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KINEMATIC CORRECTIONS TO ATOMIC BEAM EXPERIMENTS*

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We have calculated and measured the effective velocity distribution for metastable hydrogen atoms produced in a typical beam apparatus. The distribution is characterized by $U^n \exp(-U^2)$, where $n \simeq 4$ rather than $n=2$. We discuss modifications of velocity-dependent corrections to beams measurements of the Lamb shift in the first excited state of atomic hydrogen.

In many beam experiments with light atoms or molecules, a ground-state particle I is excited to a metastable state I^* by electron impact, as in Fig. 1. A well-collimated I beam from an oven at temperature T is incident at $\angle\varphi$ on a beam of electrons at energy $\mathcal{E}+\Delta\mathcal{E}$, where $\mathcal{E}$ is the $I\to I^*$ excitation threshold and $\Delta\mathcal{E}$ is a small excess energy. Afterwards, an I^* beam exits at $\angle\psi$; it is collimated into an interval $\Delta\psi$ by output slits, and then falls onto a detector. A simple calculation,[1] assuming a $U^3\exp(-U^2)$ I-beam velocity distribution [$U=V/\alpha$, where V is the atom velocity and $\alpha=(2kT/M)^{1/2}$ is the thermal velocity], shows that the most probable recoil angle $(\varphi+\psi)\simeq r/\sqrt{2}$, where $r=(m\mathcal{E}/MkT)^{1/2}$. This angle is about 7° for H at 2500°K.

We find that a combination of effects due to recoil and collimation of the I^* beam substantially affects the resultant velocity distribution which is usually assumed to be $U^n\exp(-U^2)$, with $n=2$.[2] Instead, under typical conditions, $n\simeq 4$ is a better description, and the actual distribution has upper and lower cutoff velocities which are quite sensitive to apparatus parameters. Consequently, all such experiments which depend on the assumed distribution should be checked for systematic errors. Here, we briefly describe the calculation and experiment which show these effects. We then discuss corrections to the experiments which have measured $\mathcal{S}(\mathrm{H}, n=2)$, the Lamb shift in the $n=2$ state of atomic hydrogen.

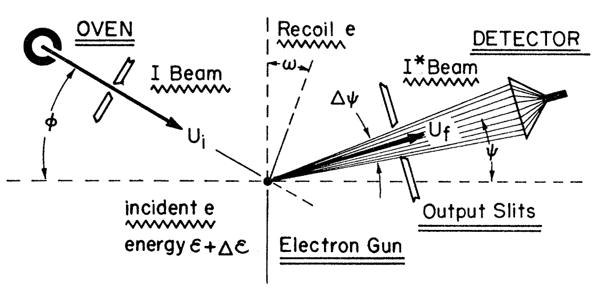

FIG. 1. Experimental schematic. A beam of ground-state particles I effuses from an oven and is incident at $\angle\varphi$ on a beam of electrons. The electron impact excites metastable I^* particles which exit toward a detector in the shape of a "cone" $(\psi,\Delta\psi)$. Generally, the angles φ and ψ are adjusted so as to provide a maximum I^* signal.

By solving the momentum- and energy-conservation equations for the situation in Fig. 1, we can relate the initial and final atom velocities. Since the experiments are usually done near threshold, where $\kappa=\Delta\mathcal{E}/\mathcal{E}\ll 1$, an approximation to $O(\kappa)$ is sufficient. We neglect transverse recoil,[3] and assume the recoil electrons are isotropically distributed.[4] Noting that the detected I^* atoms have velocities $U\sim 1$, we find that to sufficient approximation (a few percent), the initial and final velocities are equal. Next, momentum conservation gives the velocity U scattering at specific angles (φ,ψ) as

$$U=\bar{U}-\Delta U\cos(\omega+\psi),$$

where

$$\bar{U}=R\cos\psi/\sin(\varphi+\psi),\ \ \Delta U=\lambda\bar{U}/\cos\psi,$$

with

$$R=r(1+\kappa)^{1/2},\quad \lambda=[\kappa/(1+\kappa)]^{1/2}. \tag{1}$$

As the electron recoil angle ω traces out $0\leqslant\omega\leqslant 2\pi$, the velocity U at (φ,ψ) traces out $\bar{U}-\Delta U\leqslant U\leqslant\bar{U}+\Delta U$. At threshold, $\lambda=0$, the only velocity detected is $\bar{U}$ itself. If φ and ψ are adjusted to a maximum I^* signal alignment condition (MI^*SAC), as is usual, then $\bar{U}\simeq\sqrt{2}$. This is considerably faster than the most probable velocity

$U=1$ associated with a $U^2\exp(-U^2)$ distribution. We can also ask, for a given U and φ, what range in ψ is consistent with momentum conservation. We find that a range $\pm\frac{1}{2}\delta\psi$ about a mean angle $\bar{\psi}$ is allowed,[5] where

$$\delta\psi = 2\lambda R/(U^2+R^2)^{1/2}. \qquad (2)$$

Typically, near threshold and at MI^*SAC, $\delta\psi$ is "large" compared with the I^* beam collimation $\Delta\psi$. We calculate $\delta\psi = 8.6°$ for $H \to H^*$ at 2500°K and $\Delta\mathcal{E} = 2$ eV.

The collimation $\Delta\psi$ selects the velocities U of those I^* which are detected. To account for this, we must ask which "cones" $(\bar{\psi}, \delta\psi)$, or fractions thereof, fit into the detector cone $\Delta\psi$. Clearly, there are lower and upper cutoff velocities, U_l and U_u, beyond which no I^* are detected; they run into the slits. We find

$$U_{l,u} \simeq [1 \mp (\lambda/\cos\psi_M)]U_M \times\{1 \pm [U_M \tan(\Delta\psi/2)/R \cos\psi_M]\}^{-1}$$

Here, $U_M \simeq \sqrt{2}$ and ψ_M is the mean detector angle at MI^*SAC.[6] At threshold, $U_l = U_u = U_M$, as expected. Now we can derive a function Γ which is the probability that a given U is detected in $\Delta\psi$; however, Γ is a complicated function of apparatus parameters and will not be given here. We do note that $\Gamma = 0$ outside $U_l \le U \le U_u$, and that near the peak of the I^* distribution, $\Gamma = \Delta\psi/\delta\psi$ for typical experiments. The resultant distribution function, $F = \Gamma U^2 \exp(-U^2)$, shows a peak at velocity

$$U_p \simeq (\tfrac{3}{2})^{1/2}(1-\tfrac{1}{9}R^2), \qquad (4)$$

which is approximately characteristic of a U^3 distribution. However, if the apparatus is not set exactly to MI^*SAC, then U_p, as well as U_l and U_u, must be multiplied by a misalignment factor $m = 1 + (d\bar{U}/\bar{U})$, so that the corrected velocities are $U' = mU$. In particular, U_p can be changed by $\pm10\%$ for misalignments $\delta\varphi = \mp1°$.

In Fig. 2, we show a measurement of an $H \to H^*$ velocity distribution done in an apparatus similar to one previously described.[7] The apparatus was adjusted to within $\delta\varphi = 1°$ of MI^*SAC. The H^* atoms were detected directly by a Bendix Channeltron electron multiplier. A 5-μsec pulse was applied to the electron gun, and—at a variable delay time (0-100 μsec) later—a single-channel scaler was gated open for 5 μsec to count the detector pulses. Background counts were subtracted out by taking the signal difference when a suitable electric quenching field was applied to the beam. The observed time-of-flight spectrum was converted to the velocity distribution of Fig. 2. There, the data are compared with curves $U^n\exp(-U^2)$ for $n=2$ and 4. Clearly, the detected H^* are considerably faster than the $n=2$ distribution; the effective distribution is more nearly $n=4$. Observed characteristic velocities, as compared with calculations from Eqs. (3) and (4), are as follows, in units of 10^5 cm/sec:

H^* velocity	V_l	V_p	V_u
Observed	5.5 ± 0.5	10.2 ± 0.4	19.5 ± 1.5
Calculated	5.4 ± 0.4	9.3 ± 0.5	17.9 ± 1.2

The deficiency of slow H^* is particularly marked. Such deficiencies have been noticed in other experiments with light metastable atoms[8]; usually they are attributed to such causes as poor definition of oven temperature or self-scattering in the beam.[9] We believe that the present kinematic effects are, in fact, a major cause in distorting detected metastable velocity distributions.

If so, we can estimate changes in velocity-dependent corrections to those $H \to H^*$ experiments measuring the hydrogen $n=2$ Lamb shift, $\mathcal{S}$(H, $n=2$.[10] The assumed H^* distribution was $U^2 \times\exp(-U^2)$ in both the Triebwasser-Dayhoff-

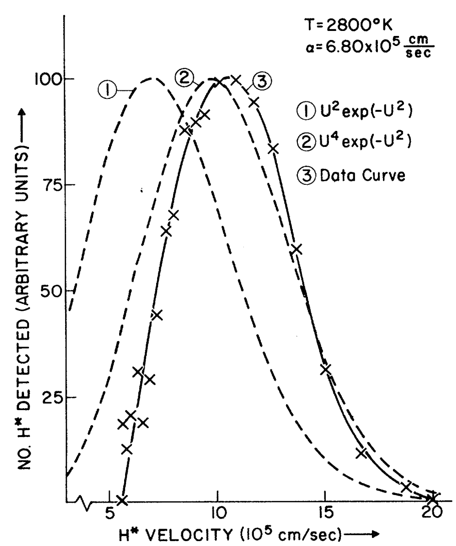

FIG. 2. Measured H^* velocity distribution. Apparatus parameters were $\mathcal{E} = 11.4$ eV, $\Delta\mathcal{E} = 2.9\pm0.5$ eV, $\varphi = 6.3\pm0.3°$ (vs $\varphi_M = 7.3°$), $\psi = 0$, $\Delta\psi = 0.764°$, and flight path 60 cm. A comparison of observed with predicted characteristic velocities is given in Eq. (5). Note that the observed distribution peaks near $V_p = \sqrt{2}\alpha$, and is reasonably well approximated by a $U^4\exp(-U^2)$ distribution function.

Lamb (TDL) and Cosens-Robiscoe (CR) experiments.[11] Correcting this to an approximate U^4 distribution in the CR work is unambiguous, as it was done in an apparatus virtually identical to that used to measure the distorted distribution of Fig. 2.[12] We find that the CR motional field-asymmetry correction is changed so that the H(605) and H(538) $\mathcal{S}$ values are raised by 0.040 MHz. The corrected mean value is

$$\mathcal{S}'(\mathrm{CR}) = 1057.90 \pm 0.10 \text{ MHz}, \qquad (5)$$

where the statistical error is 2σ(mean).[13]

Correcting the velocity distribution for the TDL work is less certain. The apparatus geometry was rather different, and beam alignment and collimation were less critical than in the CR work.[14] The velocity distribution was distorted by motional field quenching (as in the CR work); in fact, this effect was used to measure indirectly the effective n in an assumed U^n distribution by determining the inverse first moment of the distribution.[15] Measurements of this quantity in hydrogen were consistent with a $U^n\exp(-U^2)$ distribution over $0 \leq U \leq \infty$ for $n \simeq 2.5$. However, the analysis used is rather sensitive to the assumed velocity cutoffs (e.g., if $0.5 \leq U \leq \infty$, $n \simeq 1.8$, while if $0 \leq U \leq 1.5$, $n \simeq 3.6$). Unless the beam geometry during this measurement was identical (in alignment and collimation) to that used during the $\mathcal{S}$ measurements, it is possible that velocity-selection effects were introduced inadvertently. If they were as large as to require using a U^4 rather than U^2 distribution to analyze the data, then the $\mathcal{S}$(TDL) value would be raised by about 0.09 MHz.[16] But since there is no direct evidence to support this revision of $\mathcal{S}$(TDL), we believe that such a correction should be considered at most as a possible source of previously unaccounted systematic error.

If we compare the experimental $\mathcal{S}$ value in Eq. (6) with Erickson's latest theoretical value,[17]

$$\mathcal{S}(\mathrm{theor}) = 1057.56 \pm 0.10 \text{ MHz}, \qquad (6)$$

we note a discrepancy which amounts to about 7 times the statistical standard deviation of the experimental number, and more than 3 times the upper limit of error attributed to the theoretical calculation. We believe such a discrepancy should be taken seriously, in spite of the difficulties with the experimental analysis, and that it represents a possible need for revision of the theory.[18,19]

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[1]See W. E. Lamb, Jr., and R. C. Retherford, Phys. Rev. 79, 549 (1950), Appendix III. This paper is H I in the hydrogen series. Others are: H II, W. E. Lamb, Jr., and R. C. Retherford, ibid. 81, 222 (1951); H III, W. E. Lamb, Jr., ibid. 85, 259 (1952); H IV, W. E. Lamb, Jr., and R. C. Retherford, ibid. 86, 1014 (1952); H V, S. Triebwasser, E. S. Dayhoff, and W. E. Lamb, Jr., ibid. 89, 98 (1953).

[2]See Sec. 79 of H IV, Ref. 1. Also, R. T. Robiscoe and B. L. Cosens, Phys. Rev. Letters 17, 69 (1966), and Sec. III A of R. T. Robiscoe, Phys. Rev. 168, 4 (1968).

[3]For typical experiments, the $I \to I^*$ excitation takes place mainly in a plane, due to collimation of the incoming and outgoing beams. Thus, we ignore momentum components perpendicular to the plane of Fig. 1.

[4]The assumption of electron-recoil isotropy is good sufficiently near threshold, and for those $I \to I^*$ processes which are predominantly S wave above threshold (e.g., He → He*).

[5]$\overline{\psi}$ can be found from $\tan(\varphi+\overline{\psi}) = R\cos\varphi/(U - R\sin\varphi)$. This, as well as $\delta\psi$ of Eq. (2), is a generalization of the calculation carried out in Eqs. (83)-(85) of Appendix III of H I, Ref. 1.

[6]For oven orientation $\angle\varphi = 0$, $\psi_M \simeq \tan^{-1}(R/\sqrt{2})$. If $\psi = 0$, then the MI^*SAC oven position is found from $\varphi_M \simeq \sin^{-1}(R/\sqrt{2})$.

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[9]P. M. Marcus and J. H. McFee, in Recent Research in Molecular Beams, edited by I. Estermann (Academic, New York, 1959), p. 43.

[10]See H V, Ref. 1, and Robiscoe, Ref. 2.

[11]See Sec. 89 of H V, Ref. 1, and Sec. IV A of Robiscoe, Ref. 2.

[12]For apparatus parameters in the CR work, see Ref. 7, especially Fig. 4.

[13]This correction revises the $\mathcal{S}$(CR) values appearing in Table II of Robiscoe, Ref. 2.

[14]For apparatus parameters in the TDL work, see H II, Ref. 1, especially Fig. 27. With nominal alignment, the TDL beam was less tightly collimated. This tends to reduce velocity-selection effects.

[15]The method used is described in Appendix VII of H V, Ref. 1.

[16]Using a U^4 rather than U^2 distribution, we find that the TDL Stark-effect corrections (Table XX of H V, Ref. 1), which scale as $\langle U^2 \rangle$ (see Sec. 57 of H III, Ref.

1), are changed by a factor of about 5/3. More accurate calculations, based on the method outlined in Sec. 64 of H III, show that the H(αe) $\mathcal{S}$ value is raised by 0.222 MHz, while the H(αf) result is lowered by 0.049 MHz, which gives a mean $\mathcal{S}$ value of 1057.86 MHz.

[17]G. W. Erickson, in Proceedings of the Arnold Sommerfeld Centennial Memorial Meeting and Symposium on the Physics of One- and Two-Electron Atoms, Munich, Germany, 9-14 September 1968 (North Holland, Amsterdam, to be published).

[18]This point has been discussed in the recent review article on fundamental constants by B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969). See, particularly, Sec. IV C 1 and VII 5.

[19]A recent calculation by T. Appelquist and S. J. Brodsky, following Letter [Phys. Rev. Letters 24, 562 (1967)], has revised $\mathcal{S}$(theor) to 1057.91 ± 0.16 Mhz, in good agreement with $\mathcal{S}'$(CR) of Eq. (6).

ORDER α^2 ELECTRODYNAMIC CORRECTIONS TO THE LAMB SHIFT*

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The fourth-order radiative correction to the slope at $q^2=0$ of the Dirac form factor of the free-electron vertex is calculated using computer techniques. The result,

$$m^2\partial F_1^{(4)}(0)/\partial q^2 = (\alpha/\pi)^2[0.48 \pm 0.07],$$

disagrees with previous calculations, and implies a new theoretical value for the order $\alpha^2(Z\alpha)^4mc^2$ contribution to the Lamb shift. The new values for the $2S_{1/2}$-$2P_{1/2}$ separation in H and D are increased by (0.35 ± 0.07) MHz and are in good agreement with the results of recent experiments.

It is rather ironic that the only tests of quantum electrodynamics which still show a serious discrepancy between theory and experiment are the $2S_{1/2}$-$2P_{1/2}$ and $2P_{3/2}$-$2S_{1/2}$ separations in atomic hydrogen and deuterium—precisely the levels measured by Lamb[1] and co-workers which gave the theory its start. The disagreement (>200 ppm) has become more acute with recent measurements and refinements by Robiscoe and Shyn and Cosens[2] of the Lamb interval in H and D, and three measurements this year[3] of the $2P_{3/2}$-$2S_{1/2}$ interval in H. The results are tabulated in Table I.

The only experimentally relevant contribution to the theoretical value of the Lamb shift not checked by independent methods is the fourth-order self-energy correction to the energy levels of the bound electron.[4] The leading contribution, of order $\alpha^2(Z\alpha)^4mc^2$, to the level-shift formula may be obtained directly[5] from the Dirac form factor of the free electron in fourth order[6]:

$$\Delta E^{(4)}(n,j,l) = \delta_{l0}\frac{4(Z\alpha)^4mc^2}{n^3}m^2\frac{\partial F_1^{(4)}}{\partial q^2}\bigg|_{q^2=0}.$$

This contribution comes from the same set of fourth-order Feynman diagrams (see Fig. 1) which give the well-known $0.328\,47\cdots(\alpha/\pi)^2$ contribution to the electron magnetic moment.[7]

In this paper we report the results of a new computation of the slope at $q^2=0$ of the Dirac form factor in fourth order. In the calculation, all traces, projections, and reductions to Feynman parametric form are performed automatically by REDUCE, an algebraic computation program written by Hearn.[8] The integrals over the Feynman parameters (up to five dimensions) are performed numerically to a typical precision of 0.1 % using a program based on work by Sheppey.[9] The integration method is basically a computation of the Riemann sum, but on successive iterations the integration grid is modified by the program to minimize the variance of the integrand within each hypercube.

Our results for each graph are shown in Table II along with those of the previous analytic calculation of Soto.[4] Except for a discrepancy in overall sign, our results for the individual contributions are consistent with the asymptotic infrared behavior of the individual amplitudes given in Refs. 4 and 5, as well as the expectation that the separate sums of ladder-plus-crossed-ladder contributions [Table II, Figs. 1(a) and 1(e)] and corner-plus-self-energy contributions [Table II, Figs. 1(b) and 1(d)] are finite as the photon mass $\lambda \to 0$. The results shown in Table II are obtained from several types of least-squares fits to the results of numerical integration of the individual amplitudes for $10^{-6} < \lambda^2 < 10^{-2}$. In addition to the discrepancy in overall sign, our