ELECTRON HEATING BY ELECTRON-ION BEAM INSTABILITIES

R. C. Davidson, N. A. Krall, K. Papadopoulos, and R. Shanny
University of Maryland, College Park, Maryland 20742, and Naval Research Laboratory, Washington, D. C. 20742
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Rapid electron heating by two counterstreaming ion beams has been observed in computer experiments. The results are compared with the predictions of quasilinear theory and found to be in substantial agreement.

The study of unstable beam-plasma systems has been given new impetus by the development of numerical simulation techniques.[1] Early results in this field have confirmed that weak beam-plasma systems are unstable, but detailed comparison with predictions of mathematical plasma theory have been lacking.[2,3] In this Letter we investigate in one dimension a beam-plasma system [Fig. 1(a)] sufficiently uncomplicated to allow a detailed comparison with the theory of the early time behavior of fields and space-average properties of the particle distributions. This plasma geometry has not been investigated previously.

The initial velocity-space configuration is shown in Fig. 1(a). The equidensity ion beams are relatively cold, and symmetric about $v=0$ with $f_1(v,t=0)=f_2(-v,t=0)$. Their relative speed is $2V_d(0)$. The initial electron thermal speed is small in comparison with ion beam speeds, $K_e(0) \ll m_e V_d^2(0)$, where $K_e(0)$ is the initial electron kinetic energy. Moreover, the electron distribution is taken to be symmetric about $v=0$, with $f_e(v,t=0)=f_e(-v,t=0)$. Such a configuration is linearly unstable with the phase velocities ω_k^+/k and ω_k^-/k of the unstable waves in the ranges indicated in Fig. 1(a). For the symmetric case considered here the oscillation frequencies and growth rates associated with the two unstable branches satisfy $\omega_k^+=-\omega_k^-$ and $\gamma_k^+=\gamma_k^-$. In response to the unstable field fluctuations the electrons rapidly gain kinetic energy on the time scale of a few maximum growth periods associated with the initial state, i.e.,

$$\tau_H \simeq \frac{1}{\gamma_{\max}} = \omega_{pe}^{-1}\frac{2}{\sqrt{3}}\left(\frac{4m_i}{m_e}\right)^{1/3}. \quad (1)$$

When $K_e \simeq m_e V_d^2(0)$ the modes considered are no longer unstable. In Eq. (1) m_e and m_i are the electron and ion mass, respectively, and ω_{pe} is the electron plasma frequency. During this process the mean ion-beam speeds decrease slightly with a small increase in ion kinetic energy relative to the mean. The results of computer simulation for various times are shown in Fig. 1 for the case $m_e/m_i = 1/27$, $K_e(0)/m_eV_d^2(0)=1/25$, and symmetric initial excitation of the spectral energy density associated with the two unstable branches, i.e., $W_k^+(0)=W_k^-(0)$. The time scale of the analysis (theory and experiment) presented here is limited to a few times τ_H.

Theory.—The theoretical model used in describing the initial stages of electron heating and stabilization of the electron-ion beam instability consists of the one-dimensional quasilinear equations[4] in the electrostatic approximation. The jth-component spatially averaged distribution function $f_j(v,t)$ evolves according to $(\partial/\partial t)f_j = (\partial/\partial v)[D_j(\partial/\partial v)f_j]$ where the diffusion coefficient is

$$D_j(v,t) = (8\pi e_j^2/m_j^2)\sum_{\alpha=+,-}\sum_k{}' W_k^\alpha \gamma_k^\alpha[(\omega_k^\alpha - kv)^2 + \gamma_k^{\alpha 2}]^{-1}.$$

The summation is over unstable branches $\alpha=+,-$ with $\gamma_k^\alpha > 0$. The energy density $W_k^\alpha(t) = |E_k^\alpha(t)|^2/8\pi$ in the electric field fluctuations satisfies $(\partial/\partial t)W_k^\alpha(t) = 2\gamma_k^\alpha(t)W_k^\alpha(t)$, and the growth rate $\gamma_k^\alpha(t)$ and oscillation frequencies $\omega_k^\alpha(t)$ are determined adiabatically in terms of $f_j(v,t)$ from the linear plasma dispersion relation. Each ion beam ($j=1,2$) and the electrons ($j=e$) are considered as separate plasma components. Useful quantities for present purposes are the spatially averaged mean velocity $\{V_j(t)=\int dv\, v f_j(v,t)\}$ and the kinetic energy relative to the mean velocity $\{K_j(t)=m_j\int dv[v - V_j(t)]^2 f_j(v,t)\}$ for each plasma component. The appropriate velocity moments of the diffusion equation for f_j give

$$n_j m_j \frac{d}{dt}V_j = \omega_{pj}^2 \sum_{\alpha=+,-}\sum_k{}' 2\gamma_k^\alpha W_k^\alpha \int dv f_j\, 2k\frac{(\omega_k^\alpha - kV_j) - k(v-V_j)}{[(\omega_k^\alpha - kv)^2+\gamma_k^{\alpha 2}]^2}, \quad (2)$$

$$\frac{n_j}{2}\frac{d}{dt}K_j = \omega_{pj}^2 \sum_{\alpha=+,-}\sum_k{}' 2\gamma_k^\alpha W_k^\alpha \int dv f_j \frac{(\omega_k^\alpha - kV_j)^2 + \gamma_k^{\alpha 2} - k^2(v-V_j)^2}{[(\omega_k^\alpha - kv)^2+\gamma_k^{\alpha 2}]^2}, \quad (3)$$

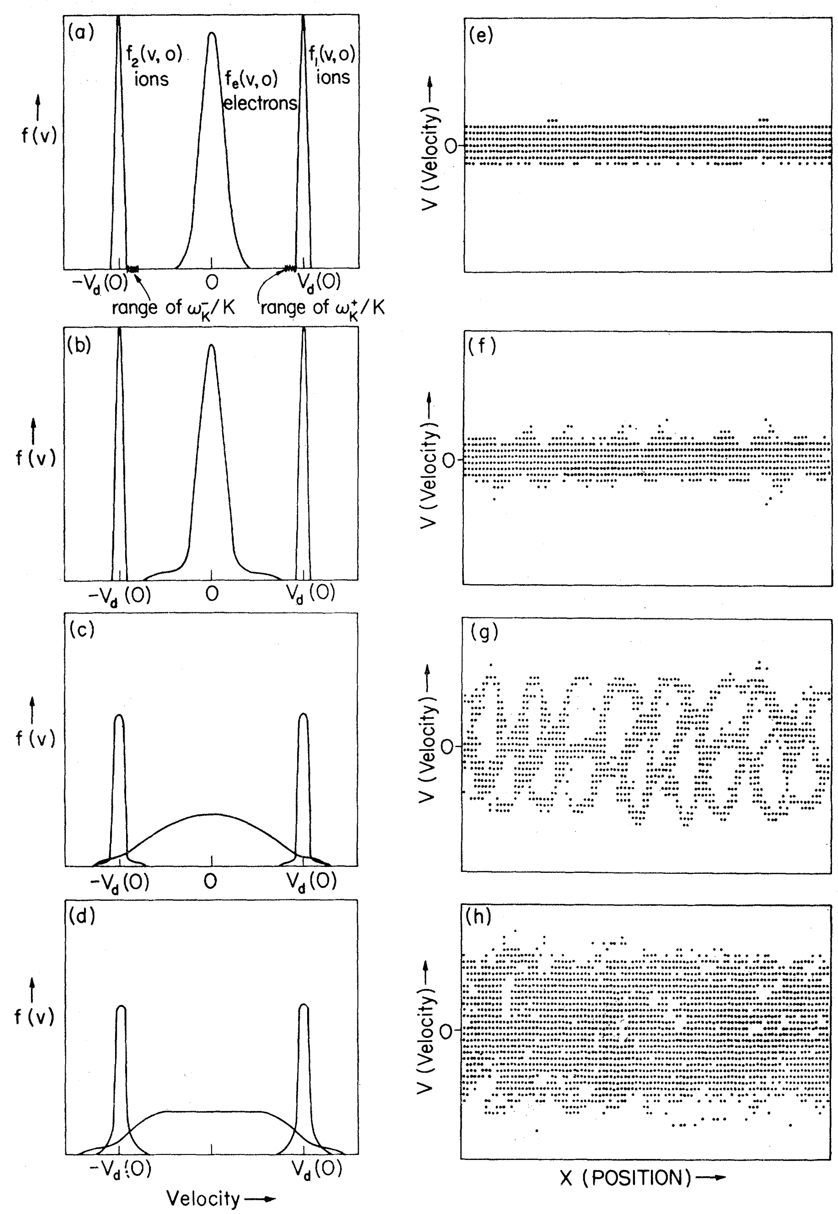

FIG. 1. Spatial average distribution functions at (a) $t=0$, (b) $t=3\tau_H$, (c) $t=7\tau_H$, (d) $t=11\tau_H$. Electron phase space at (e) $t=0$, (f) $t=3\tau_H$, (g) $t=7\tau_H$, (h) $t=11\tau_H$ [$m_e/m_i=1/27$; $K_e(0)=1/25$].

where $\omega_{pj}^2=4\pi n_j e_j^2/m_j$ and n_j is the average density of the jth component (for the equidensity ion-beam case $n_1=n_2=n_e/2$). Since $|\omega_k^\alpha/k-V_j|\gg(K_j/m_j)^{1/2}$ initially for the unstable waves, it is apparent from Eq. (2) that a particular plasma component will be accelerated or decelerated, $(d/dt)V_j>0$ or <0, according as the unstable waves have phase velocities greater or less than the component's mean velocity. Consequently for the configuration shown in Fig. 1(a) the ion beams begin to move together as a reaction to the instability. Moreover, we see from Eq. (4) that K_j increases for every plasma component during the initial stages. However, the electrons gain energy preferentially in this regard as will become apparent in the estimates which follow.

For the symmetric initial conditions with $W_k^+(0)=W_k^-(0)$ it is simply shown that $f_2(v,t)=f_1(-v,t)$, $f_e(v,t)=f_e(-v,t)$, and $W_k^+(t)=W_k^-(t)$ at subsequent times. Correspondingly $V_e(t)=0$ and $-V_2(t)=V_1(t)[\equiv V_d(t)]$, and $\omega_k^+(t)=\omega_k^-(t)$ and $\gamma_k^+(t)=\gamma_k^-(t)$ for the oscillation frequencies and growth rates. Estimates for the increase in electron and ion kinetic energy K_j and ion momentum change during the initial stages may be obtained directly from Eqs. (2) and (3) using the values $(k_0, \gamma_{k_0}^{\pm}, \omega_{k_0}^{\pm})$ corresponding to maximum growth.[5] In particular,

$$|k_0| \simeq \omega_{pe}/V_d(0),$$
$$\gamma_{k_0}^{\pm} \simeq \tfrac{1}{2}\sqrt{3}\,\epsilon^{1/3}\omega_{pe} = \gamma_{\max},$$
$$|\omega_{k_0}^{\pm}/k_0 \mp V_d(0)| \simeq \tfrac{1}{2}\epsilon^{1/3}V_d(0), \qquad (4)$$

where $\epsilon = m_e/4m_i \ll 1$. Using Eq. (4) to estimate the integrands in (2) and (3) gives

$$\frac{n_e}{2}\frac{d}{dt}K_e \simeq \frac{d}{dt}W_F \equiv \frac{d}{dt}\sum_{\alpha=+,-}\sum_k{}' W_k^{\alpha}(t); \qquad (5)$$

$$n_j\frac{m_i}{2}\frac{d}{dt}V_d^2 \simeq -\frac{d}{dt}W_F, \quad j=1,2; \qquad (6)$$

$$\frac{n_j}{2}\frac{d}{dt}K_j \simeq \epsilon^{1/3}\frac{d}{dt}W_F, \quad j=1,2. \qquad (7)$$

Equations (5)-(7) remain good approximations as long as $|\omega_k^{\pm}/k| > (K_e/m_e)^{1/2}$ and $|\omega_k^{\pm}/k \mp V_d| > (K_i/m_i)^{1/2}$. Equation (5) indicates that during the initial stages energy goes into the field fluctuations and the electrons at approximately equal rates. From Eq. (7) the ions heat a small amount during this process. The energy source is the decrease in ion-beam kinetic energy of mean motion. It is apparent from the numerator of the integrand in Eq. (3) that the electrons gain energy until K_e reaches $k^2K_e/m_e \simeq \omega_k^2 + \gamma_k^2$ where k is a wave number typical of the excited spectrum. Taking $|k| \simeq |k_0|$ corresponding to maximum growth, this reduces to

$$K_e \simeq m_e V_d^2(0) \qquad (8)$$

for saturation. We note that the fractional change in mean ion-beam velocity is small during this process. Combining (5) and (6) gives $m_i V_d \Delta V_d \simeq -\Delta K_e$, and for $\Delta K_e \simeq m_e V_d^2(0)$ we have

$$\Delta V_d/V_d(0) = -m_e/m_i. \qquad (9)$$

Computer simulation. – The electrostatic numerical simulation model consists of finite-size particles in one dimension. (This method is distinct from other commonly used simulation methods,[6] e.g., cloud-in-cell, particle-in-cell, etc.) To a large extent it suppresses short-wavelength density fluctuations and electric fields and associated collisional effects. Periodic boundary conditions are employed so that a particle leaving one end of the system is reintroduced at the other. For a more detailed discussion of the simulation model see Ref. 6. In all experiments we use a system consisting of 10^4 electrons and 10^4 ions with initial distribution functions of the general form given in Fig. 1(a). The system is $256\lambda_D$ long (λ_D = initial electron Debye length) with 512 grid points. The particle size is $1\lambda_D$, and $n_e\lambda_D = 40$ initially. Several values of the ratios m_e/m_i and $K_e(0)/m_eV_d^2(0)$ have been investigated. Figures 1 and 2 show the typical time development of the system for $m_e/m_i = 1/27$ and $m_e/m_i = 1/100$, respectively. It is apparent from the phase-space diagram, Fig. 1(f), that by the time $t \simeq 11\tau_H$, K_e may be identified with electron temperature T_e. By this time the total field-energy density in the system is beginning to decline (Fig. 3). During the initial stages the total field-energy density

$$W_F = \sum_{\alpha=+,-}\int dk\, W_k^{\alpha}(t)$$

grows exponentially at the rate $0.7(2\gamma_{k_0}^{\pm})$ where $\gamma_{k_0}^{\pm}$ is the theoretical maximum growth rate given in Eq. (4).[7] The multiplicative factor (0.7) is expected to be less than unity since the waves making up the field-energy density are not all growing with the maximum growth rate. From Fig. 2 we see that the electrons gain energy during this "linear-growth" stage; moreover, energy goes into the field fluctuations and the electrons at approximately equal rates (for $\tau_H \lesssim t \lesssim 5.5\tau_H$) in agreement with Eq. (5). The electrons continue to gain energy after the linear growth stage until $K_e \simeq m_eV_d^2(0)$ (see Fig. 2) as predicted by Eq. (8). The time scale for this to occur is a few times τ_H. When K_e is sufficiently large the field energy saturates and the nonreso-

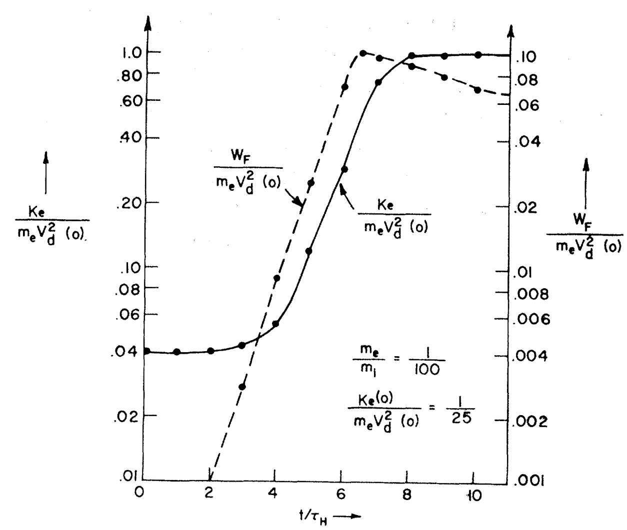

FIG. 2. Plot of electron kinetic energy and total field energy versus time.

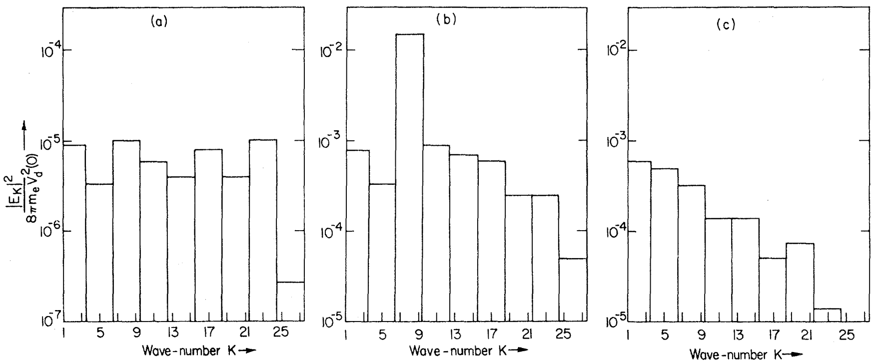

FIG. 3. Plot of field energy versus wave number for (a) $t=0$, (b) $t=6\tau_H$, (c) $t=11\tau_H$ [$m_e/m_i=1/27$; $K_e(0)/m_eV_d^2(0)=1/25$].

nant instability stops, at least on the τ_H time scale. The overshoot in W_F (Fig. 2) has not yet been explained satisfactorily. We note from Fig. 1 that for the ions, K_j ($j=1, 2$) increases by a small amount [see Eq. (7)]. Moreover, the ion-beam speeds decrease slightly. For $m_e/m_i=1/27$ the measured fractional change in beam speed is approximately 2/53 which is in excellent agreement with Eq. (9). We also followed the time development of the spectral energy density as a function of wave number k. Initially the k spectrum is relatively flat as shown in Fig. 3(a). At later stages [Fig. 3(b)] the energy density has grown by approximately three orders of magnitude, peaking around the most unstable wave numbers $\pm|k_0|$ given by Eq. (4). At the final stages [Fig. 3(c)] it flattens again but at a higher value than the initial level.

The experiments were carried out for several values of $K_e(0)/m_eV_d^2(0)$ and give results similar to those described above for $K_e(0)/m_eV_d^2(0)<1/6.25$ (cold electrons). Furthermore the experiments have been repeated for several mass ratios and indicate that the time scaling as $(m_i/m_e)^{1/3}$ [see Eq. (1)] is indeed appropriate. For smaller values of m_e/m_i the computer experiments show that the increase in ion K_j ($j=1, 2$) and fractional decrease in mean ion-beam speed are accordingly smaller, in good agreement with Eqs. (7) and (9).

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[7]In plotting the total field energy versus time in Fig. 2, we have averaged over rapid oscillations ($\omega\sim 2\omega_{pe}$) which appear as a small-amplitude modulation in the development of $W_F/m_eV_d^2(0)$ vs t. For a more detailed discussion see J. M. Dawson, R. Shanny, and T. J. Birmingham, Phys. Fluids 12, 687 (1969).