M. Jones and R. Miller for technical assistance.

†Work performed under the auspices of the U. S. Atomic Energy Commission.

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OBSERVATION OF ρ-ω INTERFERENCE IN ANTIPROTON ANNIHILATIONS*

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(Received 29 December 1969)

Evidence for structure near the ω^0 mass in the $(\pi^+\pi^-)$ mass spectrum from the reaction $\bar{p}p \to 2\pi^+2\pi^-$, at incident momenta from 1.26 to 1.65 GeV/c, is presented. We interpret this structure as arising from ρ-ω interference in the 2π decay mode, with a statistical significance of about 3.5 standard deviations. The effect can be well fitted by a partial width $\Gamma(\omega\to 2\pi)$ in the range 0.18 to 5.3 MeV (95% confidence level). Alternatively, if $\Gamma(\omega\to 2\pi)$ is taken from e^+e^- colliding-beam results, our data can be used to study both the relative phase and coherence of ρ and ω production in $\bar{p}p$ annihilation.

Several experiments[1-3] have recently reported observations of the G-parity-violating 2π decay mode of the ω. In each case, the amplitude was seen in interference with the broad $\rho\to\pi^+\pi^-$ decay spectrum. The effect may be described in terms of a rate $\Gamma(\omega\to 2\pi)$, an interference phase φ, and a coherence factor for the ρ and ω amplitudes α. However, the partial rate $\Gamma(\omega\to 2\pi)$ cannot be uniquely determined unless the coherence factor is known. With the exception of the e^+e^- colliding-beam experiment,[3] the production coherence is unknown because the experiments sum over several dynamical variables (including the incident beam momentum), and consequently yield only limits for $\Gamma(\omega\to 2\pi)$.

In this experiment, we have analyzed the $\pi^+\pi^-$ mass spectrum obtained from 1448 events of the reaction

$$\bar{p}p\to\pi^+\pi^-\pi^+\pi^- \quad (1)$$

in a search for ρ-ω interference effects. The data were obtained from the analysis of 122 000 pictures in the Argonne-MURA 30-in. hydrogen bubble chamber, covering a range of incident $\bar{p}$ momenta from 1.26 to 1.65 GeV/c. The experimental mass resolution is a critical factor in an experiment of this type, so that a careful evaluation of the $(\pi^+\pi^-)$ mass resolution in the ρ region was made. This evaluation yielded a full width at half-maximum (FWHM) of 10 ± 1 MeV, and was confirmed by a study of $K^0\to\pi^+\pi^-$ decays. K^0 decay and $\omega\to\pi^+\pi^-\pi^0$ decay also allow a check on the absolute mass scale to an accuracy of $\lesssim 1$ MeV. The strong magnetic field (32 kG), the high optical precision of the 30-in. chamber, and the use of four-constraint fits are all factors in obtaining good resolution.

Figure 1 shows the $\pi^+\pi^-$ mass spectrum from Reaction (1) in the region of the ρ, with no cuts applied to the data. The spectrum shows statistically significant evidence for structure near 780 MeV, which we interpret as a manifestation of ρ-ω interference in the 2π decay mode. The ρ- and ω-production reactions which contribute can be written as

$$\bar{p}p\to\pi^+\pi^-\rho \quad (2)$$

and

$$\bar{p}p\to\pi^+\pi^-\omega, \quad (3)$$

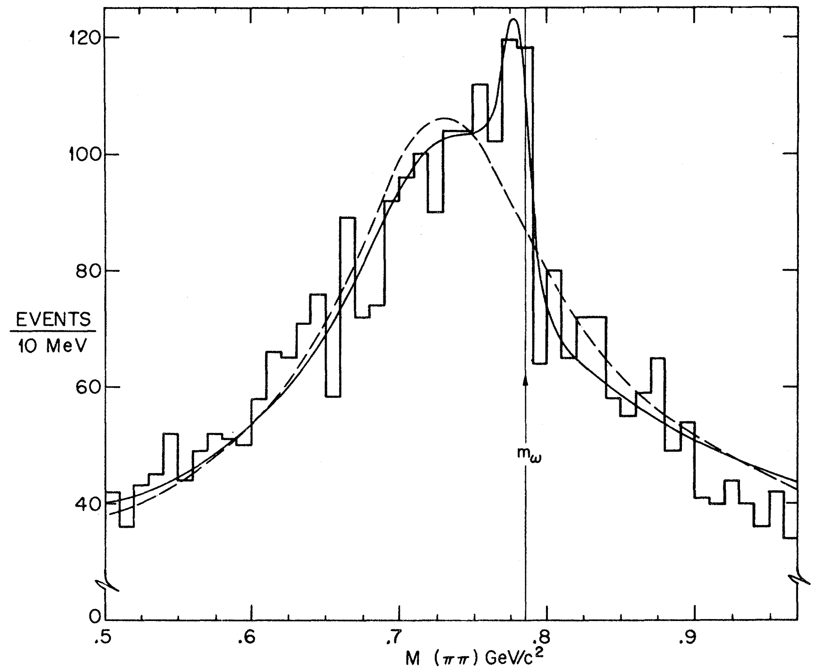

FIG. 1. Part of the mass spectrum of $\pi^+\pi^-$ from Reaction (6). The dashed curve is a fit to the histogram without interference terms. The solid line is a fit with interference terms whose parameters correspond to the dashed line on Fig. 2.

with average cross sections of 2.3 ± 0.3 and 3.0 ± 0.2 mb, respectively, in this experiment (they both decrease with momentum). We will return to a description of the properties of these production reactions, after describing the theory and procedures used for fitting the $\pi^+\pi^-$ mass distribution.

Several theoretical papers[4-6] have discussed the ρ-ω mixing problem and they differ in certain respects. We have followed the formulations of Harte and Sachs[4] and Gourdin, Stodolsky, and Renard[5] and shall outline the basic equations here.

In the absence of electromagnetic effects, the states ρ_0 and ω_0, defined as states of pure G parity +1 and −1, would decay independently into the 2π and 3π decay channels, respectively. The most important effect of turning on G-violating electromagnetic interactions is to introduce a small real off-diagonal term in the ρ_0, ω_0 mass matrix. This mass matrix describes the time development of the free ρ_0, ω_0 states, and the effect of the off-diagonal element is to couple their decay.

The physical ρ, ω states are obtained by diagonalizing the mass matrix. The eigenvectors then represent states that decay independently and, neglecting terms of order ϵ^2, are given by

$$|\rho\rangle = |\rho_0\rangle - \epsilon|\omega_0\rangle, \tag{4}$$

and

$$|\omega\rangle = \epsilon|\rho_0\rangle + |\omega_0\rangle, \tag{5}$$

where

$$\epsilon = \delta/(m_{\rho_0} - \tfrac{1}{2}i\Gamma_{\rho_0} - m_{\omega_0} + \tfrac{1}{2}i\Gamma_{\omega_0}),$$

and

$$\delta = -\langle\rho_0|M|\omega_0\rangle. \tag{6}$$

The decay rate of the physical ω to 2π is then given by

$$\frac{\Gamma(\omega\to 2\pi)}{\Gamma(\rho\to 2\pi)} = \frac{\Gamma(\rho\to 3\pi)}{\Gamma(\omega\to 3\pi)} = |\epsilon|^2, \tag{7}$$

simply as a result of the mixing. This depends on the assumption that δ is real, i.e., that the direct decay $\omega_0 \to 2\pi$ is negligible.[5]

If the physical ρ and ω amplitudes in the 2π channel are represented by A_ρ and A_ω, the 2π mass spectrum is given by

$$dN/dm = f_{\text{ps}}(m)[A_\omega^2|f_{\text{BW-}\omega}(m)|^2 + A_\rho^2|f_{\text{BW-}\rho}(m)|^2 + 2aA_\omega A_\rho \,\text{Re}(e^{i\varphi} f_{\text{BW-}\omega}(m) f_{\text{BW-}\rho}{}^*(m))], \tag{8}$$

where $f_{ps}(m)$ is a phase-space factor, $f_{BW-\omega}(m)$ and $f_{BW-\rho}(m)$ are Breit-Wigner amplitudes for the ρ and ω, and α is the coherence factor of the two amplitudes ($0 \leq \alpha \leq 1$). Equation (8) may be derived by considering the general ρ-ω density matrix

$$\begin{pmatrix} A_\omega^2 & A_\omega A_\rho \alpha e^{i\varphi} \\ A_\omega A_\rho \alpha e^{-i\varphi} & A_\rho^2 \end{pmatrix}. \tag{8a}$$

Insofar as $\alpha \sim 1$, the phase of the correlation term φ is just the relative phase of A_ρ and A_ω.

We fitted our data using the expression in Eq. (8) with an additional incoherent "phase-space" background. For the function $f_{ps}(m)$, a two-out-of-four–body phase space was used. To check that this is a reasonable representation of the actual background, which is mainly combinatorial, we examined the doubly charged two-pion invariant mass distributions, and found that $f_{ps}(m)$ was indeed very similar to these in the mass region of our fit. A P-wave Breit-Wigner resonance with mass-dependent width was used for the ρ:

$$f_{BW-\rho}(m) = \Gamma_\rho^{1/2}(m)/[(m_\rho^2 - m^2) - i m_\rho \Gamma_\rho(m)], \tag{9}$$

$$\Gamma_\rho(m) = \Gamma_0 (m_\rho/m)(q/q_\rho)^3. \tag{10}$$

A constant-width Breit-Wigner formula, with the fixed parameters $m_\omega = 784$ MeV and $\Gamma_\omega = 13$ MeV, was used for the ω. We fitted the data over the invariant-mass region from 0.50 to 0.97 GeV in 10-MeV bins, with the 10-MeV FWHM resolution folded into the theoretical curves. The ρ parameters were determined by fitting to a single resonance plus background without using data from the three bins in the interference region. This procedure gave a low mass value (743 MeV) and large width ($\Gamma_\rho = 180$ MeV) which, however, were also found in fits to both the charged and neutral $\pi\pi$ spectrum in the five-pion final states in this exposure.[7] Furthermore, m_ρ and Γ_ρ were relatively insensitive to the use of the data in the interference region.

With these parameters for the ρ fixed, the spectrum was fitted using Eq. (8) and a phase-space background.[8] The three important parameters involved in fitting the interference pattern are A_ω, φ, and α. In fact, only two of these parameters are essentially independent. This can be seen by noting that the interference signal consists in essence of a part that is odd about m_ω, proportional to $\alpha A_\rho A_\omega \sin\varphi$, and an even part, proportional to $\alpha A_\rho A_\omega \cos\varphi$ and A_ω^2. With the limited resolution and statistics in experiments such as this, these even and odd parts are the two measurable quantities, and do not suffice to determine the three unknown parameters uniquely. Only the condition $\alpha \leq 1.0$ allows us to set upper and lower limits on A_ω. Since we also observe 1250 events of Reaction (3) where ω decays to 3π, we can relate A_ω directly to the quantity $\Gamma^{1/2}(\omega \to 2\pi)$.

Figure 2 shows the minimum for χ^2 as a function of $\Gamma^{1/2}$, as well as the fitted values for φ and α, where α was constrained to be ≤ 1.0. The statistical significance of the effect is evaluated by noting that χ^2 decreases by 23 when interference terms are included in the fit, corresponding to an effect of more than 3.5 standard deviations. The parameters of the interference fit shown in Fig. 1 are indicated by a dashed line on Fig. 2. The 95% confidence limits for $\Gamma^{1/2}(\omega \to 2\pi)$ as shown in Fig. 2 are 0.42 and 2.3 MeV$^{1/2}$. The corresponding lower limit for the data of Goldhaber et al.[2] is 0.17 MeV$^{1/2}$. For the colliding-beam experiment,[3] $\Gamma^{1/2}(\omega \to 2\pi)$ is found to be 0.63 ± 0.23 MeV$^{1/2}$, which is in agreement with the SU(3) predictions based on the experimental knowledge

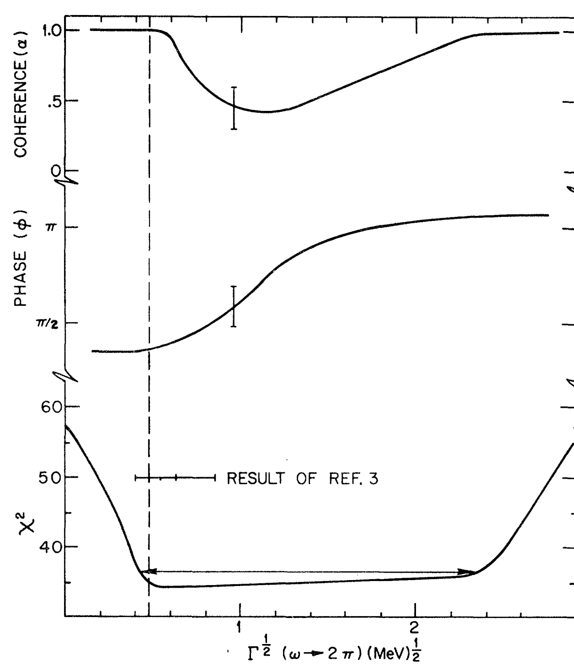

FIG. 2. The best value of χ^2, the corresponding phase φ, and coherence α for different values of $\Gamma^{1/2}(\omega \to 2\pi)$. The horizontal arrow indicates the 95% confidence limits for $\Gamma^{1/2}(\omega \to 2\pi)$ ($\Delta\chi^2 = 2.7$). The fitted curve on Fig. 1 corresponds to the values given by the dashed line. Typical uncertainties on φ and α are shown.

of the ρ^+/ρ^0 and K^{*-}/K^{*0} mass differences,[9] but there is serious disagreement with the predicted phase.[5]

Taking $\Gamma^{1/2}$ in the range 0.4-0.8 $\text{MeV}^{1/2}$ in agreement with theory and other experiments, we see from Fig. 2 that the coherence is probably greater than 0.9 and is greater than ~0.4 with 90% confidence. At first sight, such a large coherence seems surprising, since we are summing over recoil mass, helicity states, production angle, and beam momentum. In an attempt to place an upper limit on the coherence, we have examined the following features of the ρ- and ω-productions reactions:

(1) Recoil mass distributions. The mass spectra of the $\pi^+\pi^-$ system recoiling against the ρ and ω are shown in Fig. 3(a). (The ω events used are those exhibiting the normal 3π decay mode, of course.) The spectra show ρ and f peaks of comparable magnitude.

(2) Production angular distributions. These are shown in Figs. 3(b) and 3(c). The two histograms show similar features.

(3) Helicity-state populations. We computed the density matrices of the ρ and ω in the helicity frame, and obtained, for ω,

$$\rho_{11}=0.40\pm0.01), \quad \rho_{1-1}=0.09\pm0.02,$$
$$\text{Re}\rho_{10}=0.01\pm0.01;$$

for ρ,

$$\rho_{11}=0.40\pm0.01, \quad \rho_{1-1}=0.02\pm0.02,$$
$$\text{Re}\rho_{10}=0.01\pm0.01.$$

These errors are statistical and do not include systematic uncertainties in the background subtraction.

(4) Cuts. We have examined the effect of cuts on the distribution shown in Fig. 1. We made cuts on beam momentum, recoil $\pi\pi$ mass, and production angle, but saw no clear evidence for a change in the appearance of the structure in the ω mass region, within the reduced statistical accuracy of such cut samples.

None of these analyses allows us to put a useful upper limit on the coherence – even though we are summing over many dynamical variables.

Again taking $\Gamma^{1/2}$ in the range 0.4-0.8 $\text{MeV}^{1/2}$, we can use our data to determine that the effective phase of Eq. (8) lies in the range $\varphi\approx60°$-$90°$. Further, since the coherence is near 1, we interpret this value of φ as a measurement of the dominant phase of the ρ_0 component of physical

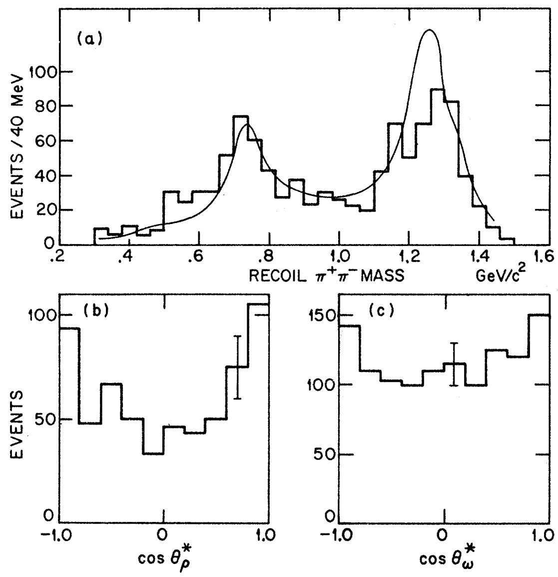

FIG. 3. (a) Histogram of the $\pi\pi$ mass spectrum recoiling against ω (background subtracted). ω cut = 0.775-0.810 GeV/c^2. The smooth curve represents the $\pi\pi$ mass spectrum recoiling against ρ. Due to background considerations, this cannot be obtained from simple cuts and has been reconstructed from detailed likelihood fits for the 4π channel. The area is normalized to equal that of the histogram. (b) Center-of-mass production angular distribution for ρ (background subtracted). The statistical error is shown. (c) Center-of-mass production angular distribution for ω (background subtracted).

ω relative to ρ. Then from Eqs. (4)-(6), if δ is positive,[9] the phase of ϵ is in the range 90°-125°, which implies that the production phase of ρ relative to ω is between 0° and 65°.

In summary, this experiment provides evidence that ρ and ω production in $\bar{p}p$ annihilations in this energy range take place with a fairly large coherence and dynamical similarity. This situation resembles that for ρ/ω production in K^-p collisions at high energy, where the independent-quark model provides a simple way to interpret the observed ρ/ω similarity.[10] For the present case, the measured coherence and phase for ρ/ω production should provide a useful test for models of annihilation. For example, a reaction mechanism involving a sequence of nonoverlapping S-channel resonances of pure G parity would lead to incoherent ρ/ω production.

We would like to thank the team which developed the automatic measuring machine, POLLY, used in this experiment; in particular J. Loken, B. Musgrave, D. Hodges, F. Beck, R. Royston, and R. Sunde. This work is part of a larger study of all antiproton reactions in this momentum range and we would like to acknowledge the work of L. G. Hyman, W. E. Manner, and L. Voyvodic on the experiment as a whole, together with the zero-gradient synchrotron and bubble chamber crews and our scanning team. We thank E. Berger for useful discussions.

*Work supported by the U. S. Atomic Energy Commission.

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$$S(\pi^+\pi^-)=\frac{A(\rho)T(\rho\to\pi\pi)}{m\rho-m-i\Gamma_\rho/2}\left(1+\frac{A(\omega)}{A(\rho)}\,\frac{1}{m_\omega-m-i\Gamma_{\omega/2}}\delta\right).$$

This is the same as our formula if our Eq. (6) is replaced by

$$\epsilon=\delta/(m_\rho-m-\tfrac{1}{2}i\Gamma_\rho),$$

which may be obtained by neglecting Γ_ω and using the approximation $m\approx m_\omega$.

[7]The results of such fitting to the 5π final state are, for the ρ^0, mass 752 ±3 and width 187 ±12 MeV; for the $\rho^\pm$, mass 750 ±3 and width 184 ±12 MeV. Similar values for the ρ mass in $\bar{p}p$ annihilations are reported by G. Kalbfleisch, R. Strand, and V. Vanderburg, Phys. Letters 29B, 259 (1969).

[8]We have also considered the analysis of the interference allowing the mass and width of the ρ as free parameters. We found that the interference variables were essentially independent of the ρ parameters used. The χ^2 decreased by 2, consistent with the inclusion of two more free parameters in the fit:

	χ^2	$\Gamma^{1/2}$
No interference	55	(0.0)
Coherent fit	32	0.57 ± 0.11 MeV$^{1/2}$

The corresponding fitted ρ parameters were consistent with those discussed in the text.

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REGGE CUTS, SCHMID LOOPS, AND RESONANCE BUMPS

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(Received 29 December 1969)

The effects of the cuts correction to Regge-pole amplitudes is investigated at intermediate energies. It is shown that, as a consequence of this correction, "resonance" bumps appear in total cross sections.

Some time ago Schmid[1] pointed out that the partial-wave projection of a crossed-channel Regge pole can give rise to counterclockwise circles in the Argand diagram. In many cases these circles strongly resemble[1,2] the "resonance circles" found in actual phenomenological partial-wave analysis. At this stage the most obvious conclusion is that the Regge-pole amplitude "contains" most of the s-channel resonances, or, in other words, the relation

$$A^{\mathrm{Regge}}(s,t)=A^{\mathrm{Res}}(s,t)$$

holds not only in the finite-energy sum rule (FESR) average sense,[3] but even locally. The most obvious criticism[2,4] of this idea is that the smooth Regge-pole amplitude does not contain second-sheet poles. Furthermore the maxima in the various partial-wave cross sections are correlated in such a way that the resulting total cross section is a structureless function of the energy; this last feature is not characteristic of the "true" resonances.

However, it must be remembered that from general principles, like Lorentz symmetry or