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## ISOSPIN POLARIZATION IN THE NUCLEAR MANY-BODY PROBLEM

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For nuclei with  $N > Z$ , isospin invariance is explicitly broken in the Hartree-Fock (HF) approximation. It is shown that isospin polarization correlations in the HF ground state restore this symmetry and lead to a natural framework for a microscopic description of analog states. We discuss the approximations under which the Lane potential model is obtained.

In this note we discuss the question of isospin invariance in connection with the shell-model description of analog states in heavy nuclei. If all electromagnetic effects (including the neutron-proton mass difference) are suppressed, the nuclear part of the many-body Hamiltonian  $H$  commutes with all components of the isospin operator. In particular,  $[T_-, H] = 0$ . If  $|\varphi_0\rangle$  is any eigenstate of  $H$  (for example, the ground state corresponding to a particular neutron number  $N$  and proton number  $Z$ ), this immediately implies that the analog state  $T_-|\varphi_0\rangle$  is also an eigenstate with the same energy eigenvalue. This property, and with it the whole framework of isospin invariance, may be lost in any approximate treatment of the eigenstates of  $H$ . The Hartree-Fock (HF) approximation for a system with  $N > Z$  is a case in point. The HF ground state  $|0\rangle$  does not have good isospin since (even without electromagnetic interactions) equivalent neutron and proton states have different wave functions on account of the presence of the symmetry-potential terms associated with the excess neutrons. The breaking of isospin invariance has to do with the approximations introduced, and not with the forces.

We show below that this trouble develops in HF due to the neglect of neutron-proton correlations in  $|0\rangle$ , and that an isospin-conserving description of at least the ground state is recovered by including such correlations within the framework of the random phase approximation (RPA).<sup>1</sup>

Consider a system of  $N$  neutrons and  $Z$  protons with a general two-body interaction which we write in the form  $V = V^0 + V^T P^T$  after separating off the charge exchange part ( $P^T$  is the usual charge-exchange operator). The HF states  $a, b, \dots$  for neutrons and  $\alpha, \beta, \dots$  for protons may be used as basis for the definition of creation and destruction operators (e.g.,  $n_a^\dagger$  creates a neutron in state  $a$ , etc.).

The Hamiltonian may then be written as

$$H = E_0 + \sum \epsilon_\alpha^p : p_\alpha^\dagger p_\alpha : + \sum \epsilon_a^n : n_a^\dagger n_a : + \frac{1}{2} \sum V_{\alpha\beta\gamma\delta} : p_\alpha^\dagger p_\beta^\dagger p_\delta p_\gamma : + \frac{1}{2} \sum V_{abcd} : n_a^\dagger n_b^\dagger n_d n_c : \\ + \sum \tilde{V}_{\alpha\alpha\beta\beta} : n_a^\dagger p_\alpha^\dagger p_\beta n_b :, \quad (1)$$

where  $\epsilon_a^n$  and  $\epsilon_\alpha^p$  are the HF single-particle energies, the colons denote normal ordering of operators with respect to the HF vacuum  $|0\rangle$ , while  $E_0 = \langle 0|H|0\rangle$  is the HF ground-state energy of the parent system (i.e., the one with  $N$  neutrons and  $Z$  protons), and  $\tilde{V}_{\alpha\alpha\beta\beta} = V_{\alpha\alpha\beta\beta}^0 - V_{\alpha\alpha\beta\beta}^\tau$ . The last three terms in (1) represent interactions which are not included in the HF fields. Note that since both sets of HF states are complete, one may, for example, also define an operator which creates a proton in a neutron single-particle state,  $p_a^\dagger = \sum_\alpha p_\alpha^\dagger \langle \alpha|a\rangle$ , which will in general be a linear combination of creation operators  $p_\alpha^\dagger$  with the same angular momenta but different radial quantum numbers.

Proton-particle, neutron-hole excitations.—The state obtained by operating on  $|0\rangle$  with the isospin lowering operator  $T_-$  will not have the same energy as  $|0\rangle$  whereas we know that the true analog state  $T_-|\varphi_0\rangle$  and the parent ground state  $|\varphi_0\rangle$  must have exactly the same energy (when electromagnetic interactions are suppressed). Since the isospin lowering operator

$$T_- = \sum_a p_a^\dagger n_a = \sum_\alpha p_\alpha^\dagger n_\alpha = \sum_{a,\alpha} \langle \alpha|a\rangle p_\alpha^\dagger n_a \quad (2)$$

has the structure of creating protons and neutron holes in the parent ( $N, Z$ ) in order to produce the analog ( $N-1, Z+1$ ), we shall examine the general particle-hole excitation of this type which is linear in particle-hole operators. Applying the equation-of-motion method<sup>1</sup> to the pair operator  $p_\gamma^\dagger n_c$  we find, after linearization,<sup>2</sup> that

$$[H, p_\gamma^\dagger n_c] \approx (\epsilon_\gamma^p - \epsilon_c^n) p_\gamma^\dagger n_c + (\rho_\gamma^p - \rho_c^n) \sum_{b\bar{b}} \tilde{V}_{c\bar{b}b\gamma} p_b^\dagger n_{\bar{b}}, \quad (3)$$

where  $\rho_\gamma^p = \langle 0|p_\gamma^\dagger p_\gamma|0\rangle$  and  $\rho_c^n = \langle 0|n_c^\dagger n_c|0\rangle$  denote the proton and neutron densities in the HF vacuum  $|0\rangle$ . Since both protons and neutrons have sharp Fermi surfaces in the HF approximation, Eq. (3) couples proton-neutron hole pairs on opposite sides of their respective Fermi surfaces only. We therefore look for oscillatory solutions of the usual form<sup>2</sup>

$$O_\lambda^\dagger = e^{-i\omega_\lambda t} \left( \sum_{\bar{b}b} x_{\bar{b}b} p_{\bar{b}}^\dagger n_b - \sum_{\bar{b}b} y_{\bar{b}b} p_{\bar{b}}^\dagger n_b \right), \quad (4)$$

where the barred (unbarred) subscripts now refer to states which are occupied (unoccupied) in  $|0\rangle$ . The operator  $O_\lambda^\dagger$  thus creates excitations in the analog system when acting on the correlated ground state  $|\bar{0}\rangle$  of the parent system. The conditions  $[H, O_\lambda^\dagger] = \omega_\lambda O_\lambda^\dagger$  and  $[H, O_\lambda] = -\omega_\lambda O_\lambda$ , together with the subsidiary condition

$$O_\lambda |\bar{0}\rangle = 0, \text{ for all } \lambda, \quad (5)$$

i.e., that  $|\bar{0}\rangle$  does not contain any of the excitations  $\lambda$ , lead to RPA-like equations for the eigenfrequencies  $\omega_\lambda$  and amplitudes  $x$  and  $y$ :

$$(\epsilon_\gamma^p - \epsilon_{\bar{c}}^n - \omega_\lambda) x_{\gamma\bar{c}} - \sum_{\bar{b}b} \tilde{V}_{\bar{b}\gamma\bar{c}b} x_{\bar{b}b} - \sum_{\bar{b}b} \tilde{V}_{b\gamma\bar{c}\bar{b}} y_{\bar{b}b} = 0, \quad (6a)$$

$$(-\epsilon_{\bar{\gamma}}^p + \epsilon_c^n + \omega_\lambda) y_{\bar{\gamma}c} - \sum_{\bar{b}b} \tilde{V}_{b\bar{\gamma}c\bar{b}} y_{\bar{b}b} - \sum_{\bar{b}b} \tilde{V}_{\bar{b}\bar{\gamma}c\bar{b}} x_{\bar{b}b} = 0. \quad (6b)$$

The eigenfrequencies  $\omega_\lambda$  are excitation energies of the analog system as measured from the ground state of the parent system.

Now the  $T_-$  operation has the structure of the operator  $O_\lambda^\dagger$  with eigenfrequency  $\omega_\lambda = 0$  if we truncate it in the same way, i.e., replace Eq. (2) by

$$T_- \approx \sum_{\bar{a}} (\alpha|\bar{a}) p_{\bar{a}}^\dagger n_{\bar{a}} + \sum_{\bar{a}} (\bar{\alpha}|a) p_{\bar{\alpha}}^\dagger n_a. \quad (2')$$

The isospin lowering operator thus creates an eigenvibration (a monopole isospin vibration) in the analog system provided that

$$x_{\alpha\bar{a}} \sim (\alpha|\bar{a}), \quad y_{\bar{\alpha}a} \sim -(\bar{\alpha}|a) \quad (7)$$

are solutions of Eqs. (6) at frequency  $\omega_\lambda = 0$ . This is indeed the case. By completing the sums [the minus sign introduced by Eq. (7) allows one to sum over complete sets of single-particle states with impunity] and using the fact that the HF Hamiltonians  $\mathcal{H}_{\alpha\beta}^p$  and  $\mathcal{H}_{ab}^n$  for protons and neutrons are diagonal and do not connect occupied and unoccupied states, one finds that the interaction terms in Eq. (6a) reduce to

$$-\sum_{b\bar{b}} \tilde{V}_{b\bar{b}} \gamma \bar{c} \beta (\beta | \bar{b}) + \sum_{b\bar{b}} \tilde{V}_{b\bar{b}} \gamma \bar{c} \bar{\beta} (\bar{\beta} | b) = -\sum_b \tilde{V}_{b\bar{b}} \gamma \bar{c} \bar{b} + \sum_{\bar{b}} \tilde{V}_{\bar{b}b} \gamma \bar{c} \bar{b} = -(\mathcal{H}_{\gamma\bar{c}}^p - \mathcal{H}_{\gamma\bar{c}}^n) = -(\epsilon_\gamma^p - \epsilon_{\bar{c}}^n)(\gamma | \bar{c}),$$

which just cancels the first term. Notice that this result does not obtain in the Tamm-Dancoff approximation (TDA) where ground-state correlations are neglected.<sup>3</sup>

We have thus shown that  $|\tilde{0}\rangle$  and  $T_-|\tilde{0}\rangle$  have the same energy, i.e.,  $T_-$  creates the analog state of  $|\tilde{0}\rangle$ . Furthermore,  $T_+|\tilde{0}\rangle \equiv T_-^\dagger|\tilde{0}\rangle = 0$  according to the subsidiary condition (5). Thus (a)  $|\tilde{0}\rangle$  has a good isospin  $T$  equal to its maximum projection  $T_3 = T_0 = \frac{1}{2}(N-Z)$ , i.e.,  $(T, T_3) = (T_0, T_0)$ , and (b)  $T_-|\tilde{0}\rangle$  must have isospin quantum numbers  $(T_0, T_0-1)$ .

The fact that ground-state correlations restore the isospin invariance which is violated in the uncorrelated ground state is of course no surprise. Exactly the same thing happens for example in the case of the center-of-mass motion of a nucleus where the spurious  $J=1^-, T=0$  state appears. The introduction of ground-state correlations shifts this state to zero excitation energy.<sup>4</sup> There is one important difference in our case, however. The analog state, being in a neighboring nucleus, is not a spurious state of the parent system but rather a collective isospin oscillation.

We note two further properties of the RPA-type solution we have constructed: (c) Since  $T_-|\tilde{0}\rangle$  has a good isospin, one knows, quite generally, that the normalization integral

$$\langle \tilde{0} | T_+ T_- | \tilde{0} \rangle = \langle \tilde{0} | [T_+, T_-] | \tilde{0} \rangle = 2T_0 = N-Z. \quad (8)$$

Calculating explicitly from (2'), one finds exactly

$$[T_+, T_-]_{\text{RPA}} \approx \sum_{\alpha\bar{a}} |(\alpha | \bar{a})|^2 - \sum_{\bar{\alpha}a} |(\bar{\alpha} | a)|^2 = N-Z. \quad (9)$$

(d) The remaining states  $O_\lambda^\dagger|\tilde{0}\rangle$  with  $\omega_\lambda \neq 0$  all have isospin  $(T_0-1, T_0-1)$ . We prove this by demonstrating that  $T_+O_\lambda^\dagger|\tilde{0}\rangle = [T_+, O_\lambda^\dagger]|\tilde{0}\rangle = 0$  for  $\omega_\lambda \neq 0$ . The commutator is

$$[T_+, O_\lambda^\dagger]_{\text{RPA}} \approx \sum_{\alpha\bar{a}} x_{\alpha\bar{a}} (\alpha | \bar{a})^* - \sum_{\bar{\alpha}a} y_{\bar{\alpha}a} (\bar{\alpha} | a)^*,$$

which vanishes by the orthogonality condition on two RPA solutions.<sup>4</sup>

**Isospin polarization.**—We investigate the modification of single-particle motion due to isospin core polarization within the RPA. Microscopically, the incident proton excites particle-hole excitations in the  $|\tilde{0}\rangle$  core of a proton-neutron hole character by charge exchange, i.e., an isospin core polarization results.<sup>5</sup> As a consequence, the proton single-particle states are redistributed over the analog and antianalog states of the compound system. The effects of the polarization process on single-particle motion may be described by constructing the proton-particle, neutron-hole Green's function, or polarization propagator:

$$-iF(1234; \omega) = \int_0^\infty dt e^{i\omega t} \langle \varphi_0 | n_2^\dagger(t) p_1(t) p_3^\dagger(0) n_4(0) | \varphi_0 \rangle \quad (10)$$

in the ladder approximation.<sup>2</sup> The associated diagram summation techniques are standard (see, e.g., Ref. 2 for technical details) and we shall mainly quote results. In our case the ladder approximation to  $F(1234; \omega)$  consists of summing, to all orders, a selected set of diagrams of which the first three terms are shown in Fig. 1(a). Both charge-exchange and -nonexchange diagrams are included, the hole lines always being neutrons and particle lines protons. Reserving odd (even) numerals for proton (neutron) states, the summation of the series in Fig. 1(a) leads to the equation

$$F(1234; \omega) = \left[ \frac{(1-\rho_1^p)\rho_2^n}{\epsilon_1^p - \epsilon_2^n - \omega - i\delta} - \frac{(1-\rho_2^n)\rho_1^p}{\epsilon_1^p - \epsilon_2^n - \omega + i\delta} \right] [\delta_{13}\delta_{24} + \sum_{5,6} \tilde{V}_{1652} F(5634; \omega)], \quad (11)$$

where  $\delta$  is a positive infinitesimal and where  $\tilde{V}_{1652}$  is again  $V_{1652}^0 - V_{1652}^\tau$ . As is well known,<sup>2</sup> the homo-

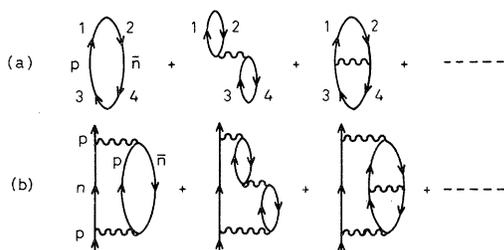


FIG. 1. Lowest order diagrams for (a) isospin polarization propagator, and (b) single-proton Green's function.

geneous part of Eq. (11) is identical in content and structure with the RPA equations (6a) and (6b). It is useful to expand  $F(1234; \omega)$  in RPA eigenstates:

$$F(1234; \omega) = -\sum_{\lambda} \frac{\langle \bar{0} | n_2^\dagger p_1 | \omega_{\lambda} \rangle \langle \omega_{\lambda} | p_3^\dagger n_4 | \bar{0} \rangle}{\omega - \omega_{\lambda} + i\delta}. \quad (12)$$

The modification to the single-particle motion of a proton due to the core polarization it induces is directly expressed in terms of the proton self energy  $\Sigma_{\alpha}(E)$ . The function  $\Sigma_{\alpha}(E)$  is given approximately by summing the diagrams shown in Fig. 1(b) to all orders. Clearly all this does is to replace the bubble in the first diagram of Fig. 1(b) by the full polarization propagator. The result, for the proton self-energy, is

$$\Sigma_{\alpha}(E) = \sum_{a, \lambda} (E - \epsilon_a^n - \omega_{\lambda} + i\delta)^{-1} \left| \sum_{\beta \bar{b}} V_{\bar{b} \alpha \beta a}^{\tau} x_{\beta \bar{b}} + \sum_{\bar{\beta} b} V_{b \alpha \bar{\beta} a}^{\tau} y_{\bar{\beta} b} \right|^2 \quad (13)$$

after using Eq. (12) and identifying  $x_{\beta \bar{b}} = \langle \bar{0} | n_{\bar{b}}^\dagger p_{\beta} | \omega_{\lambda} \rangle$ , etc. Only the charge-exchange part of the two-body force excites the core excitations shown in Fig. 1(b).

The self-energy (13) contains complete information on the modification of the single-particle state  $\alpha$ . We read off from  $\Sigma_{\alpha}(E)$  (a) the effective matrix element for exciting core vibrations,

$$\mathcal{V}_{\alpha, a \lambda} = \sum_{\beta \bar{b}} V_{\bar{b} \alpha \beta a}^{\tau} x_{\beta \bar{b}} + \sum_{\bar{\beta} b} V_{b \alpha \bar{\beta} a}^{\tau} y_{\bar{\beta} b}; \quad (14a)$$

(b) the proton-quasiparticle spectrum given by the roots  $E_I$  of

$$E_I - \epsilon_{\alpha}^p - \Sigma_{\alpha}(E_I) = 0, \quad (14b)$$

(c) the renormalization of the single-particle state,

$$Z_I = [1 - \partial \Sigma_{\alpha} / \partial E]_{E=E_I}^{-1}; \quad (14c)$$

(d) the contribution to the optical potential for an unbound proton of energy  $E$  coming from isospin polarization excitations of the core,

$$\text{Im} \Sigma_{\alpha}(E) = -\pi \sum_{a, \lambda} |\mathcal{V}_{\alpha, a \lambda}|^2 \delta(E - \epsilon_a^n - \omega_{\lambda}). \quad (14d)$$

These items are discussed fully in a forthcoming publication. Here we only wish to point out briefly the connection with the Lane model<sup>6</sup> for the isospin splitting of single-particle states in heavy nuclei. We first note that the number of solutions of Eq. (14b) depends on the number of core vibrations  $|\omega_{\lambda}\rangle$  and unoccupied neutron states  $|a\rangle$  which are allowed to contribute to  $\Sigma_{\alpha}(E)$ . We now argue that (i) the matrix element  $\mathcal{V}_{\alpha, a \lambda}$  will be largest for the collective state among the vibrations  $|\omega_{\lambda}\rangle$ , i.e., for the analog state with  $\omega_{\lambda} = 0$ , and (ii) the charge-exchange matrix elements  $V_{\bar{b} \alpha \beta a}^{\tau}$  will be largest if the proton and neutron states  $|\alpha\rangle$  and  $|a\rangle$  have the same number of radial nodes, i.e., are "principal partners." Thus, keeping only the analog state in the sum over  $\lambda$  and the principal partner of  $\alpha$  in the sum over  $a$ , we have

$$\Sigma_{\alpha}(E) \approx \frac{|\mathcal{V}_{\alpha, a \lambda}|^2}{E - \epsilon_a^n + i\delta}, \quad \mathcal{V}_{\alpha, a \lambda} = (2T_0)^{-1/2} \left[ \sum_{\bar{b}} \mathcal{V}_{\bar{b} \alpha \bar{b} a}^{\tau} - \sum_{\bar{\beta}} V_{\bar{\beta} \alpha \bar{\beta} a}^{\tau} \right], \quad (15)$$

the latter expression being the result of inserting the (normalized) coefficients (7) for the analog state into Eq. (14a).

The essential feature of the Lane model, viz. that the transition matrix element  $\mathcal{V}_{\alpha, a \lambda}$  be proportional to the symmetry energy  $(\epsilon_{\alpha}^p - \epsilon_a^n)$  of the principal partner levels, now emerges if we calculate this

energy difference in the Hartree approximation and also assume that  $|\alpha\rangle$  and  $|a\rangle$  have the same spatial wave functions. Then the symmetry energy ( $U_{\alpha a}$  say, a positive quantity) is given by  $\epsilon_{\alpha}^p - \epsilon_a^n = -u_{\alpha a} \approx -(2T_0)^{1/2} u_{\alpha, a, \lambda}$  on comparison with the second member of Eq. (15). Equation (14b) now reduces to a  $2 \times 2$  determinantal equation for the roots  $E_i$ . To leading order in  $(2T_0)^{-1} \ll 1$  one has  $E_A - \epsilon_a^n \approx u_{\alpha a}/2T_0$  and  $E_{AA} - \epsilon_a^n \approx -U_{\alpha a} - U_{\alpha a}/2T_0$  with renormalization coefficients  $Z_A \approx (2T_0 + 1)^{-1}$  and  $Z_{AA} \approx 2T_0(2T_0 + 1)^{-1}$ , respectively. It is clear that under these approximations the quasiparticle states at energies  $E_A$  and  $E_{AA}$  correspond to the analog and antianalog states of the Lane model, containing, respectively,  $Z_A$  and  $Z_{AA}$  of the single-particle strength, and therefore that these states have good isospin  $(T_0 + \frac{1}{2}, T_0 - \frac{1}{2})$  and  $(T_0 - \frac{1}{2}, T_0 - \frac{1}{2})$ .

The ingredients of our discussion—an exact symmetry of the nuclear Hamiltonian, its breaking by the HF vacuum, and the occurrence of a zero-energy analog excitation—are strongly reminiscent of the situation encountered in relativistic field theory<sup>7</sup> and in nonrelativistic many-body problems.<sup>8</sup> The precise relationship, if any, between the analog excitation and the massless gauge fields and Goldstone bosons is being investigated.

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## SOLUTIONS OF TWO PROBLEMS IN THE THEORY OF GRAVITATIONAL RADIATION\*

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The evolution of an elongated rotating configuration by gravitational radiation and the possibility of a secular instability being induced by it are considered in the context of the classical homogeneous figures of Maclaurin and Jacobi. The triaxial Jacobian ellipsoid evolves in the direction of increasing angular velocity and approaches (exponentially) the point of bifurcation where it ceases to radiate. Further, radiation reaction does not make the Maclaurin spheroid secularly unstable past the point of bifurcation.

In a recent paper,<sup>1</sup> the equations of hydrodynamics governing a perfect fluid have been derived, consistently with Einstein's field equations of general relativity, to sufficient approximation that the terms representing the reaction of the fluid to the emission of gravitational radiation are explicitly present [see Eq. (8) below]. With the aid of these equations, two problems (which have arisen in current discussions of the pulsars<sup>2-4</sup>) in the theory of gravitational radiation can be solved. In this Letter, we formulate these problems, present their solutions in a simple realizable context, and conclude with some brief

comments on the bearing of the results derived for the problem of gravitational collapse.

The formulation of the problems.—The first problem concerns the evolution of a rotating fluid mass emitting gravitational radiation. The second problem concerns the possibility that the dissipation of energy by gravitational radiation induces "secular instability" in the manner that viscosity sometimes does.

We shall consider the foregoing two problems in the context of the classical theory of uniformly rotating homogeneous fluid masses.<sup>5</sup> In this theory it is known that along the sequence of axially