tism (Wiley, New York, 1965).

[13] The basic idea of this topology is strongly suggested by the data in Refs. 3 and 4 and has been discussed by R. C. Richardson, private communication, and C. M. Varma, private communication, among others.

[14] The system of equations giverning the behavior of a multiple-bath system is discussed carefully in Ref. 1.

[15] A quantitative discussion of the specific-heat anomaly in mixtures depends upon the magnitude of the ^{3}He-^{4}He tunneling rate J'. Experimental information on J' will be available from the experiments of Richardson and co-workers on mixtures. We will discuss the specific-heat anomaly in detail when the mixture experiments have been completed.

"SHAPE-DEPENDENT" SPECIFIC HEAT OF CEROUS MAGNESIUM NITRATE*

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The specific heat of two spheres and of one cylinder of compacted powdered cerous magnesium nitrate has been measured from $T^* = 3.5$ to 20 mK. Above 6.5 mK a correction of +0.3 mK must be added to convert T^*(sphere) to the thermodynamic temperature T, and +1.1 mK to convert T^*(cylinder) to T. Below 6.5 mK, T^*(sphere of powder) + 0.3 = T^*(cylinder of powder) + 1.1 mK = T^*(single-crystal sphere) + Θ(Weiss constant). The specific heat of cerous magnesium nitrate is given by $C/nR = 6.4/T^2 - 12/T^3$.

Although nuclear magnetic resonance and Josephson junctions are promising techniques for temperature measurement in the millikelvin range, the susceptibility of cerous magnesium nitrate (CMN) is still the primary thermometric parameter because of ease and precision of measurement. The latter feature is essential where small temperature increments are being measured. In spite of the increased activity in this temperature region and of the importance of the temperature scale, the T-T^* correlation[1-5] (thermodynamic-magnetic temperature) below 6 mK has not been established with certainty nor has the demagnetization correction for shapes other than ellipsoids of revolution been evaluated. Hudson[6] has recently published an excellent article which summarizes the present status of the CMN temperature scale and clearly points up the confusion that exists. In the attempt to bring a little order into the chaos, we have measured the specific heat of two spheres and of one cylinder, the three specimens machined from compacted powdered CMN. By using the specific heat as a transfer parameter, we have established the shape correction from a cylinder and a sphere of powder to single-cyrstal sphere, as well as the coefficient of the second term in the specific-heat expansion.

A mixture of 80% by volume powdered CMN and 20% by volume powdered AgCl was compacted around a fan of silver wires. The resultant cylinder (2.54 cm diam × 2.54 cm high) was machined into either a sphere (1.892 cm diam) or a cylinder (1.892 cm diam × 1.892 cm high). The specimens were inserted, in turn, into the same coil form for measurement of the specific heat. The coil form was made of epoxy and was wound with an inner secondary of approximately 2000 turns, a compensating secondary of approximately 1000 turns, and a shimmed primary to produce a uniform field around the specimen. A heater (100 Ω nominal resistance) of Pt-5%W alloy was wound on the silver wire tail which was used to provide thermal contact to a chrome-alum guard pill through a Pb superconducting switch.

The temperature scale for each specimen was set up by calibrating the mutual-inductance bridge[7] against the vapor pressure of pure ^{3}He with the specimen in situ. Calibration commenced at 600 mK and proceeded in five equal intervals of $1/T$ to 1.6 K, the limit set by the quantity of ^{3}He in the system. Cathetometer readings for each manometer leg and the bridge readings were plotted as a function of time; the calibration point was interpolated to the midpoint of the measurement period. Pressures were corrected to Torr; drifts during calibration were never more than 1 mK/min. A least-squares fit to the form $M - M_0 = A/T$ was made for the constants M_0 and A which now determined the temperature scale; M is the mutual-inductance bridge reading.

The experiment was performed by raising a superconducting solenoid around the chrome alum and energizing the magnet to produce a field of 18 kOe. The initial cooldown to 1.2 K and the dissipation of the heat of magnetization were achieved with a small auxiliary ^{3}He evaporator. The chrome alum was demagnetized in 7 min to 35% of the field, then the magnet was centered around the CMN. After the heat of magnetization had been removed through the Pb switch, the field was reduced to zero and the magnet lowered away from the specimen. With this technique a minimum T_s^* of 2.8 mK was obtained with the spheres and a minimum $T_{\rm cyl}^*$ of 2 mK with the cylinder.

After the magnet was lowered the temperature (mutual inductance) was followed for at least 20 min before adding heat. The heating period was controlled by an electronic timer to be 100.00 sec and the energy input selected to give a temperature rise of 20% of the initial temperature. During the heating the potential drop across the heater and the current through the heater were measured with a Leeds and Northrup type-K3 potentiometer. Equilibrium was established within three minutes but the temperature was recorded every minute for an additional ten minutes to establish the after drift. The data were analyzed by fitting a function $M = a + bt + ct^2$ by least squares to the (mutual inductance versus time) readings, then extrapolating M to the midpoint of the heating period before converting to temperature. Curvature correction for the finite temperature interval was applied to the specific-heat data. By plotting the data so obtained, it was apparent that 0.8 mK had to be added to all points taken with the cylinder to superimpose them on the points taken with the spheres; this correction, incidentally, is the difference between the observed minimum $T^* = 1/\chi = A/(M - M_0)$ for these shapes. We conclude without further analysis that $T_s^* = T_{\rm cyl} + 0.8$ mK. The results are displayed in Fig. 1 where the specific heat C/nR is plotted against the thermodynamic temperature T obtained as described below. Also included in the graph are the earlier data of Abraham and Eckstein[8] which have been corrected for finite temperature rise as was done for these data. The size of the points indicates the probable error.

It is now generally accepted[9] that the temperature dependence of the specific heat of CMN is represented by a series expansion,

$$C/nR = (b_1/T^2)[1 + (b_2/b_1)/T + (b_3/b_1)/T^3 + \cdots]. \qquad (1)$$

Two types of measurements have been used to evaluate the coefficient b_1: caloric measurements[1,4,5,8,10] and susceptibility measurements.[5,11]

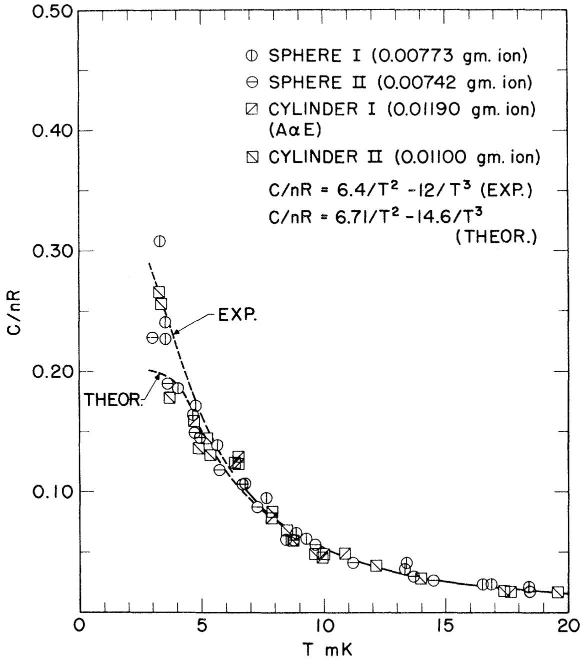

FIG. 1. The specific heat C/nR of cerous magnesium nitrate as a function of the thermodynamic temperature T in millikelvins.

The former, which have been fitted by a truncated version of Eq. (1), $C/nR = b_1'/T^2$, give too low a value for b_1 because b_1' is evaluated as b_1 times some average value of the bracket in Eq. (1). Susceptibility measurements, on the other hand, give more nearly the correct value because the leading term is the only one contributing in the temperature range in which such measurements are made. Using the susceptibility method of Casimir and du Pre,[12] Mess et al.[5] obtained the value 6.2 mK^2 for b_1 and Abraham et al.[11] the value 6.4 mK^2, which are in excellent agreement, and are to be compared with the theoretical values of Cooke, Duffus, and Wolfe[13] (6.75), Daniels[14] (6.6), Peverley and Meijer[15] (6.5), and Wong, Dembinski, and Opechowski[16] (6.8). The latter value, 6.8 mK^2, is high because incorrect lattice constants were used. The coefficient b_1 can now be considered well established at 6.4 ± 0.2 mK^2.

The measurements above 6.5 mK were combined by adding 0.8 mK to those obtained with the cylinder, and fitted by least squares with Eq. (1), from which we found that $C/nR = (6.6 \pm 0.3)/T^2 - (15.4 \pm 3)/T^3$; the data were not precise enough to determine the next term. At 7 mK the error in the T^{-3} term introduces an error of 7% in the calculated specific heat; further, b_1' can with these numbers take a minimum value of 5.4, close to most calorimetric values.

It is unreasonable to expect that a spherically shaped agglomerate of randomly oriented particles such that $g_{\rm eff}=\frac{2}{3}g_\perp$ will reach a lower temperature than a properly oriented single-crystal sphere for which $g_{\rm eff}=g_\perp$. In loosely packed powder such as this (packing fraction about 80% by volume) with a highly anisotropic g value, one would not expect the particle demagnetization correction to average to zero over the sphere. We therefore fixed b_1 at the experimental value 6.4 and applied a chi-square test to arrive at the best b_2 and Δ in the equation

$$C/nR=6.4/(T_s^*+\Delta)^2+b_2/(T_s^*+\Delta)^3. \qquad (2)$$

The Δ includes both the demagnetization correction Δ' and the Weiss constant Θ which, according to Peverley and Meijer,[15] has the value −0.3 mK. From the analysis we find that $b_2=-12.0\pm3.0$ mK^3 and $\Delta=+0.3\pm0.2$ mK. We conclude that $T_s^*+0.3=T_{\rm cyl}^*+1.1=T$(thermodynamic) above 6.5 mK and $T_s^*+0.6=T^*$(single-crystal sphere) below 6.5 mK. When T^* values are corrected as indicated, then the minimum T^* for the two shapes becomes consistent with the minimum T^*(single-crystal sphere) observed by Daniels and Robinson,[1] Frankel, Shirley, and Stone,[3] and Abeshouse *et al.*[17] with single-crystal spheres.

The specific-heat equation $C/nR=6.4/T^2-12/T^3$ is plotted in Fig. 1 as the solid line for $T>6.5$ mK and as the dotted line (Exp) for lower temperatures to emphasize that these points were not included in the numerical analysis. The experimental data below 6.5 mK have been corrected with the correlation between T and T^*(single-crystal sphere) of Frankel, Shirley, and Stone by applying the correction separately to the initial and final temperatures for each point, *not* to the average temperature. The dotted curve (Theor) was calculated from

$$C/nR=6.71/T^2-14.6/T^3, \qquad (3)$$

where the coefficients are the theoretical values from Wong, Dembinski, and Opechowski[16] using their exchange parameter $w=-0.252$. The fit to the experimental data above 8 mK is indistinguishable from Eq. (2). Although Eq. (3) fits the data remarkably well to 4 mK, this must be regarded, in part, as fortuitous since the next higher term must make a contribution and the fit is dependent on the correlation between T and T^*(single-crystal sphere). Nor can the agreement with theory be used to say anything definitive about the ferromagnetic versus antiferromagnetic transition in CMN[5,17] since a negative (ferromagnetic) exchange parameter compensates a positive $1/T^4$, the next term in the expansion.

In conclusion, we have established the following with these measurements:

(1) Complete consistency between temperatures measured with powders in spherical and cylindrical configurations and temperatures measured with single-crystal spheres. Contrary to the conclusion of Abel and Wheatley[18] that $T_{\rm cyl}^*$(powder) $=T$ for all temperatures, a conclusion which requires a *mécanisme inconnu* and a fortuitous happenstance, we find that

$$\begin{aligned} &T_s^*(\text{powder})+0.3\\ &\quad =T_{\rm cyl}^*(\text{powder})+1.1=T, \quad T\geq 6.5\text{ mK},\\ &\quad =T^*(\text{single-crystal sphere})-1.1,\\ &\qquad\qquad\qquad\qquad\qquad\qquad T<6.5\text{ mK}.\end{aligned}$$

(2) The relation

$$C/nR=6.4/T^2-12/T^3, \quad T\geq 6.5\text{ mK}.$$

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SPLITTING OF CONDUCTION-ELECTRON SPIN-RESONANCE LINE IN INDIUM ANTIMONIDE IN QUANTIZING MAGNETIC FIELDS

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Splitting of the conduction ESR spectrum in InSb ($N = 1\times 10^{14}$ cm^{-3}) has been observed at 105 GHz and at 1.6°K. Two resonance lines have been obtained instead of the previously reported single resonance line. Experimental g factors and their shift associated with hot-electron effects fit well to the theoretical g values calculated for $n = 0$ and $n = 1$ Landau levels.

The magnetic quantization of electron energy levels in indium antimonide can be achieved at relatively low magnetic fields. In principle, with properly chosen experimental conditions, one can simultaneously populate two adjacent Landau levels. Because of the nonparabolicity of the conduction band in this substance, the effective mass and the g factor for different Landau levels is not the same. For this situation, the conduction electron spin resonance (CESR) spectrum should be composed of two distinct resonance lines, each corresponding to the Zeeman splitting of the appropriate Landau level.

The CESR spectra in InSb which have been reported previously[1-6] were composed only of a single resonance line. This resulted from the fact that most experiments performed so far have been done at frequencies not high enough to fulfill the conditions $\hbar\omega_c > kT$, $\omega_0\tau > 1$. Furthermore, the temperature was too low to populate the higher Landau level to an extent which would enable presently available EPR spectrometers to detect the resonance. Another restriction, which may eventually put a lower limit upon the frequency at which the separate lines for different Landau levels can be observed, is the effect of motional narrowing.[7]

We have considered all these aspects in order to make the experimental conditions most favorable for the apparatus which we had. It was found that for the purest available sample, with carrier concentration $N = 1\times 10^{14}$ cm^{-3}, the $n = 0$ and $n = 1$ Landau-level lines should be simultaneously observable at temperatures between 4 and 6°K and at frequencies greater than 100 GHz. It should be pointed out, however, that by "temperature" we mean here the kinetic or electron temperature. The transition probability for spin resonance is proportional to the incident microwave power. On the other hand, the conduction electrons in InSb are very sensitive to heating. Therefore, it is preferable to work with lattice temperature as low as possible and attain the required population of the higher Landau level by heating the electrons in the microwave field.

The spin-resonance experiments were performed at 105 GHz and at liquid-helium temperatures. By use of a simple microwave train consisting of a millimeter-wave klystron, an attenuator, and suitable oversized waveguide, the system was arranged so that the microwave power absorbed in the sample could be precisely regulated.

We developed a special contactless detection system consisting of a flat coil which together with a small capacitor formed a rf resonant circuit immersed in the liquid-helium bath. The sample in the form of a very thin slab was kept near the coil surface in the rf field by a mica sheet fixed to the coil by silicon grease. Audiofrequency magnetic-field modulation and synchronous second detection were used to obtain the magnetic-field derivative of the resistivity.

Figure 1 shows representative spectra of the derivative of CESR absorption recorded at lattice temperature of about 1.6°K. Different micro-