,i}(t) \rangle. \quad (2)$$