

FIG. 2. Measurements of the contribution of the one-phonon scattering to the structure factor  $S(Q)$  at  $1.1^\circ\text{K}$ . The results obtained with the triple-axis spectrometer are shown by solid circles and have smaller errors than those obtained with the rotating-crystal spectrometer shown by open circles. The solid line at low  $Q$  is the theoretical limit for  $S(Q)$  at  $0^\circ\text{K}$ .

tional to  $T^{-5}$  rather than to the  $T^{-7}$  found<sup>3</sup> for a finite  $\gamma$ . Similarly  $\beta = \frac{5}{2} \delta (\hbar\bar{\omega}/c)^4 \tau$  instead of  $\frac{3}{2} \gamma (\hbar\bar{\omega}/c)^2 \tau$ , where  $\hbar\bar{\omega} = 3k_B T$ . In the absence of the cubic term,  $\beta$  is inversely proportional to temperature in agreement with the findings of Abraham et al.<sup>1</sup> although its magnitude is too small by a factor  $\sim 50$ . The agreement with the temperature dependence does, however, show that these results may be an aid to understanding at least some of the previously unexplained results on the ultrasonic properties of liquid helium.

The integrated intensities of the one-phonon neutron groups are shown in Fig. 2. The absolute intensity of the scattering was deduced from

measurements at large momentum transfers where the total scattering function  $S(Q)$  is unity. The straight line through the origin is the theoretical limit of  $S(Q)$  as  $Q \rightarrow 0$ . This line and the more accurate triple-axis measurements agree for  $Q \leq 0.3 \text{ \AA}^{-1}$ . The first moment of the scattering function  $S(Q, \omega)$  is given by<sup>2</sup>

$$\int \omega S(Q, \omega) d\omega = \hbar Q^2 / 2M,$$

where  $M$  is the mass of the helium atom. If the velocity of the phonons is a constant (which is approximately the case for  $Q < 0.5 \text{ \AA}^{-1}$ ), then departures from linearity in  $Q$  for the one-phonon neutron cross section must be accompanied by additional scattering. This additional scattering has been observed and the results are in good agreement with the above predictions as we plan to describe in detail in a more comprehensive paper on neutron scattering from liquid helium.

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<sup>3</sup>See I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965).

<sup>4</sup>Y. Disatnik, Phys. Rev. **158**, 162 (1967).

<sup>5</sup>B. N. Brockhouse, in *Inelastic Scattering of Neutrons in Solids and Liquids* (International Atomic Energy Agency, Vienna, 1961), p. 113.

<sup>6</sup>A. D. B. Woods, E. A. Glaser, and R. A. Cowley, in *Inelastic Scattering of Neutrons* (International Atomic Energy Agency, Vienna, 1968), Vol. II, p. 281.

<sup>7</sup>See Ref. 1 and other references therein.

<sup>8</sup>L. van Hove, Phys. Rev. **95**, 249 (1954).

## EXISTENCE OF ION MOBILITY DISCONTINUITIES IN SUPERFLUID HELIUM\*

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Previous investigators have reported the existence, at certain values of the drift velocity, of discontinuities in the mobility of positive and negative ions in superfluid helium. A different experimental technique gives results which seem to indicate that such discontinuities are probably spurious.

The study of positive and negative ion complexes in superfluid helium has over the last decade yielded a variety of interesting results. Perhaps the simplest thing one can do in this area is to measure the equilibrium drift velocity  $v_D$  of such

ions under the action of a uniform electric field  $E$ . Such experiments have given a great deal of information on the nature of the ion complexes and on their interaction with the normal-fluid excitations.

One of the oddest phenomena which has been reported<sup>1-5</sup> is the existence of rather sudden changes in the mobility  $\mu = v_D/E$  at certain values of  $v_D$ . Such an effect implies an abrupt change either in the nature of the ions or in their interaction with the surrounding fluid. Several authors<sup>6-8</sup> have proposed phenomenological explanations involving the creation of quantized vortex rings at the apparent "critical velocities."

To study this problem more closely we have used an experimental technique which is radically different from those used in the previous investigations. The experimental cell immersed in the helium consists of a 20- $\mu$ Ci Am<sup>241</sup>  $\alpha$  source which creates the ions, a gating grid located 0.2 cm in front of the source, a 1.9-cm ion drift space, and a collector with a guard grid located 0.1 cm in front of it. By means of properly biased guard rings the electric field in the drift space is kept uniform to within  $\frac{1}{4}\%$ . The cell is normally kept in the conducting state by a forward bias between the source and the gating grid and the ion current is periodically switched off<sup>9</sup> by a rectangular reverse-voltage pulse applied to this region. The off pulse then propagates across the drift space and the collector region. Collected current is detected by an operational amplifier<sup>10</sup> and is digitally signal averaged, the trigger being provided by the start of the off-voltage pulse. The electronic rise time of this detecting system was measured to be about 50  $\mu$ sec.

Figure 1 shows a typical output, read onto an X-Y recorder from the signal averager.<sup>11</sup> This particular sample of the raw data is for positive ions at 1.10°K, with a field of 11 V/cm. The base line corresponds to full current. Since the operational amplifier inverts the signal, an off pulse appears as a positive-going signal on the trace. To minimize the effect of electronic rise time, the time of flight is determined by drawing a straight line through the leading edge of the pulse and finding its intercept with the base line. Because of residual noise in the trace, the accuracy of this graphical procedure is estimated to be  $\pm\frac{1}{2}\%$  at worst.

Several comments about the raw data and our method of analysis should be made here. First, the time of flight defined in this way is actually the time required to reach an effective collector position close to the position of the guard grid. Even if this grid perfectly screens the drift space from the collector, an induced collector current will appear as soon as the ion pulse passes the guard grid. This effect is also responsi-

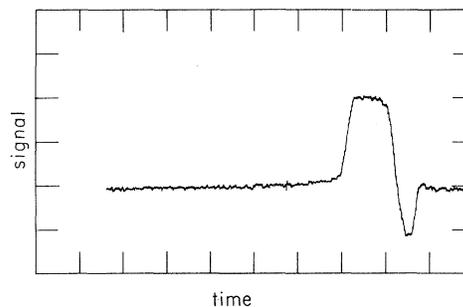


FIG. 1. Typical signal trace showing arrival of off pulse after moving across the drift space. Each division in time is 8.5 msec for this particular example. Signal scale is in arbitrary units.

ble for the finite slope of the pulse edges. The slope is definitely not caused by ions moving at different velocities, by diffusion of the ions, or by other effects which would blur the sharpness of the actual pulse edge. Similar experiments with a 30-cm drift space have shown that such effects are completely negligible. As long as the field in the collector region is kept equal to that in the drift space, our method of analysis yields results which have high relative accuracy, but are subject to a geometrical correction for the effective position of the collector.

The values of  $v_D$  derived in this way are independent of the voltages used in the source region, as well as pulse duration, pulse repetition rate, or signal strength. The shape of the received signal pulses remains substantially unchanged as the temperature or the field is varied, although of course the time of flight is strongly affected. Repeatability of the measurements apparently is limited by the accuracy of the graphical analysis.

Finally, with this setup one can observe and identify many effects which are potential sources of error in experiments where the signal is integrated and measured with a dc electrometer. As simple examples, one can see in Fig. 1 a gradual fall in current near the front edge of the off pulse, and a sharp peak immediately after the current is turned back on. The first of these is due to incomplete guarding of the collector because of our use of a rather coarse guard grid, and can easily be eliminated by using a finer grid. The second is due to ions which are trapped in the source region by the off pulse and concentrated near the source without being effectively re-collected. This peak disappears when higher off voltages are used. At lower temperatures, where the current can be composed of ions, quantized vortex rings created at the source, and

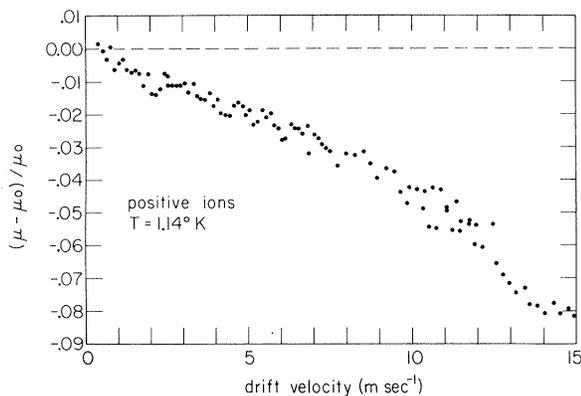


FIG. 2. Fractional deviation of the mobility from its zero-field value, for positive ions at 1.14°K.

quantized vortex rings created in the liquid, one sees quite complicated pulse shapes representing a variety of effects. Looking at such signals by time averaging can produce spurious results or wrong interpretations. For example, Cunsolo, Maraviglia, and Ricci<sup>12</sup> have observed a "limiting drift velocity" of 4.8 m/sec for negative ions at temperatures below 0.53°K, whereas even at lower temperatures we have found no difficulties in seeing pure negative-ion currents at much higher velocities.

Because of the high relative accuracy, the good repeatability, and the ease of interpretation of our data, we expected to obtain new insight into the mobility discontinuities. The experimental results, however, were disappointingly simple. We failed to observe the slightest evidence for the existence of sudden changes in either the positive- or negative-ion mobilities when plotted against drift velocity or field. Figures 2 and 3 are typical of our results, where the fractional deviation of the mobility from the zero-field limit is plotted against the drift velocity. According to the previous reports, there should be sudden drops of from 3 to 10 vertical divisions at  $v_D \sim 5$  and 10 m/sec in Fig. 2, and  $v_D \sim 2.4$ , 4.8, and 7.2 m/sec in Fig. 3. Clearly no such effects are present. Smooth lines can be drawn through the data with virtually all experimental points lying within  $\pm \frac{1}{2}\%$  of the lines. The figures do show an interesting difference in the field dependence of positive- and negative-ion mobilities similar to what has been observed at lower temperatures.<sup>13,14</sup> This is under further study.

The single-gate method used in various forms in the previous experiments where discontinuities were observed<sup>15</sup> is subject to a great num-

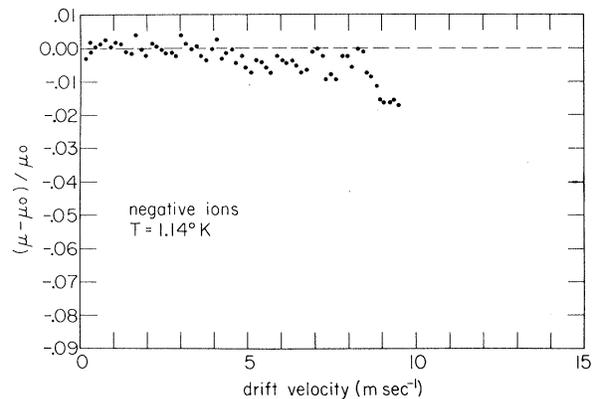


FIG. 3. Fractional deviation of the mobility from its zero-field value, for negative ions at 1.14°K.

ber of possible sources of error, including large buildups of space charge, generation of second-sound resonances in the drift space by eddy currents, frequency-dependent dc electrometry, and (as we have discussed) various spurious components in the actual signal. On the other hand the direct time-of-flight technique, which we have also used in a number of previous experiments,<sup>13,14</sup> suffers from none of these disadvantages. It is also worth emphasizing that in their classic double-gate experiments, Reif and Meyer<sup>16</sup> failed to observe any sudden changes in the mobility, although their accuracy and resolution were comparable with later work. Finally, all of the theoretical attempts to deal with the mobility discontinuities have been entirely phenomenological, and in themselves present no compelling reasons for the existence of such an effect. In view of all this, it appears probable that discontinuities in the mobility do not in fact exist.

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eliminated.

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<sup>13</sup>K. W. Schwarz and R. W. Stark, Phys. Rev. Letters 21, 967 (1968).

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## ULTRASONIC MEASUREMENTS NEAR THE CRITICAL POINT OF He<sup>4</sup>

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Low-frequency sound-velocity measurements in the range 1.5-50 kHz have been made along several isochores near the critical point of He<sup>4</sup> using a resonance technique. The adiabatic compressibility along the critical isochore diverges as  $(T-T_c)^{-\alpha}$  with  $\alpha$  larger than zero. The data are consistent with  $\alpha \approx \frac{1}{8}$ . The temperature dependence of the measured dispersion agrees with a recent calculation by Kawasaki.

In this Letter, measurements of the sound velocity  $u$  in He<sup>4</sup> along isochores near the critical point are reported. These indicate that the divergence of the adiabatic compressibility  $\kappa_s$  as a function of  $t \equiv (T/T_c) - 1$  is stronger than logarithmic for  $T > T_c$ . In addition, low-frequency dispersion measurements are presented which agree with the temperature dependence predicted by a recent mode-mode-coupling calculation of Kawasaki.<sup>1</sup> The divergence of the constant-volume specific heat  $C_v$  which has been observed in several fluids near the critical point<sup>2-5</sup> implies the vanishing of the isentropic sound velocity.<sup>6</sup> Ultrasonic measurements in He<sup>4</sup> at 1 MHz<sup>7</sup> and in CO<sub>2</sub> at 1 kHz<sup>8</sup> have been interpreted in terms of a logarithmic divergence of  $\kappa_s$ . In these and other previous investigations,<sup>9</sup> however, the behavior of  $u$  very near the critical point was obscured by effects of gravity or dispersion.

In the present work, corrections due to gravity and dispersion are minimized by performing low-frequency velocity measurements in a resonator of small vertical height. The resonant frequencies  $f$  of a cylindrical resonator containing a homogeneous fluid are given by

$$f_{p mn} = \frac{1}{2} u [(p/l)^2 + (\alpha_{mn}/a)^2]^{1/2}, \quad (1)$$

where  $u$  is the sound velocity,  $p$  is a non-negative integer,  $l$  and  $a$  are the length and radius, respectively, and  $\alpha_{mn}$  is a solution of the relation  $d[J_m(\pi\alpha)]/d\alpha = 0$ , where  $J_m(\pi\alpha)$  is a Bessel

function of the first kind. The resonator was designed to excite either the "plane-wave" modes ( $\alpha_{mn} = 0$ ) or the "Bessel-function" modes ( $p = 0$ ).<sup>10</sup>

In the presence of gravity, the divergence of the isothermal compressibility near the critical point produces a significant density gradient in an experimental cell of finite height. In this case, the local velocity is a function of height and the resonant frequencies no longer satisfy Eq. (1). If the equation of state is known, however, the density distribution can be determined, and the resonant frequency calculated numerically. Such an analysis using a "scaled" equation of state<sup>11</sup> will be reported elsewhere.<sup>12</sup> In this paper, the results of the gravity analysis are presented in the limit of small density gradients.

The sound velocity was measured in a cylindrical resonator, 0.49 cm high and 5 cm in diameter, which was terminated with similar electrostatic transducers.<sup>13</sup> Commercial helium gas purified in a charcoal trap at 4.2°K was stored in a room-temperature reference volume before being condensed into the resonator. The pressure in the resonator was measured with a sensitivity of 0.01 Torr through a small filling capillary which was vacuum jacketed and heated to eliminate condensation. The density was changed by exchanging helium gas between the resonator and reference volume. Relative density changes were accurate to within  $\pm 0.3\%$ . Temperatures, measured with a germanium thermometer in di-