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¹⁰U. Fano and J. W. Cooper, *Phys. Rev.* **137**, A1364 (1965).

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RAMSAUER EFFECT AS A RESULT OF THE DYNAMIC STRUCTURE OF THE ATOMIC SHELL

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We show a way of attacking elastic atomic collisions on the grounds of classical physics.

The Ramsauer effect is commonly recognized as inherently connected with the "wave" properties of matter and as constituting a conspicuous example of the uselessness of the deterministic concepts of classical physics as regards microcosmic phenomena.¹ Attempts undertaken in the twenties to explain this effect and, moreover, to construct a classical theory of elastic low-energy atomic collisions have, despite the discovery of certain regularities, entirely failed. It now appears that this failure was the result of inconsistencies committed within the framework of classical physics itself.

It is evident that the conglomerate of charges formed by a positively charged nucleus (nuclei) and electrons can, according to Coulomb's law and Newtonian dynamics, remain in a state of dynamic equilibrium only. The resulting time-varying field of such a conglomerate, if expanded as a power series in terms of the distance from the system, can be completely described by a set of coefficients, both independent and varying with time, which are closely related to the multipole moments of the system. Using the results of the potential theory and Fourier analysis, the series determining the field of such a system can be expressed in the form

$$\varphi(r, \theta, \phi, t) = \sum_k \sum_n \frac{A_{nk}(\theta, \phi)}{r^n} e^{-i\omega_k t}, \quad (1)$$

where A_{nk} represents the component corresponding to the frequency ω_k in the Fourier expansion of a multipole moment of the n th order.

Approximate analysis of the scattering problem, performed on the basis of classical small-

angle deflection theory,² leads to the following expression for the scattering angle Θ arising from the presence of the term A_{nk} :

$$\tan\Theta \propto \frac{A_{nk}}{D^{n+1}v^2} e^{-\omega_k D/v}, \quad (2)$$

where D is the collision parameter, and v is the scattered particle velocity. It is evident from the above that, irrespective of the multipole order involved, there exists a limit to the interaction range of the scattering. This limit is determined by the frequency of the system. It is easy to show on the basis of the small-angle approximation that the results of scattering on a dynamic system are formally the same as in the case of a system with a potential

$$\varphi \propto \frac{1}{r^n} e^{-r/r_0} \quad (3)$$

with r_0 being dependent on v . This allows one to suppose that the short range interactions—that is, interactions diminishing exponentially with distance—reflect the dynamic, periodic in time, structure of scattering objects.

Having the relation between the angle Θ and the collision parameter D as given by Eq. (2), one can easily derive a formula for the elastic-scattering cross section:

$$Q_{sc} \propto 1/v^{4/(n+1)} \text{ for } v > v_1, \\ \propto v^2 \text{ for } v < v_1, \quad (4)$$

where the boundary velocity v_1 depends on the frequency of the system,

$$v_1 \propto \omega_k.$$

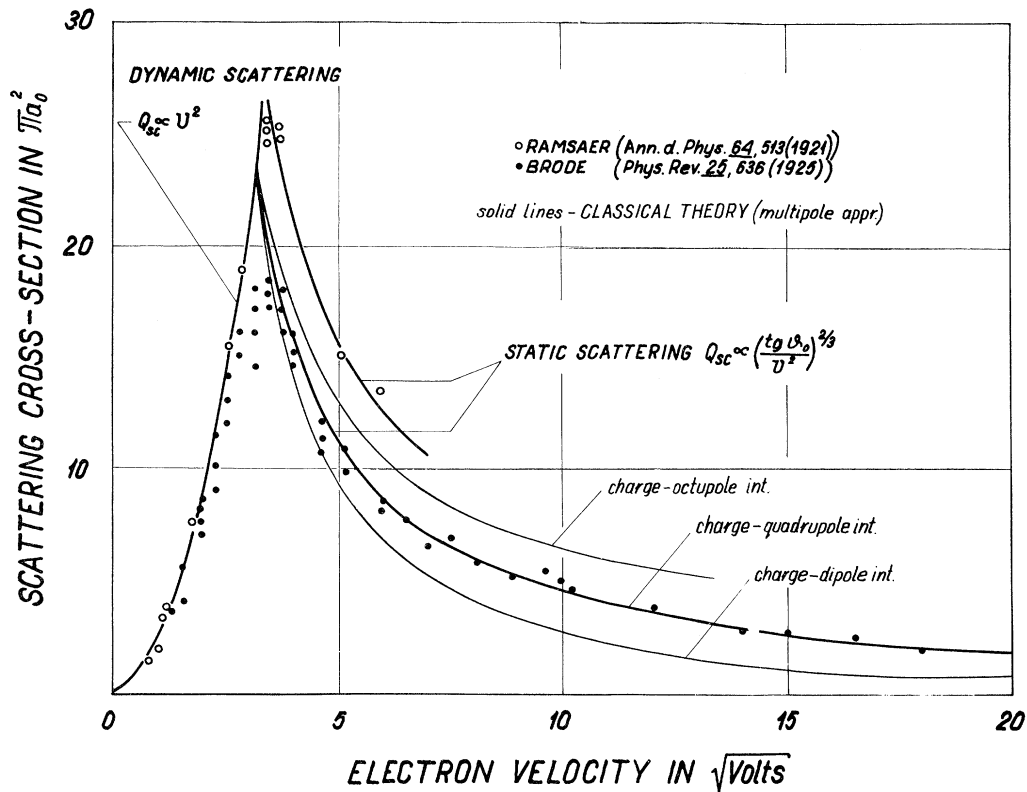


FIG. 1. Theoretical and experimental results of low-energy electron scattering from argon. The thick solid line represents the scattering on a dynamic (periodic in time) quadrupole moment, while the thin lines represent the scattering on dipole and octupole moments. The decrease of cross section at very low velocities is characteristic for dynamic scattering (scattering from fields periodic in time). In this range, the cross section practically does not depend on the geometry (angular aperture, $\tan\theta_0$) of the experimental apparatus. In the high-energy range (static scattering) the cross section is quite sensitive to the apparatus geometry.

With $\omega_k = 0$, which corresponds to scattering on a static multipole, the cross section in question depends monotonically on velocity and is deprived of any maxima.

The Ramsauer effect should therefore be considered as the manifestation of the dynamic character of the electronic shell of the atom. The final shape of the curve Q_{sc} is of course determined by all multipole moments contributing to the potential $\varphi(\vec{r}, t)$. Nevertheless some of them, depending on the symmetry of the atomic shell, will play the decisive role in the scattering. The analysis of the results for electron scattering on argon (Fig. 1) enables one to draw the conclusion that the external shell of argon, like the shells of other noble gases, possesses a dynamic quadrupole moment. The absolute value of the quadrupole moment ${}_2Q \approx 4.6ea_0^2$ deduced on the basis of the theory, and its variation period $\omega_k \approx 0.66\omega_B$, in no way contradict other experimental facts (e.g., concerning shell dimension and binding en-

ergies), and are furthermore consistent with the author's previously expounded classical concept of shell structure.³⁻⁵ According to this concept, the existence of both constant and varying multipole moments is evident. Long-range molecular interactions (van der Waals forces, resonance problems, etc.) can therefore be explained on the grounds of the concepts of classical physics in a very natural way. A detailed paper treating elastic scattering and based on the multipole representation of atomic fields will be published in the near future.

In the light of the above considerations it is supposed that the negation of shell dynamics underlying the theoretical considerations of the twenties, and later the usage of the quantum-mechanical atomic model with a static and spherically symmetrical potential arising from electron "spread" over the space, have been responsible for the discrepancies between the pseudo-classical theories and experiment.

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²See, for example, L. Landau and E. Lifshitz, Mechanics (Addison-Wesley Publishers, Reading, Mass., 1960), pp. 73-75.

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USE OF LIQUID-DROPLET MODEL IN CALCULATIONS OF THE CRITICAL EXPONENT δ *

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The liquid-droplet model is used to derive a formula relating the exponent δ and the compressibility factor $P_c/k_B T_c \rho_c$ at the critical point of a fluid. For fluids (the van der Waals continuum, the Bragg-Williams lattice gas, and Ising lattice gases) where both these quantities are known, the proposed relation is in agreement with experiment and theoretical prediction. Therefore, in other cases this relation may be used to predict δ from the known P_c , T_c , and ρ_c . On this basis we predict a δ between 4.2 and 4.5 for nonpolar molecules, about 5 for hydrocarbons, and larger than 5 for polar molecules.

Three basic assumptions were used in Fisher's liquid-droplet model¹ in order to formulate the cluster theory in mathematical form: (1) It is a classical model. (2) The excluded-volume effect between droplets is neglected. (3) The most probable surface area \bar{S} of a cluster of size l will vary as

$$\bar{S}(l) \approx a_0 l^\sigma \quad (l \rightarrow \infty)$$

with, necessarily for $d \geq 2$, $0 < \sigma < 1$, where $a_0 = a_0(T)$. With these assumptions, the pressure and density of an infinite system for the liquid-droplet model can be expressed as

$$\frac{P}{k_B T} = g_0 \sum_{l=1}^{\infty} l^{-\tau} x^{l^\sigma} y^l, \quad (1)$$

$$\rho = g_0 \sum_{l=1}^{\infty} l^{1-\tau} x^{l^\sigma} y^l, \quad (2)$$

where

$$x = \exp[-a_0(w - \omega T)/k_B T], \quad (3a)$$

$$y = z \exp[E_0/k_B T + S_0/k_B], \quad (3b)$$

and k_B = Boltzmann constant, w = surface energy arising through loss of binding energy by particles near the surface of the cluster, ω = corresponding entropy per unit of cluster surface, z = fugacity, E_0 = bulk energy per particle in the cluster, S_0 = bulk entropy per particle in the cluster, and g_0 is a constant. Here τ , $2 < \tau < 2.5$,² is a parameter due to the "close" effect on the surface of the clusters near the critical point.¹ At

the critical point $x = 1$, and $y = 1$, and hence

$$\frac{P_c}{k_B T_c} = g_0 \sum_{l=1}^{\infty} l^{-\tau} = g_0 \zeta(\tau), \quad (4)$$

$$\rho_c = g_0 \sum_{l=1}^{\infty} l^{1-\tau} = g_0 \zeta(\tau-1), \quad (5)$$

where $\zeta(x)$ is the zeta function of x . Therefore the compressibility factor at the critical point can be written as

$$P_c/k_B T_c \rho_c = \zeta(\tau)/\zeta(\tau-1). \quad (6)$$

Also, from Fisher's analysis,¹ the critical exponent δ , which describes the behavior of the critical isotherm as $|P - P_c| \sim |\rho - \rho_c|^\delta$, at $T = T_c$, can be expressed in terms of τ as

$$\delta = 1/(\tau-2), \quad (7)$$

From Eq. (6) it appears that the compressibility factor at the critical point is simply a function of τ only. Therefore, by the known P_c , T_c , and ρ_c , with Eqs. (6) and (7), we are able to calculate the value of τ , as well as the critical exponent δ , or for given δ , we can predict the compressibility factor at the critical point.³ On this basis, we have investigated the δ for fluids with given compressibility factors at the critical point, and the compressibility factor at the critical point for the van der Waals continuum, Bragg-Williams lattice gases and Ising lattice gases from the known δ values.

From the experimental point of view, it is eas-