ly, Eq. (10) and property (iii) imply that as $\omega_{n,\,n+2}{}^{l}\rightarrow 0$, $\Omega_{n,\,n+2}{}^{l}\rightarrow 0$. Hence, $\mathfrak{F}_n(\Omega)$ cannot be analytic about the origin. In fact, the origin is a nonisolated singularity of the energy eigenvalues.

The nonanalyticity we have observed in the above example was easy to establish because Ω was given as an analytic function of a finite number of energy levels. The general argument includes the possibility that Ω is not simply expressible in terms of the energy levels.

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[4]Renormalization is not an answer to the problem of extracting physical information from divergent series. One must either take the coupling constant very small and use an asymptotic approximation or else introduce summability methods when this is not possible. Padé techniques have been used to sum the perturbation expansion in the anharmonic oscillator. For a discussion of these techniques and a verification of some of the properties of the anharmonic oscillator discovered in Refs. 2 and 3 see B. Simon, "Coupling Constant Analyticity for the Anharmonic Oscillator" (to be published), and J. J. Loeffel, A. Martin, B. Simon, and A. S. Wightman, "Padé Approximants and the Anharmonic Oscillator" (to be published).

PROTON-PROTON ELASTIC SCATTERING AT 15, 20, AND 30 BeV/c *

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Proton-proton elastic scattering has been measured at 15.2, 20.0, and 29.7 BeV/c. The data at 20.0 BeV/c confirm the presence of a break at $-t \cong 1.2$ (BeV/c)^2. The data at 29.7 BeV/c show essentially the same behavior. The cross section is still falling with increasing energy in this $-t$ range.

In the course of an extensive wire-plane experiment at the Brookhaven alternating-gradient synchrotron (AGS), our group accumulated data on elastic and inelastic p-p scattering at a number of angles and energies. The inelastic-scattering data along with details of the apparatus and the technique have already been reported.[1] A distinctive feature of these data is the presence of sharp breaks in the cross sections for single isobar production near $-t = 1$ (BeV/c)^2. Recent proton-proton elastic-scattering measurements[2] near 20 BeV/c also exhibit a pronounced break at $-t \simeq 1.2$ (BeV/c)^2. Many models have suggested that a dip or break might occur in this region. Some of these, based on the optical model proposed by Yang and his collaborators,[3] give an asymptotic form which is related to the proton electromagnetic form factor. Others, such as the Regge-pole model of Frautschi and Margolis,[4] are able to predict the energy dependence of the cross section. To distinguish between the predictions, it is important to obtain proton elastic-scattering data at as high an energy as possible in order to observe how the break changes with energy. This Letter presents elastic cross-section measurements at 15.2, 20.0, and 29.7 BeV/c, making it possible to compare directly with the data of Allaby et al.[2] and also to observe the change of the cross section with increasing energy.

The cross sections were measured by detecting the high-energy proton with a magnetic spectrometer which utilized wire planes connected "on-line" to a PDP-6 computer. A beam of 10^5 to 10^8 protons per pulse was obtained by diffraction scattering at one degree from an internal

Table I. Parameters for the least-squares fits of the elastic cross section in the small $-t$ region by the form $e^{a+bt+ct^2}$. The $-t$ interval is given below the incident beam momentum. The cross section at $t=0$ in mb/(BeV/$c)^2$ is given below a. b is in units of (BeV/$c)^{-2}$ and c is in units of (BeV/$c)^{-4}$.

Experiment	Beam momentum and t range	a and $d\sigma/dt(0)$	b	c
This experiment	15.1	4.08 ± 0.14	7.89 ± 0.59	-0.43 ± 0.59
	$0.22 < -t < 0.78$	59		
Harting et al.[a]	18.4	4.18 ± 0.08	8.58 ± 0.24	
	$0.2 < -t < 0.5$	65		
Foley et al.[b]	19.84	4.19 ± 0.15	8.68 ± 0.79	0.70 ± 0.92
	$0.2 < -t < 0.8$	66		
This experiment	20.0	4.23 ± 0.10	9.15 ± 0.45	0.72 ± 0.45
	$0.21 < -t < 0.8$	69		
Foley et al.[b]	24.63	4.09 ± 0.30	7.97 ± 1.56	0.82 ± 1.83
	$0.25 < -t < 0.75$	60		
This experiment	29.7	3.76 ± 0.12	8.02 ± 0.60	-0.64 ± 0.65
	$0.21 < -t < 0.73$	43		

[a]Ref. 6. [b]Ref. 7.

target in the AGS. This system had an overall momentum resolution of ±0.3% and an angular resolution of ±0.4 mrad at 30 BeV/c. Based on these numbers, the resolution in t is estimated to be ±0.03 (BeV/$c)^2$ at 30 BeV/c and $-t=1.2$ (BeV/$c)^2$, with the dominant contribution resulting from the beam angular divergence. Empty-target runs were taken for nonhydrogen background subtractions.

A correction has been made for background due to the tail of the inelastic scattering under the elastic peak and multiple interactions in the 9-in. liquid-hydrogen target. For low momentum transfer, the correction is small. For example, at $-t \cong 0.4$ (BeV/$c)^2$ and 30 BeV/c it is 2%. The correction averages (25± 15)% at 130 BeV/c in the $-t$ region near 1.0 (BeV/$c)^2$ and (15± 10)% at 20 BeV/c. The relative uncertainty between 20 and 30 BeV/c due to nonelastic background subtraction is at most 10%. The variations in this correction in the region 0.8 (BeV/$c)^2 < -t < 2.0$ (BeV/$c)^2$ are smaller than the statistical errors and can be neglected.

The cross-section values below $-t=0.8$ (BeV/$c)^2$ have been fitted with the form $e^{a+bt+ct^2}$ following Foley et al.[5] These parameters are given in Table I, along with parameters from several similar fits. Our values are consistent to within statistics. There is a tendency for all of these fits to extrapolate below the cross section at $t=0$ determined from the total-cross-section measurements of Foley et al.,[7] particularly at the highest energy. This behavior might indicate that the logarithmic slope of the cross section increases below $-t=0.2$ (BeV/$c)^2$. Such a behavior is consistent with the recent Serpukhov results[8] at very small $|t|$. Although there is a tendency for the fits beyond $-t=0.2$ (BeV/$c)^2$ to extrapolate below the optical point, this does not imply a large reduction in the elastic cross section because a large fraction of the cross section is in the range $0.0 < -t < 0.2$ (BeV)/$c)^2$.

The data have been plotted in Fig. 1 along with the small-$|t|$ fits. In addition the four-parameter Orear parametrization[9] has been shown for the incident momenta of this experiment. The Orear parametrization approximately follows the gross energy dependence of these data. The data in the region of the break have been plotted in Fig. 2 along with the data of Allaby et al. at 19.2 BeV/c. The errors shown are statistical only. The 20.0-BeV/c data are in good agreement with Allaby et al. both in magnitude and shape. A clear break occurs at $-t \simeq 1.2$ (BeV/$c)^2$. The 30-BeV/c cross section has the same shape but is on the average 0.5 as large as the 20-BeV/c data, indicating that the cross section is not yet approaching an asymptotic limit but is still falling with increasing energy. The break still occurs at $-t \cong 1.2$ (BeV/$c)^2$. Although the $-t=1.3$-(BeV/$c)^2$ point is distinctly low, it is not possible to show statistically that the break has deepened into a dip.

The logarithmic slopes after the breaks are 1.5 (BeV/$c)^{-2}$ at 20.0 BeV/c and 1.3 (BeV/$c)^{-2}$ at 29.7 BeV/c. As we have noted in our earlier paper,[1] these logarithmic slopes are very similar to the logarithmic slopes of the 1.52- and 1.69-BeV isobars in the same t region [$b=1.5$ (BeV/$c)^{-2}$]. This is unlike the situation for $-t$

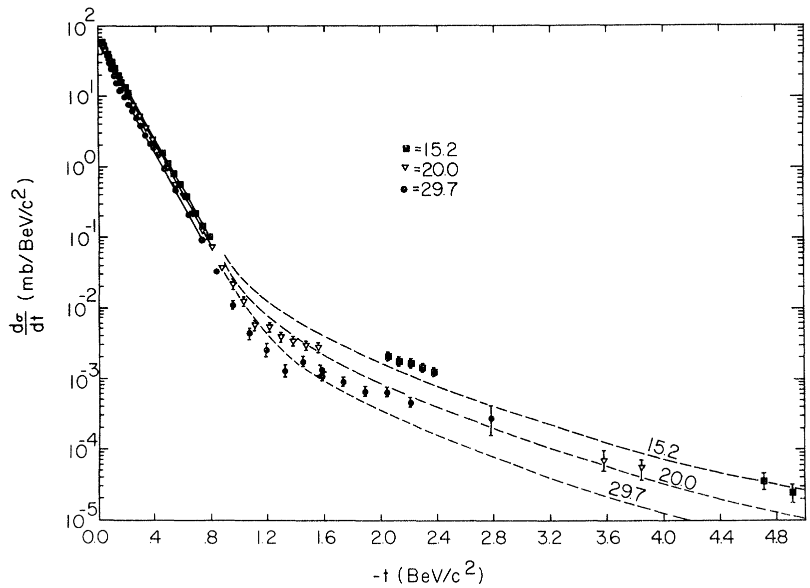

FIG. 1. p-p elastic cross section as measured in this experiment. The solid lines are the fits from Table I. The dashed lines are the predictions of the universal Orear parametrization.

<1.0 (BeV/c)^2 where the logarithmic slopes for these isobars are a factor of 2 smaller than the elastic logarithmic slopes.

A number of theories have been proposed for the intermediate $-t$ region. Optical models, based in part on the original Wu-Yang theory, show diffractive minima which are somewhat filled in by contributions from the real part of the scattering amplitude and spin effects. The requirements of unitarity appear to separate the minima for the imaginary and real part so that complete zeros never occur at present energies.[10] Typically the cross section changes slowly with energy. The asymptotic cross section of Abarbanel, Drell, and Gilman[3] has been plotted in Fig. 2 as an example of a diffractive theory. Their model is particularly intended to describe the very large $-t$ region. In Regge-pole models the zeros of the real and imaginary parts can occur at the same t, leading to the possibility of deep dips in the cross section. In addition, Regge theories characteristically predict a stronger energy dependence for the cross section than the optical theories. The 30-BeV/c prediction of Frautschi and Margolis has been plotted in Fig. 2 to illustrate a Regge-pole model. The curve shown utilizes typical parameters for illustrative purposes and is not intended as a fit. There have been some promising attempts to relate the Regge-pole and optical approach in a hybrid model.[11] These are also capable of producing relatively

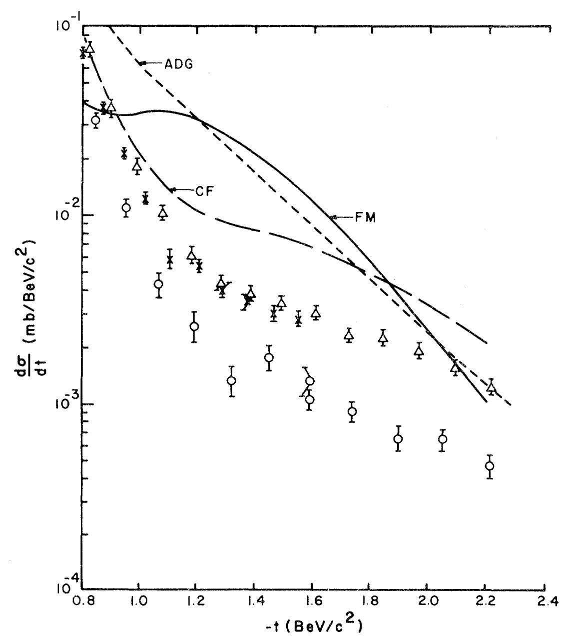

FIG. 2. p-p elastic cross section as a function of t in the region of $|t| = 1.0$ (BeV/c)^2. Circles are the 29.74-BeV/c points, crosses are the 20.0-BeV/c points. Triangles are the data of Allaby et al. (Ref. 2) at 19.2 BeV/c. The solid line is the 30-BeV/c prediction of Frautschi and Margolis. The short-dashed line is the asymptotic cross section of Abarbanel, Drell, and Gilman (Ref. 3). The long-dashed line is the 25-BeV/c cross section of Chiu and Finkelstein. [Note that the y axis is at $-t = 0.8$ (BeV/c)^2, not at $t = 0.0$ (BeV/c)^2.]

deep dips. Figure 2 includes the 25-BeV/c prediction of Chiu and Finkelstein as an example of a hybrid model. This theory reproduces the general shape of the intermediate $-t$ region and also gives a cross section which falls with increasing energy approximately in the way our cross-section measurements do.

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