the critical behavior is different from that in typical order-disorder transitions.

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NEUTRON-MATTER RADII FROM AN ANALYSIS OF PIONIC ATOMS*

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We use the low-energy pion-nucleus interaction as a means for investigating the distribution of nucleons in nuclei. An optical-model analysis of existing experimental data on pionic atoms is used to study possible differences in the rms radii of the neutron and proton densities. The difference in these radii, assumed to be independent of mass number, was found to be -0.01 ± 0.16 F. The absence of any significant difference in these radii should be considered an average behavior for a wide range of nuclei.

The present situation concerning possible differences in the rms radii of the neutron and proton distributions in nuclei is not clear. For example, Greenlees, Pyle, and Tang[1] used an optical-model analysis of proton-nucleus elastic scattering for several nuclei from Ni^{58} to Pb^{208}, and found that the rms radius of the neutrons is about 0.6 F larger than the rms radius of the protons. This result is supported by shell-model analyses of single-particle energies,[2] although a more recent study[3] indicates that the difference in these radii approaches zero in the lighter nuclei. In contrast to these studies, investigations of Coulomb displacement energies[4] lead to the general conclusion that the rms radius of the neutron distribution is just slightly larger than that of the proton distribution.

In this Letter we use pionic atoms because of the strong pion-nucleus interaction which furnishes an additional means for studying the distribution of neutrons and protons in nuclei. The possibility of using pionic atoms as a tool for the investigation of nucleon distributions has previously been recognized,[5] although in these studies no attempt was made to examine possible differences in the nucleon densities. However, the utility of this probe has been exploited in a study of pion atomic transitions in the nickel isotopes,[6] and it was found that the experimental data are consistent with equal rms radii for the neutron and proton densities.

Experimentally, the pion atomic-transition energies and the energy-level widths (due to nuclear absorption of the pions) are measured, and these quantities provide information concerning the pion-nucleus interaction. The energy levels and widths can be calculated by finding the (complex) eigenvalues E of a modified Klein-Gordon equation

$$\{\nabla^2 + 2m_r[(E - V_{em}) + (E - V_{em})^2/(2m_\pi)]\}\Psi = 2m_r V_{opt}\Psi, \quad (1)$$

where m_π and m_r denote the pion and pion-nucleus

reduced mass, respectively. The electromagnetic potential V_{em} is taken to include both the Coulomb and vacuum-polarization potentials. The strong pion-nucleus interaction is represented by an optical-model potential[7] that has both local and nonlocal contributions,

$$V_{opt}(r) = V(r) - (2m_r)^{-1}\nabla\cdot\alpha(r)\nabla. \quad (2)$$

In order to emphasize the dependence of this potential on the neutron, proton, and matter densities denoted, respectively, by ρ_n, ρ_p, and $\rho_m = \rho_p + \rho_n$, we write

$$V(r) = -(4\pi/2m_r)\{[b_0\rho_m + b_1(\rho_n - \rho_p)] + i[B_0\rho_m\rho_p + B_1(\rho_n - \rho_p)\rho_p]\}. \quad (3)$$

Similarly, for the nonlocal part we use

$$\alpha(r) = \alpha_0(r)/[1 - \tfrac{1}{3}\alpha_0(r)],$$

where

$$\alpha_0(r) = -4\pi\{[c_0\rho_m + c_1(\rho_n - \rho_p)] + i[C_0\rho_m\rho_p + C_1(\rho_n - \rho_p)\rho_p]\}. \quad (4)$$

This optical-model potential has eight free parameters if we assume that the nucleon densities are known. Our analysis indicates, however, that the absorptive parameters B_1 and C_1 are not well determined, and are consistent with values of zero. Accordingly, we have set these parameters equal to zero, thus reducing the number of parameters from eight to six. In order to determine these six quantities, the pion transition energies and level widths were computed using Eq. (1) for 38 nuclei ranging from B^{10} to Bi^{209}. The neutron and proton densities were assumed to have the same shape, and for each nucleus a Fermi or modified Fermi distribution was used, with geometrical parameters obtained from electron-scattering analyses and muonic-atom x-ray data.[8] The density parameters were corrected for the finite size of the proton. The calculated results were compared with a weighted average of existing experimental data[9] and a least-squares search routine was used to determine the optical-potential parameters which best fit the experimental data.

The resulting parameters are presented in Table I. These quantities are in good agreement with those deduced in previous studies[5] and the parameters referring to the real part of the potential are in good agreement with theoretical predictions.[5,7]. As in previous analyses the absorptive permit a quantitative description of the experiments, but are about twice as large as theoretical predictions.

Table I. Optical-potential parameters.

$b_0 = -0.033 \pm 0.001 m_\pi^{-1}$	$c_0 = 0.200 \pm 0.010 m_\pi^{-3}$
$b_1 = -0.117 \pm 0.019 m_\pi^{-1}$	$c_1 = 0.139 \pm 0.107 m_\pi^{-3}$
$B_0 = 0.076 \pm 0.005 m_\pi^{-4}$	$C_0 = 0.300 \pm 0.035 m_\pi^{-1}$

The pion-nucleus optical potential has been determined, as described above, by assuming that the neutron and proton densities have the same shape. Since, however, the calculated energies and widths depend on these densities, it is relevant to determine whether or not a better description of experimental data would be obtained if these densities were not assumed to be identical. Furthermore, if the calculated results depend strongly on assumed differences in these distributions, we may expect pionic atoms to provide information on the nucleon densities.

In order to investigate these questions, we introduce a parameter Δ which represents a difference in the rms radii of the distributions of the neutrons and the protons:

$$\Delta = \langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}.$$

Since we have no basis for predicting the dependence of Δ on the mass number, we have assumed Δ to be the same for all nuclei. In the present work, this difference in the rms radii was induced by changing the half-density radius parameter in the Fermi distribution. The optical-potential parameters together with the parameter Δ were again varied to produce a set of quantities which best describe the experimental data. The resulting optical-potential parameters were essentially identical to those given in Table I and the best value of Δ was found to be

$$\Delta = -0.01 \pm 0.16 \text{ F}.$$

This result is consistent with equal rms radii for the neutrons and protons, but it must be emphasized that this result reflects only the average behavior for a very broad range of nuclei.

To demonstrate the sensitivity of pionic-atom data to possible differences in the rms radii of the neutrons and the protons, we display in Fig. 1 χ^2 as a function of Δ. The optical-potential parameters have the values given in Table I. There is clearly a very strong dependence of χ^2, and hence the calculated results, on Δ, which illustrates the practicability of pionic atoms for investigating nucleon distributions.

The pion-nucleus interaction and, in particular, its absorptive part, will have to be better under-

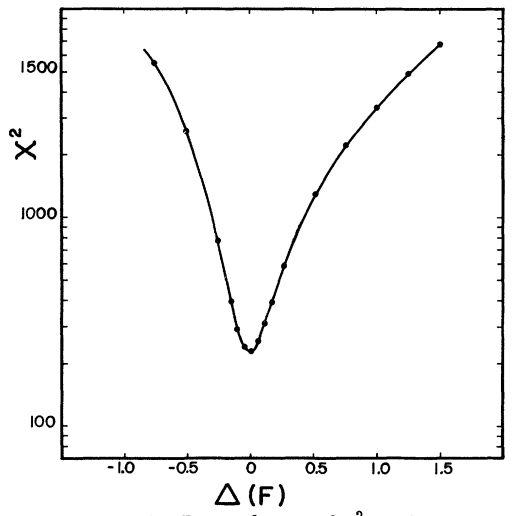

FIG. 1. Dependence of χ^2 on Δ.

stood before full advantage can be taken of the precision of π-mesonic-atom data. Our result that for nuclei in general there is no significant difference between the rms radii of the neutrons and protons is based on the assumption that this difference is independent of mass number. Although this assumption must be further investigated, this conclusion is in good agreement with similar conclusions obtained from studies of isobaric analog-state energies[4] and also with more general considerations concerning the neutron and proton distributions.[10]

A more detailed discussion of the work presented in this Letter will appear elsewhere.

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RMS CHARGE RADII OF ^{16,17,18}O BY ELASTIC ELECTRON SCATTERING*

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The charge radii of ^{16,17,18}O have been determined by elastic electron scattering using a gas target system. The result obtained is $R_{17} < R_{16} < R_{18}$.

Anomalies in the charge radii of isotopes have been observed in fairly recent measurements of muonic x rays[1] and by elastic electron scattering[2] as well as by earlier results on the optical isotope shift.[3] The charge radii of isotopes do not follow, even approximately, the $A^{1/3}$ law.[4]

The anomalies in radius are especially marked for closed-shell nuclei and the experiments on the calcium isotopes where neutrons are added to the doubly closed d shell have stimulated great theoretical interest.[5] We have therefore measured the rms charge radii of the oxygen isotopes where neutrons are added to the doubly closed p shell.

Muonic x-ray experiments are very accurate for medium and heavy nuclei; however for light nuclei ($Z \leq 20$) elastic electron scattering is a preferable technique.[1]

The experiments were performed using the electron-scattering facility at the Saskatchewan Accelerator Laboratory[6] and a gas target system developed at this laboratory.[7] The elastic-scattering cross sections for ^{16,17,18}O and hydrogen were measured at incident electron energies