$=0.006$ Hz/V cm^{-1} but since these ENDOR lines are very broad ($\Delta \sim 50$ KHz) one would expect the electric field shift (although linear in this case) to be too small to measure. Our experimental measurements were only able to set an upper limit on the EFE to be $\partial a/\partial E < 0.1$ Hz/V cm^{-1}. No attempt was made to observe the EFE on the second-shell F^{19} ENDOR lines, because of the extremely large hyperfine coupling ($a_F \sim 106$ MHz).

In conclusion, isotopic substitution has been shown to be useful for measuring electric field perturbations in a system with large hyperfine coupling.

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[3]Grown by the Harshaw Chemical Company, Cleveland, Ohio.

[4]No theoretical justification has been found for the experimentally reproducible asymmetry in the high-field ENDOR lines.

DAMPING OF A HEATED OSCILLATING CYLINDER IN He II*

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The change in the logarithmic decrement upon applying heat to a freely oscillating cylinder in He II has been measured. After corrections, caused by heat, in the normal viscous contribution to the decrement the results are used to verify the prediction by Penney and Overhauser that a torque should exist on a uniformly heated cylinder about which there is a superfluid circulation. A critical comparison is made between the results of the present experiment and the prior work of Hunt who employed a quite different method.

Penney and Overhauser[1] (PO) have recently predicted that a heated cylinder, immersed in He II, and about which a superfluid circulation Γ exists should experience a torque τ given by

$$\tau=\Gamma\dot{Q}/2\Pi s_n T, \qquad (1)$$

where T is the Kelvin temperature, s_n is the specific entropy of the normal fluid, and $\dot{Q}$ is the total power dissipated at the surface of the cylinder. Hunt[2] has reported an experiment which verifies that torques exist on a heated cylinder which satisfy the power dependence of Eq. (1) and which are consistent with the temperature dependence.

It appears that an alternative method of verifying the PO theory is desirable, and so we have performed the following experiment. The physical origin of the torque discussed by PO is the conversion of moving superfluid into normal fluid which then interacts with a stationary surface and transmits its momentum to that surface, the conversion of superfluid to normal fluid being caused by the application of heat at the surface. An analogous situation occurs in the case of a heated cylinder oscillating in He II. It is well known[3] that the superfluid component is not excited into motion by an oscillating solid provided the amplitude of oscillation is not too large and its period not too short. As long as these conditions are met, a drag will be caused by the creation of stationary normal fluid at the surface of the heater which must then be accelerated to the velocity of the surface.

If a cylinder is suspended from a torsion fiber and allowed to oscillate freely, the change in the logarithmic decrement of the motion caused by the application of heat can be accurately measured. A simple calculation shows that the change in the decrement $\Delta\delta_{PO}$ due to the PO force, assuming that both the zero-power decrement and $\Delta\delta_{PO}$ are both much less than unity, is given by

$$\Delta\delta_{PO}=P\alpha^2\dot{Q}/2Is_n T, \qquad (2)$$

where P is the period, a is the radius of the cylinder, and I is its moment of inertia.

We suspended a cylinder of radius $a=0.72$ cm from fine stainless steel wires in a large bath of He II. The cylinder was a hollow glass bulb coated with a sputtered film of Nichrome. A second wire was attached to the bottom of the bulb so that the cylinder could be heated by the passage of an electrical current through the suspension wires and the thin film of Nichrome. The length of the heated section of cylinder was $h=3.4$ cm. The total length of the bulb was 3.8 cm. Its moment of inertia was measured to be $I=0.969$ g cm^2. Using different diameter suspension wires the period of oscillation was varied and periods of 39.5, 26.5, and 12.7 sec were achieved. The angular position of the cylinder was observed by reflecting a low-intensity light beam from a mirror attached to the cylinder onto a ground glass scale.

It was observed from the decay of the free oscillation of the heated cylinder that the change in the logarithmic decrement as a function of power was systematically less than the change which we had calculated from the added drag of the PO force. It was suggested that the radial motion of the normal fluid due to the application of heat might change the normal fluid viscous drag on the the cylinder.[4] The viscous drag is a shear force proportional to the gradient of the tangential component of the velocity at the surface. In He II the creation of normal fluid by heat and its streaming away by internal convection result in a diminishing of the gradient and a lessening of the viscous drag. By solving the Navier-Stokes equation for the normal fluid velocity near an infinite heated oscillating cylinder and neglecting end effects, we have been able to calculate that the change in the logarithmic decrement from viscous heating effects should be given by

$$\Delta\delta_{\mathrm{VH}}=\frac{2\pi^2a^3h\eta}{I\omega}\left[\mathrm{Re}\left\{\beta\frac{H_0^{(1)}(\beta a)}{H_1^{(1)}(\beta a)}\right\}-\mathrm{Re}\left\{\beta\frac{H_\alpha^{(1)}(\beta a)}{H_{1+\alpha}^{(1)}(\beta a)}\right\}\right], \qquad (3)$$

where $H^{(1)}$ is a Hankel function of the first kind, η is the viscosity, ω the angular frequency, $\beta=(1+i)/\lambda$, λ is the penetration depth of viscous waves, $i=(-1)^{1/2}$, $\alpha=v_na/2\nu$, v_n is the perpendicular component of the normal fluid velocity at the surface of the cylinder, $v_n=\dot{Q}/A\rho_ns_nT$, A is the total heated area, ρ_n is the normal fluid density, and ν is the kinematic viscosity.

The relative magnitudes of α and $|\beta\alpha|$ make it impossible to express Eq. (3) in a more convenient analytical form. The theoretical curves which we will display are calculated numerically by computer from the sum of Eqs. (2) and (3). We note that the term represented by Eq. (3) is a correction term to be applied to Eq. (2) and, in actual practice, amounted to from 20% to 50% of the latter, depending on the temperature and the power.

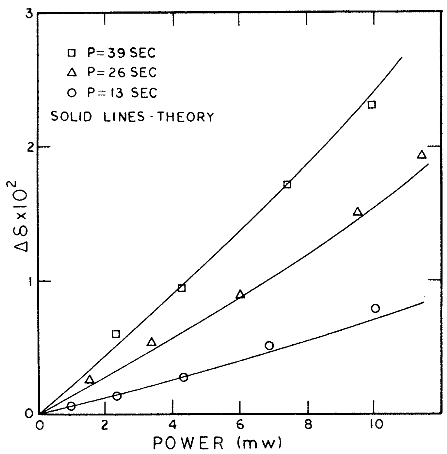

FIG. 1. The change in the decrement versus power for three different periods. $T=1.60$ K.

With the inclusion of these two effects we are able to account for our observations of the decrement over the temperature range 1.3 K to the λ point, and for the range of periods mentioned. Figure 1 shows the change in the decrement $\Delta\delta$ with power for three periods, and Fig. 2 shows $\Delta\delta$ at three temperatures for a period of 26 sec.

A fundamental difficulty with Hunt's experiment lies in the fact that the hydrodynamic flow pattern is unknown and is, in fact, impossible to predict. A similar difficulty arose in the interpretation of the Vinen vibrating-wire experiment.[5] By comparison, the superfluid "flow pattern" in the present experiment is rather precisely known; in the range of amplitudes and periods

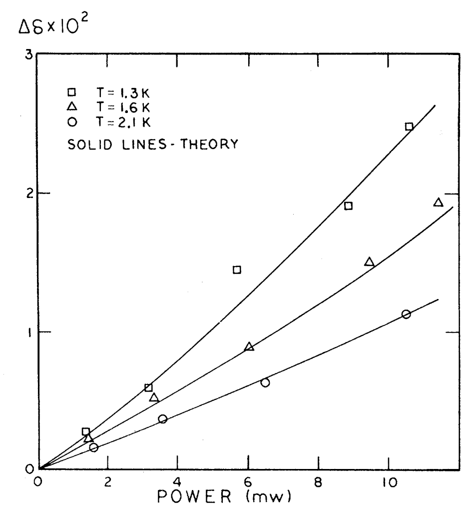

FIG. 2. The change in the decrement versus power for three different temperatures. $P=26$ sec.

used here there is strong experimental evidence[3] that the superfluid remains at rest and this is the only assumption which we have employed. In addition, we calculate that, even in the absence of any superfluid circulation in Hunt's experiment, torques proportional to power and of the magnitude measured by him could have been produced by a 10% nonuniformity across the light beam that he used for the application of heat to his apparatus. We therefore believe that our experiment provides a more certain verification of the PO theory.

*Assisted by the National Science Foundation and the Army Research Office (Durham).

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OSCILLATIONS AND DISSIPATION IN THE FLOW OF HELIUM FILM*

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The damping and frequency of oscillations in liquid level between two open reservoirs of helium II connected by film has been observed to change with time following level equilibration. A maximum in the damping was found to coincide with a maximum in $\dot{v}$. The change in frequency is interpreted as a change in the thickness of the film, a change in ρ_S, or both.

When two open containers of bulk liquid helium II in the same isothermal enclosure are positioned at different levels, flow from the one at the higher gravitational potential to the one at the lower potential occurs via a mobile surface film.[1] When the levels equalize, the inertia in the flowing film causes the reservoir levels to oscillate about their equilibrium positions. These oscillations were first studied by Atkins[2] and have since been utilized by many other investigators[3,4] to elucidate various aspects of film behavior. Picus[4] in particular noticed that the frequency of what he called the "final" oscillations was higher than that observed initially, and he suggested that this change could be most simply explained by a difference between the moving and the stationary film.

We are investigating oscillations between two reservoirs of liquid helium II connected by film along the inside surface of an inverted copper U-tube. One end of this tube is closed to form one of the reservoirs; the other end dips into a much larger container of helium II. Level differences change the capacitance of a cylindrical condenser which is located in the closed end of the U-tube. The condenser is a component of a 20-MHz tunnel-diode oscillator circuit operating at the temperature of the external helium bath, and the changes in capacitance are detected as frequency shifts of the oscillator. The stability of the system permits liquid-level differences of the order of 10^{-6} cm to be easily resolved. 5 cm above the capacitor the inside perimeter of the U-tube is reduced by a factor of 4.2 for a length of 1 cm. The diameter of the constriction is 0.4572 cm.

After temperature equilibrium is established, a level difference is produced by changing the position of a plunger in the large reservoir. Within a few seconds, a constant flow rate into or out of the capacitor reservoir is observed. This flow rate appears to be independent of the level difference between the two baths over the range investigated, namely, 0-0.25 cm.

Figure 1 shows the experimental points for the level oscillations which followed an outflow from the capacitor reservoir at 1.222 K. The curve passing through the points is obtained from a nonlinear least-squares fit of the data an exponentially damped oscillator equation. Although it is barely discernible, a close examination of Fig. 1 reveals that the oscillation frequency begins to increase after approximately 170 sec. In other longer runs at this temperature, a definite frequency shift occurs within the next cycle accompanied by a strong increase in the damping. The number of cycles through which the system oscillates at approximately constant frequency and damping following the first passage through the equilibrium level is not precisely reproducible. In general, the higher the temperature, the few-