used here there is strong experimental evidence[3] that the superfluid remains at rest and this is the only assumption which we have employed. In addition, we calculate that, even in the absence of any superfluid circulation in Hunt's experiment, torques proportional to power and of the magnitude measured by him could have been produced by a 10% nonuniformity across the light beam that he used for the application of heat to his apparatus. We therefore believe that our experiment provides a more certain verification of the PO theory.

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OSCILLATIONS AND DISSIPATION IN THE FLOW OF HELIUM FILM*

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The damping and frequency of oscillations in liquid level between two open reservoirs of helium II connected by film has been observed to change with time following level equilibration. A maximum in the damping was found to coincide with a maximum in $\dot{V}$. The change in frequency is interpreted as a change in the thickness of the film, a change in ρ_s, or both.

When two open containers of bulk liquid helium II in the same isothermal enclosure are positioned at different levels, flow from the one at the higher gravitational potential to the one at the lower potential occurs via a mobile surface film.[1] When the levels equalize, the inertia in the flowing film causes the reservoir levels to oscillate about their equilibrium positions. These oscillations were first studied by Atkins[2] and have since been utilized by many other investigators[3,4] to elucidate various aspects of film behavior. Picus[4] in particular noticed that the frequency of what he called the "final" oscillations was higher than that observed initially, and he suggested that this change could be most simply explained by a difference between the moving and the stationary film.

We are investigating oscillations between two reservoirs of liquid helium II connected by film along the inside surface of an inverted copper U-tube. One end of this tube is closed to form one of the reservoirs; the other end dips into a much larger container of helium II. Level differences change the capacitance of a cylindrical condenser which is located in the closed end of the U-tube. The condenser is a component of a 20-MHz tunnel-diode oscillator circuit operating at the temperature of the external helium bath, and the changes in capacitance are detected as frequency shifts of the oscillator. The stability of the system permits liquid-level differences of the order of 10^{-6} cm to be easily resolved. 5 cm above the capacitor the inside perimeter of the U-tube is reduced by a factor of 4.2 for a length of 1 cm. The diameter of the constriction is 0.4572 cm.

After temperature equilibrium is established, a level difference is produced by changing the position of a plunger in the large reservoir. Within a few seconds, a constant flow rate into or out of the capacitor reservoir is observed. This flow rate appears to be independent of the level difference between the two baths over the range investigated, namely, 0-0.25 cm.

Figure 1 shows the experimental points for the level oscillations which followed an outflow from the capacitor reservoir at 1.222 K. The curve passing through the points is obtained from a nonlinear least-squares fit of the data an exponentially damped oscillator equation. Although it is barely discernible, a close examination of Fig. 1 reveals that the oscillation frequency begins to increase after approximately 170 sec. In other longer runs at this temperature, a definite frequency shift occurs within the next cycle accompanied by a strong increase in the damping. The number of cycles through which the system oscillates at approximately constant frequency and damping following the first passage through the equilibrium level is not precisely reproducible. In general, the higher the temperature, the few-

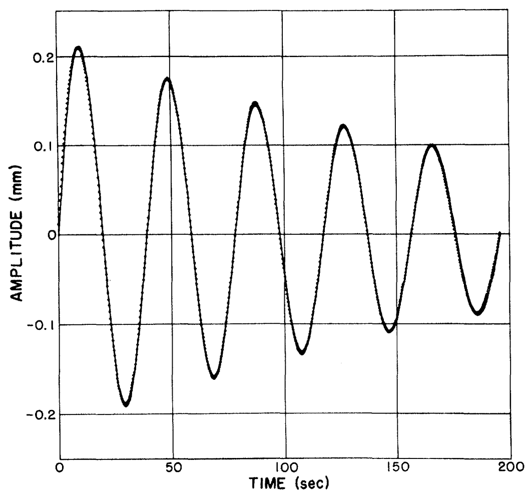

FIG. 1. Amplitude versus time for oscillations at 1.222 K.

er the oscillations before the frequency shift is observed.

Figure 2 shows this effect in detail for another run which was continued for 0.64 h. Consecutive sections of data exhibiting approximately constant decrement and amplitude were analyzed to obtain the applicable damped-oscillator–equation parameters for each section. In order to minimize computer bias, three separate analyses were carried out in which the entire run was examined using progressively smaller sections of data. Figure 2 is a composite of the results of these three examinations. No significant changes occurred as a result of the more fine-grained analysis except that the peak in the damping coefficient became sharper. The maximum in the damping coincides very closely with the maximum in $\dot{\nu}$. Since the natural frequency of a damped oscillator increases as the damping decreases, and since the damping coefficient $\mathfrak{a}_3$ of the level oscillations is also observed to decrease with time (except for the superimposed peak), it might be assumed that the measured increase in frequency is merely a consequence of the net decrease in damping. A simple calculation will show, however, that the observed effect is about 1000 times larger than that predicted. Specifically, at $t=0$ the damped frequency ν_d is 2.6063×10^{-2} sec^{-1}, the damping coefficient $\mathfrak{a}_3$ is 5×10^{-3} sec^{-1}, and the undamped frequency ν_0 is 2.6075×10^{-2} sec^{-1}. This last number should be compared with the observed frequency after the observed damping coefficient has decreased to zero, namely, 3.65 $\times10^{-2}$ sec^{-1}. Hence we conclude that the observed frequency changes must be due to changes in ν_0.

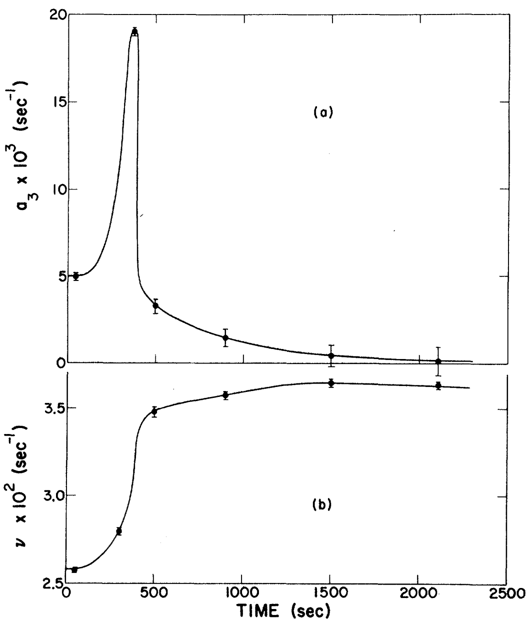

FIG. 2. Oscillation parameters at 0.975 K. Oscillations are fitted by an equation of the form $x=x_0 \times\exp(-\mathfrak{a}_3 t)\sin 2\pi(\nu t+\mathfrak{a}_5)$. Error bars of $\pm\sigma$, the standard deviation of the fit, are shown at selected points on the graph. (a) Damping coefficient $\mathfrak{a}_3$ versus time. (b) Frequency of oscillation ν versus time.

The undamped oscillation frequency is given by[2]

$$\nu_0=\left[\frac{g}{2\pi\rho A}\left(\int_C^R \frac{ds}{\rho_s r\delta}\right)^{-1}\right]^{1/2}, \tag{1}$$

where A is the cross-sectional area of the condenser annulus, r is the inside radius of the inverted U-tube, δ is the film thickness, and the integral is taken from the liquid surface in the condenser C to that in the open reservoir R along the flow path s. Clearly all the variables in this equation are fixed by the geometry except ρ_s and δ. For films as thick as those being considered here there is no evidence that ρ_s in the film is other than equal to the superfluid density for the bulk liquid. One must, however, admit to the possibility that ρ_s in the film flowing through the constriction could be significantly less than ρ_s(bulk liquid) during the dissipative level-equilibration process. After the levels equalize, ρ_s(constriction) would then be able to increase to the bulk-liquid value and the oscillation frequency should increase correspondingly. The observed frequency change must therefore be attributed to increases in δ, in ρ_s, or in both, which take place in the constriction following the initial level-equilibration process.[5] From Fig. 2 it is clear that the relaxation time for these changes is about 400 sec.

Clark[6] has predicted that the film thickness should decrease in dissipative flow. Assuming that the film thickness profile at the end of the run shown in Fig. 2 corresponds to that which would be exhibited by a static film, i.e., $\delta(\mathrm{cm}) = 3\times 10^{-6}H^{-1/3}$ at a height H centimeters above the bulk liquid, then Eq. (1) requires that the film thickness in the constriction during the initial equilibration process be reduced to $\frac{1}{3}$-$\frac{1}{2}$ of its static value if the entire change in ν is attributed to a change in film thickness.[7] If this decrease in thickness actually occurs, the superfluid velocity in the constriction during the dissipative flow regime, and for a few oscillations thereafter, must reach 2-4 m/sec.

Atkins[2] has defined a quantity $\gamma \equiv 2\pi\nu_0 x_0/\dot{x}_{x=0}$, where x is the displacement from equilibrium of the liquid level in the capacitance reservoir, $\dot{x}_{x=0}$ is the slope of the x vs t curve at its first crossing of the time axis, x_0 is the undamped amplitude, and ν_0 is the frequency. If the damping coefficient is zero or a constant, $\gamma = 1$. For the oscillations shown in Fig. 1, $\gamma = 0.9982$. In some instances, however, a small additional amount of energy (i.e., in addition to that specified by the damping coefficient) is lost from the system during the first quarter period. Thereafter, both the frequency and damping are relatively constant for several cycles before the more marked changes in damping and frequency begin to occur, as shown in Fig. 2. In these cases $\gamma \neq 1$; but generally we found $\gamma > 0.94$.

Since the level-equilibration behavior discussed earlier implies that the superfluid velocity $\vec{v}_s$ is effectively independent of the potential difference, the dissipation function governing quasisteady film flow is assumed to be of the form $\nabla\mu = \beta(\vec{v}_s - \vec{v}_{s,c})$, where $\vec{v}_{s,c}$ is the superfluid critical velocity. Because $\vec{v}_s \approx \vec{v}_{s,c}$, β must be very much larger than any practically realizable gradient in the potential. After $\Delta\mu = 0$, however, it was expected that no appreciable further dissipation would occur (dissipation due to evaporation and recondensation or heat transfer to the liquid reservoirs during the oscillations should be negligible,[8] and in any event it is difficult to see how the damping coefficient for a process such as this could change so markedly with time). The persistent-current experiments of Henkel, Kukich, and Reppy[9] also seem to support the idea that, for velocities $\vec{v} \leq \vec{v}_{s,c}$ and for temperatures not too close to T_λ, the flow is indeed frictionless. As Fig. 2(a) demonstrates, however, such behavior is not observed in this experiment. Furthermore, the fact that the decrement is relatively constant for the first few cycles and then changes markedly suggests that more than one process may be responsible for this damping. The energy loss appears to be related to the thickening of the film and the simultaneous decay of vorticity in the constriction, but the details of this process are obscure. Some of this damping does not seem to depend upon the presence of vorticity since in another experiment at 0.96 K, after the oscillations had been allowed to continue for about $\frac{1}{2}$ hour and most of the vorticity should have disappeared from the film, a displacement of the reference level by 0.004 cm (which was far from sufficient to accelerate the film to $\vec{v}_{s,c}$) nevertheless produced a new series of oscillations with a damping coefficient $\alpha_3 = 5.9\times 10^{-3}$ and a frequency of 2.81×10^{-2} sec^{-1}. Before perturbing the system in this way, $\alpha_3 \approx 0$, $\nu \approx 3.4\times 10^{-2}$ sec^{-1}, and the amplitude was about 2 μ.

We believe that the changes in ν reported here are clearly observable because of the geometry of the flow path employed. In all earlier work, the perimeter of the flow path has been relatively constant, and hence the kinetic energy of the flowing film has been uniformly distributed along the entire flow path. Since the dissipative region in the film is usually confined to only a small region of the flow path, the oscillatory behavior was always determined in previous work by the much larger nondissipative portions of the flowing film. In contrast, the geometry used in the present experiments forces that region of the flow path which predominantly determines ν to be coincident with that in which the dissipation occurs during the initial level-equilibration process.

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THERMODYNAMICS NEAR THE TWO-FLUID CRITICAL MIXING POINT IN He^3-He^4†

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The two-fluid critical mixing point in He^3-He^4 differs from ordinary critical points in that it occurs at the intersection of three lines of critical points, in a suitable variable space. A free-energy function is proposed which removes certain discrepancies between classical (Landau) theory and experimental thermodynamic measurements. Certain solid-state transitions (e.g., the metamagnetic-antiferromagnetic transition in $FeCl_2$) are thermodynamic analogs of critical mixing in He^3-He^4.

In He^3-He^4 mixtures[1] under saturated vapor pressure, the λ transition temperature decreases with increasing mole fraction x of He^3, and below a temperature $T^*=0.87$ K corresponding to $x^*=0.67$ a first-order phase separation takes place. The angular top of the two-fluid coexistence curve is the terminus of the line of λ transitions (Fig. 1) according to recent experiments,[2] in (approximate) agreement with a phenomenological argument given by Landau[3] which we shall call the "classical theory" CT. In ordinary critical phenomena CT is a valuable first guide (though its detailed predictions are often in error), and in the present instance it predicts a peculiarity which suggests that the two-fluid critical mixing in helium has features quite unlike those found at "ordinary" critical points (ferromagnets, liquid-vapor critical points, λ transition in pure He^4, etc.).

The peculiarity is seen most easily if one uses intensive thermodynamic variables, replacing x with its thermodynamic conjugate $\Delta=\mu_3-\mu_4$, the

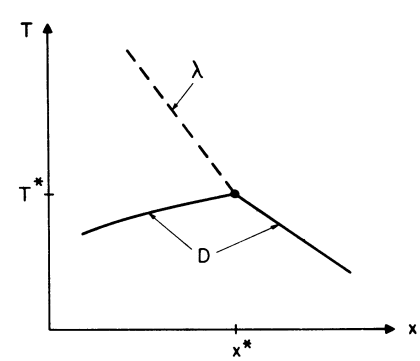

FIG. 1. Phase diagram (schematic) for He^3-He^4 mixtures near the critical mixing point. The two-fluid coexistence curve is labeled D and the dashed curve is the line of lambda transitions.