this result shows that there is a well-defined radio shower front. This front may precede the particle front by about 15 m, on the average (Fig. 2), but the possible systematic error is also 15 m.

Although we used the particle arrival directions to study the background, the identification of radio showers can be made without reference to it. Thus, we do not need the BASJE fast-timing array. We do need some particle detectors, however, to provide the trigger pulse and the timing reference pulse.

We also attempted to trigger on a coincidence of radio pulses in three antennas without a particle requirement. In addition to interference triggers, we obtained several good triggers per hour, most of which showed not only clean pulses in the three required antennas but also coincident pulses in one or more other antennas. All pulses were consistent with production by a single electromagnetic wave front. We are now studying the possibility that these events were in effect produced by extinct, near-horizontal showers.

The first three conditions for using radio pulses to measure large air showers can be met, according to the above results. On the other hand, no good correlation was found between pulse heights and such shower parameters as size and core distance. Thus, at present we can determine neither the core location nor the size of a shower from radio-pulse data alone. However, for some studies these data are not necessary. For example, the celestial arrival directions of large radio showers can be measured with our present radio-pulse techniques.

On the other hand at sea level, Vernov et al.[3] have evidence for a dependence of radio pulse heights on muon numbers and also on the distance from shower axis, promising a possible solution to part D of the problem of radio-pulse measurement of air showers.

At sea level, Allan, Jones, and Neat[4] have found increases in the proportion of radio showers both as a function of shower size and as a function of zenith angle, similar to our results.

This experiment would not have been possible without data from the BASJE group whose efforts in Bolivia were directed by Dr. K. Kamata, Dr. M. Lapointe, Dr. K. Murakami, Dr. S. Shibata, Dr. K. Suga, and Dr. Y. Toyoda.

*Research sponsored by the U. S. Air Force Office of Scientific Research, Office of Aerospace Research, under Contract/Grant No. AFOSR 68-1460 A.

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# PROTON-NEUTRON SCATTERING

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Using Glauber multiple-scattering theory, the missing-mass spectrum for protons scattered off a deuterium target is computed. The relatively clean separation of the single- and double-scattering peaks offers the possibility of determining the high-energy proton-neutron differential cross section.

Experimental data on proton-proton elastic scattering at high energies show angular distributions with interesting structure.[1] It is therefore natural to ask if similar features are present in the neutron-proton case. Unfortunately, cross-section measurements with neutron beams[2] are very imprecise, especially at the larger angles $[|t| > 1\ (\mathrm{GeV}/c)^2]$. In the absence of a free-neutron target, it has recently been suggested that the proton-neutron cross section can be deduced from observations of quasielastic scattering in proton-deuteron collisions.[3]

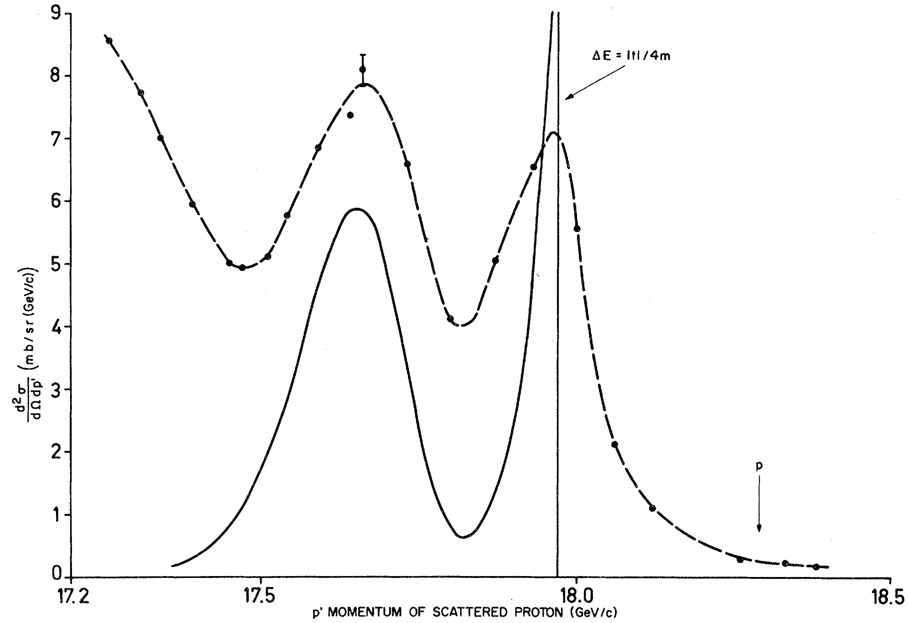


FIG. 1. Spectrum of protons scattered from deuterium [incident momentum $p = 18.29$ GeV/$c$, $t = -1.2$ (GeV/$c$)$^2$]. The dashed curve joining the experimental points (Ref. 4) is meant only to guide the eye. The solid curve is our theoretical prediction neglecting experimental resolution and meson production.

Figure 1 shows a typical momentum spectrum of protons scattered from a deuterium target[4] [incident momentum $p = 18.29$ GeV/$c$, $|t| = 1.2$ (GeV/$c$)$^2$]. In addition to a rising continuum associated with meson production, there are two distinct peaks. The second of these occurs at a momentum loss $\simeq |t|/2m$, that is at about the same position as for the scattering from free protons. Since this can be thought of as elastic scattering from a single nucleon in the nucleus, it is called quasielastic. The binding energy of the deuteron is unimportant because of the poor experimental resolution (±50 MeV/$c$), but the quasielastic peak is broadened considerably by the deuteron Fermi motion. The other bump lies close to the edge of phase space, $\Delta p \simeq |t|/4m$, where the two target nucleons are recoiling with low relative energy. The vast majority of these pairs cannot be deuterons since the elastic proton-deuteron cross section is too small.[5] The effect can be explained in classical terms as a kinematic enhancement arising from a double scattering.[6] If the proton hits both the neutron and proton in the target then, because the nucleon-nucleon amplitudes fall so fast with momentum transfer, the most likely event is that both scatterings will lie in the same plane and be of essentially the same angle. In this case the proton and neutron will come out together, as is observed.

We want now to present a more careful investigation of the spectrum on the basis of Glauber theory.[7] This allows us also to estimate the importance of interference between single and double scattering. If we neglect spin dependence and final-state interactions between the recoiling nucleons, then in the notation of Bertocchi[8] the amplitude for the scattering of a proton from deuterium is the coherent sum of single- and double-scattering contributions:

$$F = f_{pp}(t)\psi(\vec{k}_n) + f_{pn}(t)\psi(\vec{k}_p) + \frac{i}{2\pi p}\int d^2q\,\psi(\vec{q}-\vec{k})f_{pp}\left(-\left(\vec{q}-\frac{\vec{\Delta}}{2}t\right)^2\right)f_{pn}\left(-\left(\vec{q}+\frac{\vec{\Delta}}{2}t\right)^2\right), \quad (1)$$

where $\psi$ is the deuteron wave function, $f$ are the nucleon-nucleon amplitudes, $k_{p(n)}$ is the recoiling proton (neutron) momentum, and $p'$ is the momentum of the scattered proton;

$$\vec{\Delta} = \vec{p}' - \vec{p} = -(\vec{k}_p + \vec{k}_n), \quad \vec{k} = \tfrac{1}{2}(\vec{k}_p - \vec{k}_n), \quad s = (k_p + k_n)^2, \quad t = (p' - p)^2. \quad (2)$$

The integration in Eq. (1) is over the plane perpendicular to the beam direction (subscripts $l$ and $t$ will denote the components of vectors parallel and perpendicular to $\vec{p}$).

To get a simple estimate of the relative effects of these terms, let us take a Gaussian deuteron wave function[9]

$$\psi(p) = Ne^{-b_f p^2} \tag{3}$$

with $b_f = 66$ (GeV/$c$)$^{-2}$, and purely imaginary (and equal) proton-nucleon amplitudes

$$f_{pp} = f_{pn} = (ip\sigma/4\pi)e^{-b_s t/2} \tag{4}$$

with $b_s = 9.4$ (GeV/$c$)$^{-2}$. A more accurate parametrization of the amplitudes will be used later. With these assumptions the $q$ integration becomes trivial.

We now calculate the invariant mass spectrum of the neutron-proton pair[10]:

$$\frac{d\sigma}{d\Omega ds} = \int d^3k\,|F|^2\delta(s+\Delta^2-\{[m^2+(\vec{k}+\tfrac{1}{2}\vec{\Delta})^2]^{1/2}+[m^2+(\vec{k}-\tfrac{1}{2}\vec{\Delta})^2]^{1/2}\}^2). \tag{5}$$

The purely single-scattering term can then be evaluated in terms of the error function. At high energies and away from the forward direction this can be approximated by

$$\left(\frac{d\sigma}{d\Omega ds}\right)_{\text{single}} \simeq \left(\frac{p\sigma N}{2\pi}\right)^2 \frac{\pi}{8\Delta b_f}\exp\left\{b_s t - \frac{b_f}{2}\left[\Delta - \left(\left[s-4m^2\right]\left[1+\frac{\Delta^2}{S}\right]\right)^{1/2}\right]^2\right\}, \tag{6}$$

which shows the expected quasielastic peak at the point where one of the nucleons takes all the recoil momentum. The width ($\sim 1/b_f$) is determined by the Fermi motion.

As they stand, the integrals for the double scattering cannot be done analytically. Now the value of $k^2$ is greatly restricted,

$$s-4m^2 \leqslant k^2 \leqslant (s-4m^2)\,(1+\Delta^2/s); \tag{7}$$

and furthermore the double scattering is strongly enhanced towards the lower value (the momentum $\vec{\Delta}$ of the pair is then equally divided), and so we can replace the argument of the delta function in Eq. (5) by $s-4m^2-4k^2$. The same result is obtained by expanding the argument in $\vec{k}\cdot\vec{\Delta}$ and keeping only the lowest term. The same approximation is sufficient for the interference term since this turns out to be quite small. With this simplification, the purely double-scattering contribution becomes

$$\left(\frac{d\sigma}{d\Omega ds}\right)_{\text{double}} = \left(\frac{p\sigma N}{2\pi}\right)^2 D^2\left(\frac{\pi}{4}\right)^{3/2}\left(\frac{2(b_s+b_f)}{b_f^2}\right)^{1/2}\exp\left[\frac{b_s t}{2} - \frac{b_s b_f(s-4m^2)}{2(b_s+b_f)}\right]\operatorname{erf}\left[\left(\frac{(s-4m^2)b_f^2}{2(b_s+b_f)}\right)^{1/2}\right], \tag{8}$$

where

$$D = \sigma/16\pi(b_s+b_f).$$

For the interference term, there remains one integration which is done numerically:

$$\left(\frac{d\sigma}{d\Omega ds}\right)_{\text{interf}} = -\left(\frac{p\sigma N}{2\pi}\right)^2 D\left(\frac{\pi}{4}\right)(s-4m^2)^{1/2}\exp\left[\frac{3b_s t}{4} - \frac{b_f\Delta^2}{4} - \frac{b_f}{4}\left(1+\frac{b_s}{b_s+b_f}\right)(s-4m^2)\right]$$
$$\times\int_0^1 d\lambda\,\cosh[\tfrac{1}{2}b_f\Delta_b\lambda(s-4m^2)^{1/2}]\,I_0(\tfrac{1}{2}b_f\Delta_t[(s-4m^2)\,(1-\lambda^2)]^{1/2})\exp\left[-\frac{1}{4}\frac{b_f^2}{b_s+b_f}(s-4m^2)\lambda\right], \tag{9}$$

where $I_0$ is the modified Bessel function of zeroth order.

At the energy and momentum transfer of the data shown in Fig. 1, it is not a good approximation to take just one exponential in the nucleon-nucleon parametrization (4). However, for the single-scattering amplitude we can easily take a more complicated form, e.g., that of Krisch[11]; for the double scattering we mainly need the nucleon amplitudes at momentum transfer $t/4$ and for these a one-exponential fit is very reasonable. In Fig. 2 we show the three contributions to the differential cross section

$$d^2\sigma/d\Omega dp' \simeq 4md^2\sigma/d\Omega ds. \tag{10}$$

The double scattering rises extremely rapidly from threshold because of the argument in the error function of Eq. (8). The single- and double-scattering peaks are well separated and hence it is not surprising that the interference term turns out to be so small. The integrated single- and double-scattering contributions are at this value of $t$ very comparable, whereas the inter-

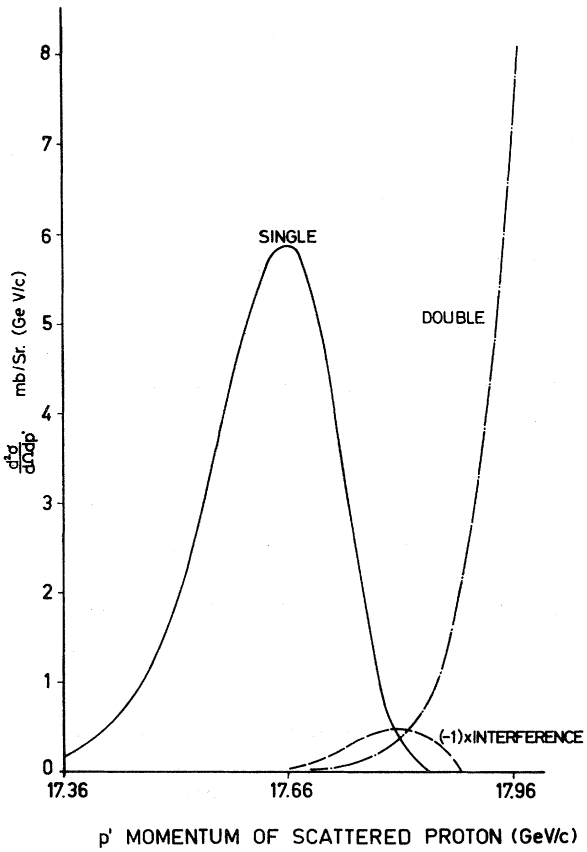


FIG. 2. Spectrum of protons scattered from deuterium [incident momentum $p=18.29$ GeV/$c$, $t=-1.2$ (GeV/$c$)$^2$]. The three curves represent the calculation of single scattering [Eq. (6)], double scattering [Eq. (8)], and interference [Eq. (9)].

ference is only about 5%. The effect would have been appreciably larger if the longitudinal momentum transfer had been neglected in the single-scattering amplitude (1).

As $|t|$ increases, the single- and double-scattering peaks move further apart. If the $N$-$N$ amplitudes were pure exponentials (4) then the single scattering (6) would decrease much faster with $|t|$ than the double (8). However, since the $N$-$N$ amplitudes fall off much slower,[11] the effect is much less marked. Indeed, at $|t|\sim 2$ (GeV/$c$)$^2$ the single scattering is more prominent than at 1.2 (GeV/$c$)$^2$. These qualitative features will not be changed by a more accurate calculation. The details of the double and interference terms are model dependent (spin effects, uncertainty in the real part of the amplitudes, inelastic intermediate states, etc.) but the single scattering is just the incoherent sum of the cross sections off a proton and a neutron[12] multiplied by the deuteron momentum distribution, in exact analogy to electron scattering (see for example Hughes et al.[13]). If we can subtract the interference and double-scattering terms even approximately, the area under the quasielastic peak gives a good determination of the sum of the proton and neutron differential cross sections. This method is said to be less sensitive to final-state interactions[14] (alternatively the peak method can be used[14]). The final-state interaction is much more important for the double scattering than for the single, where it is estimated, as for electron scattering,[15] to be of the order of 5%, because then the emerging $np$ pair has low relative energy.

In Fig. 1 is plotted our calculated spectrum and the experimental one.[4] The theoretical peaks, especially the double-scattering one, are broadened by the experimental resolution which can easily be taken into account. The pion-production continuum is much more troublesome than in proton-proton scattering. There is a large background underneath the quasielastic peak due to pion production via a double-scattering mechanism. This can be estimated, given the production rate from nucleons, but it seems likely that this is the main limitation in an accurate determination of the proton-neutron differential cross section, especially at higher momentum transfer.

The double-scattering peak contains information about the structure of the initial nucleus. This may be of interest for other very light nuclei.

In this note, we have presented a qualitative calculation showing the feasability of extracting high-energy proton-neutron differential cross sections from the deuteron breakup reaction. Further calculations, including the pion-production background, are in progress. It is to be suspected that kinematic enhancements will also be present in resonance production from deuterium and thus make missing-mass experiments difficult to interpret. The mass shift of the $N^*(1400)$ observed in deuterium[16] is, however, hard to explain this way since it shows up at very small angles. We do not care to speculate about analogous enhancements in particle physics.

We are very grateful to Dr. A. Wetherell for bringing to our attention the data of Ref. 4 and its interpretation by Karplus and Yamaguchi. We had useful discussions with Dr. W. Fischer and Dr. H. J. Gerber. It is a pleasure for one of the authors (C.W.) to thank Professor J.-P. Blaser for hospitality at Schweizerisches Institut für Nuklearforschung.

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# PHOTON CROSS SECTIONS AND VECTOR DOMINANCE*

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We calculate (1) incoherent photoproduction of $\rho^0$ mesons, (2) total photon-nucleus cross sections, and (3) incoherent photoproduction of positive pions within the framework of simple vector dominance. Comparison is made with experiment.

The multi-GeV elastic scattering of photons by nuclei and the incoherent photoproduction of $\rho^0$ and $\pi$ mesons provide interesting tests of vector dominance. There have been calculations of these processes using eikonal methods which will be referred to below, and now a considerable amount of experimental data has become available. Although there are no photon elastic-scattering results from nuclei, there are total photon cross-section measurements, which can be simply related to the forward-scattering amplitude through the optical theorem.

The object of this Letter is to make calculations of these processes in some detail, assuming simple vector-meson dominance. We will need vector-dominance coupling constants, certain scattering and production amplitudes on nucleons, and nuclear size parameters. All these we take from other experiments. Our calculations have no free parameters then.

Incoherent photoproduction of $\rho^0$ mesons.—In a recent Letter[1] we have calculated the incoherent production of $\rho^0$ mesons on nuclei under the assumption of vector dominance using eikonal methods.[2,3] The process is calculated as a combination of (a) a one-step process corresponding to diffractive photoproduction of a $\rho^0$ meson on a nucleon accompanied by nuclear excitation, and (b) coherent photoproduction of a $\rho^0$ meson on a nucleon (no nuclear excitation) followed by incoherent scattering of the $\rho^0$ meson (nuclear excitation occurs). Appropriate diagrams are shown in Fig. 1(a). Since the detailed formulas have been written down before,[1,3,4] we only note here that we can write the incoherent cross section in the form

$$d\sigma^{(1)}(\gamma A \to \rho^0 A')/d\Omega = |f_{\gamma\rho}(t)|^2 N_{\text{eff}}, \tag{1}$$

where $N_{\text{eff}}$ is an effective nucleon number and $f_{\gamma\rho}(t)$ is the two-body photoproduction amplitude, assumed spin and isospin independent.

We have redone the calculation for incoherent $\rho^0$ photoproduction allowing for a nonzero real part for the $\rho^0$-nucleon forward-scattering amplitude, taken from pion-nucleon scattering[5] using the quark-model relation

$$f_{\rho^0 p}(0) = \tfrac{1}{2}[f_{\pi^+ p}(0) + f_{\pi^- p}(0)]$$

to determine the real part of $f_{\rho^0 N}(0)$. The total cross section for a $\rho^0$ meson on a nucleon $\sigma_{\rho N}$, related by the optical theorem to the imaginary part of the forward-scattering amplitude $f_{\rho^0 N}(0)$, has been determined using the vector-dominance relation[6]

$$\left.\frac{d\sigma_{\gamma\rho}}{dt}\right|_{t=0} = \frac{1}{16}\frac{\alpha}{4\pi}\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1}\sigma_{\rho N}^2(1+\beta^2), \tag{2}$$

where $\beta = \text{Re}\, f_{\rho^0 N}(0)/\text{Im}\, f_{\rho^0 N}(0)$ and $\gamma_\rho^2/4\pi = 0.5$, taken from the compilation of Ting.[6] The values of $d\sigma_{\gamma\rho}/dt$ are taken from a fit to data of $\rho^0$ photoproduction on hydrogen.[7] These values and other parameters for the calculation are listed in