DISTRIBUTIONS IN TRANSVERSE MOMENTUM FOR DEEPLY INELASTIC HADRON COLLISIONS*

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Explanation is given for observed sharp forward peaks in distributions of transverse momentum for hadrons produced in deeply inelastic hadron collisions. Small Q-value decay of peripherally produced nucleon resonances is the basic mechanism responsible for peaks in both π and proton spectra. Implications of this result are discussed.

Experimenters have recently made detailed measurements of cross section $d^2\sigma/dpd\Omega$ for a single observed particle emergent from high-energy hadron-hadron interactions.[1] Particularly striking is structure at very small values of transverse momentum p_T of the final pion in $pp \to \pi^\pm +$anything.[2] Indeed, as displayed in Fig. 3 of Ref. 2, for pion c.m. longitudinal momentum $p_L = 0.6$ GeV/c, data points for which $p_T^{-2} \lesssim 0.1$ (GeV/c)$^{-2}$ fall on a curve of form $\exp(-bp_T^2)$ with $b \approx 15$ (GeV/c)^2; there is a break in the distribution, and at large p_T^2 a shallow curve $[\propto \exp(-3p_T^2)]$ is measured. Suggestion of a similar forward peak is apparent also in data from $pp \to p +$anything.[3]

Because only one final particle is detected in these single-arm–spectrometer experiments, there is no empirical way to ascertain whether the observed particle is scattered directly or is the decay product of a resonance. Nevertheless, there is intuitive appeal to the idea that data at very small p_T^2 are of the direct variety[3]; in particular, the very steep forward peak might be thought to reflect simple exchange-model dynamics. In this note we develop very much the opposite point of view. Essentially, we assert that the steep forward peak in $pp \to \pi^\pm +$anything is formed from $\pi^\pm$ which are decay products of nucleon resonances produced peripherally. The exact spin structure of nucleon resonances is not crucial, nor is it terribly important how strongly peripherally they are produced. Decisive factors are small Q value in resonance decay and small value of pion mass, which enters in the ratio $(m_{\rm res}/m_\pi)^2$. As a result, decay pions emerge with smaller (not larger!) p_T than parent resonances.[4] The same mechanism is responsible for protons with small p_T, but the forward peak is predicted to be less steep than for pions. A consequence of our analysis is the important, if pessimistic, conclusion that it is impossible to isolate a direct-scattering component in deeply inelastic data obtained in single-arm experiments. Other implications and suggestions for further tests of our approach are listed at the end.

Model.—Our mechanism is sketched in Fig. 1. We imagine a process in which resonance N^* is produced along with n other particles. Symbol X does not indicate any specific exchange mechanism but merely serves to represent peripheral production. (We need not specify details about the configuration of n particles produced with N^* other than to remark that in single-arm data an implicit sum is made over all possible multiplicities and particle types.) Resonance N^* subsequently decays via $N^* \to \pi + N$ or sequentially as $N^* \to \pi + N_f^*$. We are interested in the distribution in transverse momentum of decay pions.

In order both to motivate our model and to illustrate an important aspect, we begin by discussing $pp \to n\Delta^{++}(1236)$, a special case of Fig. 1. This quasi-two-body process is known to proceed peripherally; most Δ's go forward/backward. In Fig. 2, we sketch the distribution in p_L(c.m.) vs p_T for π^+ coming from decay of forward Δ's. Pions populate a thin shell. If we ignore finite Δ width, the maximum of p_T (for pions) is $\{[m_\Delta^2-(m_N+m_\pi)^2][m_\Delta^2-(m_N-m_\pi)^2]/4m_\Delta^2\}^{1/2} \approx 0.23$ GeV/c. Moreover, for incident proton lab momentum 12 GeV/c and $p_L = 0.6$, $p_T \approx 0.2$ GeV/c. It is instructive to compare these numbers with corresponding values obtained from data.[2] Experimenters find $d^2\sigma/dpd\Omega \propto \exp(-15p_T^2)$ at small p_T and for fixed $p_L = 0.6$ GeV/c.[2] Using $d^2\sigma/dp_Ldp_T^2 = (\pi/p^2)d^2\sigma/dpd\Omega$, we deduce directly that at $p_L = 0.6$ GeV/c, the maximum of experi-

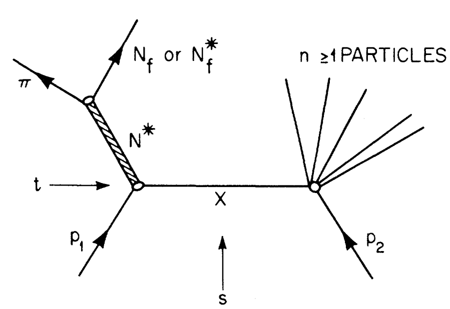

FIG. 1. Illustration of the basic production and decay mechanism responsible for final π with very small p_T.

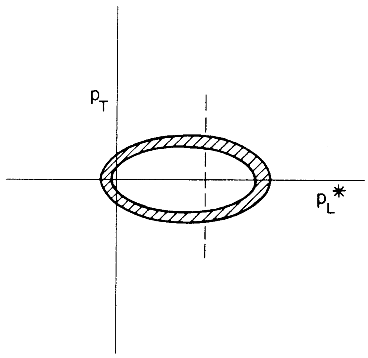

FIG. 2. Sketch showing region (cross-hatched) in p_T, p_L plane populated by π resulting from N^* decay. Dashed line indicates $p_L \approx 0.6$ GeV/c.

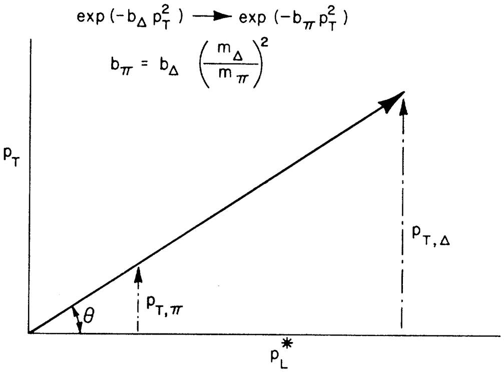

FIG. 3. Diagram (not to scale) giving relationship of transverse momentum vectors of decay π and parent resonance (Δ), assuming $Q=0$.

mental $d\sigma/dp_T$ occurs at $p_T \approx 0.2$ GeV/c (not at $p_T = 0$). Note the very close correspondence of (i) the value of p_T at which the experimental maximum occurs and (ii) the position of the thin-shell distribution produced by N^* decay. The Δ is not important per se; what is crucial is the existence of forward N^*'s which have small Q-value[5] decay modes: $N^* \to \pi N$ or $N^* \to \pi N_f^*$. Decay with small Q generates π's having p_T distributions peaked at very small values, such as observed in deeply inelastic data.[2]

Certainly not all N^* go exactly forward, as was assumed just above; the result is that there is a series of elliptically shaped thin shells of the type shown in Fig. 2 but with major axes inclined slightly with respect to the p_L axis. Moreover, for large values of multiplicity n (see Fig. 1), kinematical considerations suggest that N^* may not be produced very peripherally. We turn therefore to an examination of the relationship of production and decay distributions. One might expect intuitively that if a Δ^{++} (or, more generally, N^*) is produced with distribution $\exp(-b_\Delta p_T^2)$, then the p_T^2 distribution of its decay pion is necessarily less peripheral.[3] In other words, the pion's distribution in the N^* rest frame convoluted with the N^* production distribution should lead to an overall pion distribution in p_T^2 with diffraction constant $b_\pi < b_\Delta$. This argument is wrong in one crucial respect. A simple illustration shows the fallacy. Imagine a particle of mass M produced at center-of-mass angle θ in the reaction $pp \to M+$anything. Assume, moreover, that M decays into m_1 and m_2 with zero Q value.[5] Figure 3 is a sketch of the kinematics. Because $Q=0$, the velocities of M, m_1, and m_2 are all equal; consequently, the final momentum of m_i is $p_i = (m_i/M)p_M$ and $p_{Ti} = (m_i/M)p_{TM}$. Therefore, if production of M is described by $d\sigma/dp_T^2 \propto \exp(-b_M p_T^2)$, then the distribution for m_1 is $d\sigma/dp_T^2 \propto \exp(-b_m p_T^2)$, with $b_m = b_M(M/m_1)^2$. Observe that sharpening occurs, not smearing. The effect is tremendous if m_1 is a pion and is even considerable for a proton. In reality, of course, $Q \neq 0$, and convolution (smearing) will occur. However, smearing starts from $b_M(M/m_1)^2$, not from b_M as a naive argument would suggest. This discussion emphasizes the crucial roles of (i) small Q value and (ii) small value of the pion mass.

More quantitative study requires accommodating at the same time $Q \neq 0$, nonforward production, as well as realistic distributions in spin J and mass of N^*'s.[6] We may investigate these issues numerically by postulating the process $pp \to N^* + (\mathrm{MM})$ and choosing a suitable (if crude) production amplitude of the form

$$|A|^2 = c|A_{N^*}|^2 \exp(\lambda t). \tag{1}$$

Amplitude A_{N^*} is a function of N^* mass and scattering angle $\theta(p_1 \to \pi)$ measured in the N^* rest frame (see Fig. 1); we can represent A_{N^*} either as a sum of Breit-Wigner (BW) terms or, for $N^* \to p\pi$, by using elastic on-shell data. In Eq. (1), c is a constant, and t is defined in Fig. 1. We ignore variables internal to cluster MM. In our investigation, we varied the mass and spin of N^*, the mass of MM, and the constant λ. From Eq. (1) we compute $d\sigma/dp_T^2$ for $0.5 \le p_L \le 0.6$ GeV/c, in order to compare with the data.[2] Results give full quantitative support to the qualitative arguments given above and are also in good agreement with the data.[2] For example, if MM = neutron, $\lambda = 12$ (GeV/c)$^{-2}$, and $N^* = \Delta(1236)$ with $J = \frac{3}{2}$, $d\sigma/dp_T^2$ is well approximated by $\exp(-bp_T^2)$ with $b = 16$ (GeV/c)$^{-2}$. Insensitivity of the results to λ

is apparent: E.g., if λ is dropped to 6, $b \approx 15$. However, MM and Q values are important variables. With $N^* = \Delta(1236)$, MM = 1.5 GeV, and $\lambda = 8\ (\mathrm{GeV}/c)^{-2}$, $b = 24$. On the other hand, in process $pp \to N^* + p$, if the N^* has mass 1.6 GeV and s-wave decay to $p\pi$, $d\sigma/dp_T^2$ peaks at $p_T^2 \cong 0.3\ (\mathrm{GeV}/c)^2$ and has a pronounced minimum at $p_T^2 = 0$.

We have established that a very sharp forward peak in $pp \to \pi^\pm$ + anything is obtained naturally. We now turn to the important issues of absolute normalization and the π^+/π^- ratio; comments follow on other aspects of the data and on the relationship of our approach to possible alternative models.

(1) Normalization.—The integral under the forward π^+ peak of Ref. 2 yields a large experimental cross section of $\lesssim 10$ mb.[7] This is a substantial fraction of the total inelastic pp cross section. However, bubble-chamber data on specific final states may be combined to show that $\sigma(pp \to \Delta^{++}$ + anything) at 12 GeV/c is $\geqslant 6$ mb.[8] Moreover, additional cross section is provided by production of other N^*'s having decay modes with Q sufficiently small to contribute a steeply-peaked forward π^+ distribution. These include $N^*(1400)$ ($\sigma \gtrsim 1$ mb), for which $n\pi^+$, $\Delta^0\pi^+$, and $\Delta^-\pi^+$ modes contribute, as well as $N^*(1512)$ and $N^*(1688)$ for which $\Delta^0\pi^+$ and $\Delta^-\pi^+$ modes have small enough Q. The total cross section from all these processes is sufficient to match the effect seen in Ref. 2.

(2) Ratio π^+/π^-.—Experimentally, $d\sigma(\pi^+)/d\sigma(\pi^-) \approx 3$ at 12.2 GeV/c.[2] That is approximately the ratio expected in our model; we sketch our reasoning here. Principal processes contributing to the forward π peak are assigned to three categories: $pp \to N_{3/2}{}^{++} + N_{1/2,3/2}{}^{0}$, $N_{3/2}{}^{+} + N_{1/2,3/2}{}^{+}$, and $N_{1/2}{}^{+} + N_{1/2}{}^{+}$. Symbol N denotes a system (not necessarily resonant) made up of a nucleon plus n pions, with $n = 0$, 1, or 2. Superscripts and subscripts on N denote total charge and isospin, respectively. Isospin $\geqslant \frac{5}{2}$ is excluded because exchange of $I \geqslant 2$ would be implied. We limit $n \leqslant 2$ because otherwise mass values are so large as to make unlikely the minimum necessary peripheralism in production. Decay proceeds in all three cases via $N \to N' + \pi^\pm$. Simple arguments based on isospin coupling coefficients demonstrate that the expected (π^+/π^-) ratio is in the range 2 to 4. In order to specify an exact figure, we would have to give a detailed discussion of production mechanisms for states in each of the three categories listed; that argument is inappropriate here.

(3) Proton distribution.—Our discussion has concentrated for the most part on the reaction $pp \to \pi^\pm$ + anything. However, for $pp \to p$ + anything,[9] our mechanism predicts that the p_T^2 distribution of the final proton will also be steeply forward peaked. This conclusion is easily obtained simply by reviewing the above arguments with the decay proton rather than the pion in mind; the slope b_p of the forward peak will be less than for pions, of course, because the ratio $(m_p/m_{\mathrm{res}})^2 \gg (m_\pi/m_{\mathrm{res}})^2$. Explicit calculations at incident proton lab momentum 12 GeV/c and $p_{L,p} = 1.7$ GeV/c (which is the appropriate fixed value of $p_{L,p}$ corresponding to $p_{L,\pi} = 0.6$ GeV/c) indicate that $b_p \lesssim \frac{1}{2} b_\pi$. Data indeed suggest a forward peak for protons, and $b_p \approx 8(\mathrm{GeV}/c)^{-2}$ at $p_{L,p} \approx 1.7$ GeV/c (inelasticity 60-70%).[3] However, there are really too few experimental points at small p_T^2 to determine a precise slope.[3] An important consistency check on our model is the observation that for protons and for pions, integrated experimental cross sections in steeply peaked forward regions are equal to within a factor of 2.[7]

(4) Background spectrum.—As the Q value increases, $\langle p_T \rangle$ for decay protons and pions also increases; for large Q, the distribution $d\sigma/dp_T^2$ has a pronounced minimum near $p_T^2 = 0$. Moreover, sensitivity to the mass of the decay product decreases sharply with increasing Q. It is not unexpected, therefore, that $d\sigma/dp_T^2$ approaches the same shape at large p_T^2 for both p and π.[2,3]

(5) Overall energy dependence.—Two comments are in order. First, the length of the major axis of the decay ellipse shown in Fig. 2 increases as $p_{\mathrm{lab}}^{1/2}$. Therefore, if distributions $d^2\sigma/dp_L dp_T^2$ at two energies are to be compared, one must scale values of p_L appropriately. For example, $p_{L,\pi} = 0.6$ at $p_{\mathrm{lab}} = 12$ GeV/c corresponds to $p_{L,\pi} = 1.0$ at 30 GeV/c. Second, regarding the magnitude and shape of $d^2\sigma/dp_L dp_T^2$, unless we invoke a detailed model for N^* production, it is not possible to make definitive statements. However, inasmuch as cross sections for Δ^{++} + anything, $N^*(1688)$ + anything, etc., appear to vary very slowly as p_{lab} is changed from 10 to 30 GeV/c,[8] we expect both the slope and magnitude of the forward peak in p_T^2 (integrated over p_L) to vary correspondingly little. On the other hand, because of the dilation mentioned above, the magnitude of $d^2\sigma/dp_L dp_T^2$ at fixed (scaled) p_L should decrease roughly as $p_{\mathrm{lab}}^{-1/2}$.

(6) Experimental confirmation.—It is important that bubble-chamber data support the view de-

veloped here. Panvini has examined fitted events of the type $pp \to pN + n\pi^+ + m\pi^-$ at 28.5 GeV/c, where N is a nucleon and $n, m \geqslant 1$. A strong Δ^{++} signal is seen in these data. The distribution $d\sigma/dp_T^2$ for π^+ has both steep and shallow components. The steep component is strongly correlated with π^+ being in the Δ^{++} region of $p\pi^+$ invariant mass.[10] Panvini's result suggests that bubble-chamber and counter data could be usefully correlated in other instances also, to yield more understanding than either device alone provides.

(7) Mass effect.—In high-energy reactions it is observed that $\langle p_T\rangle$ increases systematically with increasing particle mass.[11] Our mechanism, based on decay of isobars with low Q, suggests an explanation for this effect.

(8) Exchange mechanisms.—Finally, let us mention briefly the possibility of using baryon exchange at the $p\pi^\pm$ vertex to explain the observed steep dependence on p_T^2. Such an approach is in principle dual to the one we have advanced here. In practice, however, the explanation in terms of N^* decay is quantitatively superior. First, as remarked, it was essential to our argument that the Q value of N^* decay be small; in other words, $(N_f^*\pi)$ subenergy must be small in order that p_T^2 for the decay pion be small. Experience demonstrates that exchange mechanisms give quantitatively poor representations at small values of subenergy when strong resonances are present. Second, in the π^- case where only Δ_δ exchange is involved, the quite flat behavior of backward elastic π^-p scattering[12] would seem to rule out an exchange-model interpretation for $\exp(-15p_T^2)$. However, let us recall that one can get much closer to the Δ pole in an inelastic process and, moreover, that the Δ_δ reduced residue is rapidly varying.[12] No doubt an explanation based on baryon exchange can be constructed, but that type of argument will be speculative, at best, until we know considerably more about how to extrapolate reliably from backward elastic π^-p data to the correct residue at the $\Delta_\delta(1238)$ pole position.

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[1]The most recent publications are J. L. Day, N. Johnson, A. Krisch, M. Marshak, J. Randolf, P. Schmueser, G. Marmer, and L. Ratner, Phys. Rev. Letters 23, 1055 (1969); and G. Marmer, L. Ratner, J. Day, N. Johnson, P. Kalbaci, A. Krisch, M. Marshak, and J. Randolf, Phys. Rev. Letters 23, 1469 (1969). Previous work is cited therein. Symbol p denotes the magnitude of c.m. three-momentum; $d\Omega$ is the differential element of c.m. solid angle.

[2]Day, Johnson, Krisch, Marshak, Randolf, Schmueser, Marmer, and Ratner, Ref. 1.

[3]Marmer, Ratner, Day, Johnson, Kalbaci, Krisch, Marshak, and Randolf, Ref. 1.

[4]This is contrary to the conjectures found in the concluding paragraphs of Ref. 3.

[5]$Q = m_{\rm res} - \sum_i m_i$ where m_i is mass of ith decay product.

[6]In particular, $J > \frac{1}{2}$ leads to nonuniform population in the thin shell shown in Fig. 2 and to enhanced peaking at small p_T.

[7]Experimenters do not give an integrated cross section. In order to get our number, we integrate over p_L as well as p_T^2; that is the reason for the $\lesssim$ symbol.

[8]R. Panvini, in Proceedings of the Third International Conference on High Energy Collision of Hadrons (Gordon and Breach, New York, 1970). See also Fig. 1 of W. E. Ellis et al., BNL Report No. A-227(b) (unpublished).

[9]As in Ref. 3, "anything" excludes elastic channel.

[10]R. Panvini, private communication.

[11]G. Cocconi, Nuovo Cimento 57A, 837 (1968). See also Ref. 8, Table 2.

[12]E. L. Berger and G. C. Fox, Phys. Rev. 188, 2120 (1969). This paper contains a wealth of references to data, as well as detailed fits.