ASTROPHYSICAL DETERMINATION OF THE COUPLING CONSTANT FOR THE ELECTRON-NEUTRINO WEAK INTERACTION

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The existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ weak interaction is confirmed by the results of nine astrophysical tests. The value of the coupling constant is equal to, or close to, the coupling constant of beta decay, namely, $g^2 = 10^{0\pm 2} g_\beta{}^2$.

Of all the astrophysical tests applied so far for the inference of a direct electron-neutrino interaction in nature, none has unambiguously provided a useful upper limit on the coupling constant, which in the $V-A$ theory of Feynman and Gell-Mann[1] is taken to be equal to the "universal" weak-interaction coupling constant measured from beta decays (called g_β hereafter). However, it is important to point out that these tests, made by the author and his colleagues during the past eight years, do provide a nonzero lower limit, and therefore establish at least the existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction. It should be emphasized, nonetheless, that all of these tests rely on the validity of various stellar model calculations. These models, while not subject to scrutiny in the same sense as a laboratory experiment, are based on reasonable physical assumptions, so that the input physics is presumed to be adequate at least for the desired test. But, in addition, the models should and do make a sufficient number of predictions [apart from the effects due to the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction] which can be checked satisfactorily against pertinent observational data.

This Letter reports the results of an unambiguous test for an upper limit on the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ coupling constant using the statistics of white-dwarf stars, and concludes with a summary of the results of tests providing a lower limit.

The basic premise in the case of white dwarfs is that, if the rate of neutrino emission, supposed due to processes[2] based on the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction, is very high, the cooling of bright white dwarfs should be so rapid that a marked deficiency of such objects is expected to occur in counts of white dwarfs. All earlier attempts[3-6] at setting a lower limit on the coupling constant by this method have been unsuccessful, but an upper limit has always been implicit in the observational data.

Our adopted approach will be to compare relative numbers of observed white dwarfs in successive luminosity intervals with the respective relative theoretical lifetimes, calculated with and without the inclusion of neutrino emission. In this Letter, the unmodified term "luminosity" will mean the photon luminosity L radiated by the star. The "neutrino luminosity" will be designated L_ν. Quantities referring to the sun are subscripted with an encircled dot.

The most accurate available data on white dwarfs are those collected by Eggen[7] for the two clusters Hyades and Pleiades and for the nearby general field. Of chief interest here are the hot white dwarfs, for which the observational data[7,8] have been reduced following the procedure of Van Horn.[9] The resulting luminosities are estimated to have a statistical accuracy of ±0.1 in $\log(L/L_\odot)$, which is adequate here.

Models of cooling white dwarfs have been constructed in great detail by a large number of authors. Fortunately, the stellar structure is basically so simple and straightforward (more so than for a main-sequence star) that the results of the different investigators are closely in agreement. Omitted features, such as convection in the envelope and ion crystallization, have only a small effect on the relative cooling rates.[10,11] For our purposes, the sequences of Vila and of Savedoff et al.[12] will be adopted because they have been calculated for a wide variety of masses, chemical compositions, and with and without the inclusion of plasma neutrino emission (the dominant neutrino process for luminosities fainter than $\sim 10^2 L_\odot$). Of course, these authors used the conventional value of the coupling constant in the cross section for the neutrino rate, but their results may be modified in a straightforward way to obtain approximate revised lifetimes for a different choice of the coupling constant. Since (a) the stellar models of interest here are found to have already contracted to their terminal radius, (b) the cooling rate is simply proportional to the total radiated power, $L+L_\nu$, and (c) L_ν is proportional to the square of the coupling constant g^2, it is easy to re-evaluate the time step Δt between one model and the next along a sequence, as follows:

$$(\Delta t)_{\text{new}} = (\Delta t)_{\text{old}} \frac{L + (L_\nu)_{\text{old}}}{L + (L_\nu)_{\text{old}} g_{\text{new}}^2 / g_{\text{old}}^2},$$

where L and L_ν are the average photon and neutrino luminosities between models.

Application of the various published stellar models to the three groups of observed white dwarfs will next be considered.

Hyades.—Eggen[7] has listed 21 white dwarfs in this cluster (omitting one very faint outlying object). He considers all 21 objects to be actual members, probably constituting a complete sample (at least for the hot, or blue, white dwarfs) since the proper-motion surveys extend to a fainter magnitude than the observed cutoff magnitude of the blue sequence of white dwarfs. Thus there is excellent agreement with Sandage's[13] earlier prediction of about 23 dead stars in the Hyades on the basis of an extrapolation of the observed Hyades main sequence using the normalized "universal" luminosity function. (If stars initially more massive than, say, $7M_\odot$ become neutron stars or collapsed objects, only one such object is expected in the Hyades.)

Two distinct sequences of white dwarfs appear in this cluster: a red sequence with $0.1M_\odot$ and a blue sequence with $(1.0 \pm 0.2)M_\odot$. In addition, two blue stars of intermediate mass ($\sim 0.5M_\odot$) also occur. A theoretical explanation for the existence of these sequences and for the detailed characteristics of the low-mass sequence is given elsewhere.[10] Since the neutrino processes are expected to be less effective in the low-mass white dwarfs, we consider only the ten blue stars.

Table I gives the observed and predicted luminosity functions for the blue Hyades white dwarfs. The predicted numbers are based on (a) an expectation of ten blue subluminous stars in the Hyades; (b) a cluster age of $\log[t/(1\text{ yr})] = 8.7 \pm 0.1$ (Henyey et al.[14]); and (c) the theoretical sequences of stellar models for pure-iron (Fe) and for carbon-oxygen (C/O) interiors. The carbon-oxygen sequences are a priori more realistic on purely theoretical grounds (helium interiors are not possible).[10] Ion crystallization effects, which have been neglected in the models, may alter the absolute luminosity functions slightly, but not the relative luminosity functions above the luminosity level corresponding to the cluster age. The results show that, with or without inclusion of the conventional neutrino losses, the carbon-oxygen sequences reproduce well the observed data. However, if $g^2 > 10^{2.5} g_\beta^2$, strong disagreement results, and this sets an upper limit on the coupling constant.

For definiteness, the whole procedure has been repeated, assuming no corrections at all to the observed colors of the white dwarfs (assumed to radiate like black bodies). The same basic conclusion is obtained as before.

Pleiades.—Only one white dwarf is known in this cluster.[8] Sandage[13] predicted about two, but the main-sequence turnoff in the Pleiades occurs at approximately $(5\text{-}7)M_\odot$, and this is roughly the mass at which evolution is expected to produce neutron stars or collapsed objects. The observed colors of the white dwarf lie almost exactly on the black-body line in the color-color diagram, hence no corrections were applied. The luminosity of the white dwarf is therefore $10^{-1.6} L_\odot$ and its mass $\sim 1M_\odot$. The theoretical cooling times at this luminosity are listed in Table II, to be compared with the cluster age of $\log[t/(1\text{ yr})] = 7.5 \pm 0.3$ (Ref. 14). The numbers suggest not only that plasma neutrino emission

Table I. Observed and predicted luminosity functions for the blue Hyades white dwarfs.

$\log(L/L_\odot)$	Observed number	Predicted number[a] $(\cdots)$[b] $g^2=0$	(Fe) $g^2=g_\beta^2$	(C/O) $g^2=g_\beta^2$	(C/O) $g^2=100g_\beta^2$	(C/O) $g^2=1000g_\beta^2$
>−1.50	0	0	0	0	0	0
−1.50 to −1.75	1	1	0	1	0	0
−1.75 to −2.00	2	2	0	2	1	0
−2.00 to −2.25	2	2	1	2	1	1
−2.25 to −2.50	3[c]	2	2	2	2	2
−2.50 to −2.75	2	3	2	3	4	3
<−2.75	0	0	5	0	2	4

[a]Insensitive to mass near $\sim 1M_\odot$.
[b]Insensitive to chemical composition.
[c]Including two white dwarfs of $\sim 0.5M_\odot$ (one possibly a composite).

Table II. Cooling times of white dwarfs to a luminosity of $\log(L/L_\odot) = -1.6$.

g^2	Chemical composition	$\log[t/(1\text{ yr})]$ $1M_\odot$	$0.6M_\odot$	$0.4M_\odot$
0	Fe	8.2	8.4	8.5
0	C/O	8.1	8.1	8.1
g_β^2	Fe	6.7	7.1	7.7
g_β^2	C/O	7.7	7.7	7.7
$100g_\beta^2$	Fe	4.7	5.1	5.7
$100g_\beta^2$	C/O	6.9	5.7	5.6
$1000g_\beta^2$	Fe	3.7	4.1	4.7
$1000g_\beta^2$	C/O	6.0	4.7	4.6

must occur in order to obtain agreement with the cluster age but also that too large a neutrino rate makes the white dwarf's lifetime improbably short.

This evidence, however, should be accepted with caution (unlike the situation in the Hyades). First, the white dwarf is not a confirmed member of this rather distant cluster. Second, the luminosity will be brighter (lifetime shorter) if the bolometric correction has been underestimated, as will be the case if the star appears slightly reddened by interstellar matter or by rotation (the main-sequence stars in the Pleiades are fast rotators). However, to force agreement with the cluster age for the models with no neutrino emission, the white dwarf would have to be ~3 magnitudes brighter than the value derived here.

Field stars.—It was noted earlier[5] that the brightest of 66 white dwarfs listed by Eggen and Greenstein[8] as having well-determined luminosities [on the basis of (a) a trigonometric parallax, (b) membership in a galactic cluster, or (c) a spectroscopic parallax based on the spectrum of a red-dwarf companion] form approximately the same normalized luminosity function as do the 16 listed cluster members alone. The result still holds, with the present improved data. Therefore, our conclusions relating to the Hyades white dwarfs are supported by the more numerous, but undifferentiated, statistical data for the nearby field white dwarfs.

The observational tests which lead to the determination of the existence of the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction (and a lower limit for the coupling constant) can be ordered according to quality class. By a "positive" result of a test we shall mean that the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction is inferred to exist in nature. In the first-class determinations, the result is positive even when the maximum known observational and theoretical uncertainties are applied. In the second-class determinations, the result is positive, but the observational data and/or the theoretical models have greater uncertainty. In the third-class determinations, the result is positive but ambiguous because an alternative explanation is possible, unrelated to the conditions under which the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction would apply. However, two of the third-class determinations would be definitely first class if the present alternative explanations could be ruled out. It should be added that no test so far has yielded a negative result.

First-class determination: statistics of red supergiants in young groups.[15,16] Humphreys's[17] recent enlargement of the statistical data has confirmed the positive result. The earlier well known, but spurious, negative result of Hayashi et al.[18] was due to an incorrect application of the test. However, it should be pointed out that this test involves the use of more complicated stellar models than does the test concerning white dwarfs.

Second-class determinations: (a) statistics of luminous carbon stars in young groups[16]; (b) statistics of yellow supergiants in young groups[16]; and (c) statistics of ultraviolet dwarfs (pre-white dwarfs of very blue color).[4,5,19]

Third-class determinations: (a) statistics of luminous red giants in young clusters[20] and in the field[19]; (b) statistics and observational lifetimes of planetary nebulae[4,5,19]; (c) frequency of type-II supernovae[21]; and (d) isotopic abundances of heavy elements.[22]

Using only the first-class determination, one may set a lower limit on the square of the coupling constant equal to $\sim 10^{-1.5} g_\beta^2$. This limit is based on the assumption that the phase of carbon burning in massive stars is characterized by an initial core composition of roughly one-third carbon and two-thirds oxygen, and that the Arnett-Truran rate of energy generation for carbon burning applies. More initial carbon or a larger carbon-burning rate will raise the lower limit stated above. The second-class determination (c) gives a sharpened lower limit of $\sim 10^{-1} g_\beta^2$, and the third-class determination (b) gives a lower (and upper) limit of $\sim g_\beta^2$ itself. The astrophysical data for white dwarfs (as well as for helium-burning giants[18-20]) suggest that the coupling constant (squared) probably cannot exceed $10^{2.5} g_\beta^2$. Reines[23] is currently attempting to make a laboratory measurement of the cross section for $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$, which will provide an independent determination of the coupling constant.

From the assembled astrophysical data it is concluded, first, that the $(\bar{e}\nu_e)(\bar{\nu}_e e)$ interaction does exist in nature, and, second, that the value of the coupling constant is equal to, or close to, the coupling constant of beta decay, namely, $g^2 = 10^{0\pm 2} g_\beta^2$.

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SIMPLE APPROACH TO UNITARIZATION IN HARD-MESON CALCULATIONS*

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The hard-pion effective-range formula for the pion form factor of Brehm, Golowich, and Prasad is derived from unitarity and (vector) meson dominance assuming a real tree form factor as input. The result is generalized and applied to the π-A_1-ρ system. The suitably continued finite ρ-width form factors are used to calculate the soft π^+-π^0 mass difference when (a) both tree form factors are unsubtracted in broken chiral symmetry ($\delta m \simeq 5.7$ MeV) and when (b) both tree form factors are subtracted ($\delta = -\frac{1}{2}$) ($\delta m \simeq 5.2$ MeV).

In a recent Letter[1] it was shown how the specific hard-pion current-algebra method of Ward identities[2] could be used to generate an effective-range formula for the pion form factor directly without reference to the $\pi\pi$ phase shift. Thus, the usual ordering of input and output is curiously inverted, since in this case the $T=J=1$ $\pi\pi$ phase shift δ_{11} is among the output, once the pion form factor $F(t)$ is given. Unfortunately, the derivation leading to an "on-shell dynamical equation for $F(t)$" presented in Ref. 1 largely obscures what is demonstrably an attractively simple approach to unitarization in hard-meson calculations. In our derivation it will not be necessary to tie the unitarization to any particular current-algebra procedure which produces hard-meson results. As shown below it is enough merely to require that the input hard-meson vertex, say[3]

$$\Gamma(t) = (m_\rho^2 - t)F(t), \tag{1}$$

which we shall occasionally refer to as the "tree vertex," be real. The unitarization of "tree form