either hypothesis.

[1]For an account of these and other accepted classifications see, e.g., Advances in Particle Physics, edited by R. L. Cool and R. E. Marshak (Interscience, New York, 1968), Vol. 2, especially Barbaro-Galtieri and references therein.

[2]N. Barash-Schmidt et al., Rev. Mod. Phys. 41, 109 (1969).

[3]Here "s.d." stands for "standard deviation." The symbols $\langle\,\rangle_{\rm est}$ and $\langle\,\rangle_{\rm th}$ indicate whether the standard deviation has been estimated from the sample, or calculated theoretically.

DUAL MODELS WITHOUT PARITY DOUBLING*

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The recently proposed fixed cuts are dualized. The resulting functions apparently allow construction of parity-doubling–free models for arbitrary processes. Applications are made to the reciprocal and meson bootstraps.

A very simple idea for avoiding parity doubling in fermion Reggeization has recently been proposed by Carlitz and Kislinger.[1] The point is to sum Feynman graphs with the appropriate Dirac projection operators for the desired parity. Thus in π-N scattering, keeping only natural parity, one considers sums of the form

$$\gamma_5\left[\sum_j \frac{(\not p + m_j)}{s-m_j^2} g_j^2 P_j\right]\gamma_5, \qquad (1)$$

where m_j is the jth mass and P_j is a Legendre polynomial. m_j has a branch point at $j=\alpha(0)$, where $\alpha(s)$ is the corresponding Regge trajectory, so there is a fixed cut in the j plane at the trajectory intercept.

Our task here is to dualize[2] this mechanism, that is, to investigate possible supporting dynamics in cross channels. Within the framework of our functions, we find the same branch points in all cross channels. This is uneventful for baryon channels, and allows construction of a model for the reciprocal bootstrap without parity doubling. However, such cuts appear also in meson channels. In fact, it appears that the mesonic cuts can be taken to serve the same purpose as the baryonic: Extending the projection-operator idea to all fermions, including quarks, we construct dual meson-scattering amplitudes without parity doubling.

A possible overall picture emerges: Positive-energy-projected quarks form a unified dual parity-doubling (ghost)-eliminating mechanism for all hadrons. Apparently, parity-doubling (ghost)-free models can be constructed for any process. Two points need emphasis, however. First, for reasons mentioned below, we examine parity doubling explicitly only for leading trajectories. Second, and more important, is the need to determine whether the data will support bosonic cuts. In the case of most interest, (the possible ρ cut in) π-N charge-exchange scattering, it appears however that a detailed dynamical calculation will be necessary to ascertain the strength of the cut. This we do not attempt here.

The functions we need are defined by the following integral representations:

$$B_\sigma^{(+)}(\alpha_s(s), \alpha_t(t)) \equiv \frac{1}{\Gamma(-\sigma/2)}\int_0^1 dx \int_0^1 dz\, z^{-\alpha_s(s)+\sigma/2}(1-zx)^{-t-1}(1-z)^{-1-a_t}x^{-1-a_s}(1-x)^{\sigma/2+1} \times(-\ln x)^{-1-\sigma/2}\left[-\ln\left(\frac{1-z}{1-zx}\right)\right]^{-1-\sigma/2}, \qquad (2a)$$

$$B_\sigma^{(-)}(\alpha_s(s), \alpha_t(t)) \equiv \frac{1}{\Gamma(-\sigma/2)}\int_0 dx \int_0^1 dz\, z^{-\alpha_s(s)+\sigma/2}(1-zx)^{-t}x^{-1-a_s}(1-z)^{-1-a_t}(-\ln x)^{-1-\sigma/2} \times\left[-\ln\left(\frac{1-z}{1-zx}\right)\right]^{-1-\sigma/2}(1-x)^{\sigma/2}\left(\frac{1-z}{1-zx}-x\right), \qquad (2b)$$

where the Regge trajectories in s and t are given by $\alpha_s(s) = a_s + s$, $\alpha_t(t) = a_t + t$, and, for convergence, σ is a negative real number. Changing variables on integration according to $1 - zx = u$, $(1-z)/(1-zx) = v$, one easily shows that

$$B_\sigma^{(\pm)}(\alpha_s(s),\ \alpha_t(t)) = (\pm) B_\sigma^{(\pm)}(\alpha_t(t),\ \alpha_s(s)). \tag{3}$$

Like the more conventional B functions, our new functions consist entirely of poles in s and t. This is easily seen by expanding the integrands in power series with respect to z. The residues of poles are polynomials of the correct order in t. For reference we exhibit the leading trajectory, which in the s channel is identical for $B^{(+)}$ and $B^{(-)}$:

$$B_\sigma^{(\pm)}(\alpha_s(s),\ \alpha_t(t)) \sim \sum_{j=0}^{\infty} \frac{t^J (m_{js})^\sigma}{j![j - \alpha_s(s)]}, \tag{4}$$

where $m_{js} = (j - a_s)^{1/2}$ is the jth mass in the s channel. Notice that the residue is just the usual Veneziano residue times the factor m_j^{α}.

The behavior of the function for large t can be studied through a modified Mellin transform,

$$\begin{aligned}\tilde{B}_\sigma(\alpha_s(s), \lambda) &\equiv \sin(\pi\lambda) \int_1^\infty dt\, t^{-\lambda-1} B_\sigma^{(+)}(\alpha_s(s),\ \alpha_t(-t)) \\ &\sim -\frac{\pi}{\Gamma(-\frac{1}{2}\sigma)\Gamma(1+\lambda)} \int_0^1 dx \int_0^1 dz\, z^{-\alpha_s + \sigma/2} \\ &\qquad \times [-\ln(1-zx)]^{\lambda} (1-z)^{-1-a_t} x^{-1-a_s} (1-x)^{\sigma/2+1} (-\ln x)^{-\sigma/2-1} \left[-\ln\left(\frac{1-z}{1-zx}\right)\right]^{-1-\sigma/2},\end{aligned} \tag{5}$$

where the extension of the lower limit in t to 0 does not affect the λ-plane singularities of interest. For the leading singularities in the λ plane, we need only consider $xz \sim 0$, and, more precisely, when both variables are small. Then, the integrals can easily be done, to yield the result

$$\tilde{B}_\sigma \sim -\frac{\pi}{\Gamma(1+\lambda)} \frac{1}{\lambda - \alpha_s(s)} (\lambda - a_s)^{\sigma/2}, \tag{6}$$

where we have used

$$-\ln(1-xz) \sim xz, \quad -\ln\left(\frac{1-z}{1-zx}\right) \sim z(1-x),$$

etc. This is the Regge pole with a multiplicative fixed cut at $j = a_s$. Of course, there are multiplicative fixed cuts at the intercept of each subsidiary trajectory as well.

We have not been able to construct satisfactory functions with fixed cuts only in one channel. As easily seen from Eq. (4), any simple attempt to remove, say, the t-channel cuts results in fixed poles in t. We have presented only the functions with cuts in both channels because (a) cuts are, of course, what we want – to kill parity doubling in other channels – and (b) the cuts are not required to move by unitarity.

On the other hand, it is possible to change the degree of the branch points in the two channels. For example, the function

$$\begin{aligned}B_{\sigma\tau}^{(+)}(\alpha_s(s),\ \alpha_t(t)) \equiv{}& \frac{1}{\Gamma(-\sigma/2)} \int_0^1 dx \int_0^1 dz\, z^{-\alpha_s(s)+\tau/2} (1-zx)^{(\sigma-\tau)/2-t-1} (1-z)^{-1-a_t} x^{-a_s-1} (1-x)^{1+\tau/2} \\ &\times \left[-\ln\left(\frac{1-z}{1-zx}\right)\right]^{-1-\tau/2} (-\ln x)^{-1-\sigma/2} \left(1 - \frac{1-z}{1-zx} zx\right)^{(\sigma-\tau)/2} \frac{\Gamma(1-(1+\sigma/2)zx)}{\Gamma(1-(1+\tau/2)zx)},\end{aligned} \tag{7}$$

has all the same leading trajectory properties as $B_\sigma^{(+)}$, except that now factors of $(m_{js})^\sigma$ and $(m_{jt})^\tau$ appear in the s and t channels, respectively. The antisymmetric counterpart can easily be written as well, and finally $B_{\sigma\sigma}^{(\pm)} = B_\sigma^{(\pm)}$.

As our first application of the new functions, we present a dual model of the reciprocal bootstrap without parity doubling – consisting only of s-u-channel baryon resonances in π-N scattering. To simplify matters, we take the N and Δ trajectories degenerate, and call them $\alpha(s)$ [or $\alpha(u)$]. The model is

$$A^{(1/2)}=M[C_2B_\sigma^{(+)}+2C_1B_\sigma^{(-)}]+[\tfrac{1}{3}(C_2-4C_1)B_{\sigma+1}^{(+)}-(4C_2+2C_1)B_{\sigma+1}^{(-)}],$$
$$A^{(3/2)}=M[C_2B_\sigma^{(+)}-C_1B_\sigma^{(-)}]+\tfrac{1}{3}[(C_2-4C_1)B_{\sigma+1}^{(+)}+(2C_2+C_1)B_\sigma^{(-)}],$$
$$B^{(1/2)}=C_2B_\sigma^{(-)}+2C_1B_\sigma^{(+)},\quad B^{(3/2)}=C_2B_\sigma^{(-)}-C_1B_\sigma^{(+)}, \tag{8}$$

where M is the nucleon mass, and

$$T^{(1/2),(3/2)}=\bar{u}(p')\Big\{A^{(1/2),(3/2)}(\alpha(s)-\tfrac{1}{2},\alpha(u)-\tfrac{1}{2})+\Big(\frac{\not{q}+\not{q}'}{2}\Big)B^{(1/2),(3/2)}(\alpha(s)-\tfrac{1}{2},\alpha(u)-\tfrac{1}{2})\Big\}u(p). \tag{9}$$

In the standard manner,[3] the superscripts refer to isospin in the s channel. The leading trajectory is free of ghosts if and only if,

$$C_2+2C_1<0,\quad C_1-C_2>0, \tag{10}$$

where the C's are otherwise arbitrary real parameters.

The N and Δ trajectories can be split by introducing satellite terms. Signature for the Δ trajectory, lacking in the above model, can be incorporated by adding s-t and u-t terms which contain mesons in the t channels.[3] If we use the same B functions for these terms (and we seem forced to do so), the meson channels will also contain the fixed cuts.

In fact however, the location and strength of the mesonic cuts appear to be dynamical questions. For example, using satellite terms in our functions combined with ordinary beta functions, we can push the mesonic cuts arbitrarily low. Thus it appears that theoretical calculation of meson cuts in π-N must await considerations of factorization; that is, π-N should be included in a larger bootstrap. In particular, we comment that even a large cut contribution for the pion trajectory may not be undesirable.

One last comment is in order here, before proceeding to mesons. The reader will notice that, with our functions, only the leading trajectory is obviously free of ghosts (and parity doubling). We can construct models which have $\not{p}\pm m_j$ for nonleading trajectories as well. However, this can apparently be done only for one channel at a time—the other channel picking up undesirable properties again, including fixed poles. In any case it is not clear precisely which particles on lower trajectories are ghosts, so we leave this question open for the moment.

Our next application is to the (four-point) meson bootstrap. By associating a projection operator with each quark line in the meson duality diagrams, we can project out all the parity doubling (all the ghosts) in the so-called "Born term."[4,5] Our notation will be that of Ref. 5. Let q_i be the incoming meson momenta, and define $p_s\equiv q_1+q_4$, $p_t\equiv q_3+q_4$ $(p_s^2=s,\ p_t^2=t)$. Now as a mnemonic, we first write

$$\mathrm{Tr}[\Gamma_1(q_1)\Lambda_+(p_s,m_{js})\Gamma_2(q_2)\Lambda_+(p_t,m_{jt})\Gamma_3(q_3)\Lambda_-(p_s,m_{js})\Gamma_4(q_4)\Lambda_-(p_t,m_{jt})](m_{js}m_{jt})^\kappa, \tag{11}$$

where $\Gamma_i(q_i)$ are the relevant spin factors for the incoming mesons, and

$$\Lambda_\pm(p_s,m_{js})=(1-zx)^2\frac{\pm\not{p}_s+m_{js}}{2m_{js}}+(zx)^2,\quad \Lambda_\pm(p_t,m_{jt})=z^2\frac{\pm\not{p}_t+m_{st}}{2m_{jt}}+(1-z)^2 \tag{12}$$

act as projection operators in their own channel and unity in the other.[7] m_{js} and m_{jt} are to be thought of as the jth masses in the s and t channels, respectively. κ is an arbitrary negative number, introduced so that all powers of m_{js}, m_{jt} in (11) are negative. To use the mnemonic, expand it as traces of Γ's times powers of m_{js} and m_{jt}. To realize the model, simply replace, in this sum, every term of the form $m_{js}^\sigma m_{jt}^\tau$ by $B_{\sigma\tau}^{(+)}(\alpha(s),\alpha(t))$ [with the relevant factors of $(1-zx)^2$, etc.]. The construction is evidently quite arbitrary, using subsidiary terms, and points to the need for systematic study of factorization. Our point here is only that models <u>can</u> be constructed.

To see that all parity doubling has been projected out, we factor the traces as in Ref. 5. Because of the projection operators, the relevant identity, say at a pole in s, is

$$\mathrm{Tr}[\Lambda_-(p_s,m_{js})A\Lambda_+(p_s,m_{js})B]=\mathrm{Tr}[A\pi_j(p_s)]\mathrm{Tr}[\pi_j(-p_s)B]+\mathrm{Tr}[A\rho_{j\mu}(p_s)]\mathrm{Tr}[\rho_j^{\mu}(-p_s)B], \tag{13}$$

where

$$\sqrt{2}\pi_j(q) = \gamma_5 + \gamma_5 \not{q}/m_j,$$
$$\sqrt{2}\rho_j(q) = [\gamma_\mu - q_\mu \not{q}/m_j^2] - i\sigma_{\mu\nu}q^\nu/m_j, \tag{14}$$

and A and B are arbitrary matrices in the Dirac space. Factorization in t is identical. The model is entirely free of parity doubling (ghosts), on the leading trajectories; in fact, at $l=0$, there remains only one ρ and one π—whose spin factors are (14). As in Ref. 4, the A_1 (etc.) is at $l=1$. Internal symmetry considerations may be taken unchanged.

In the language of conspiracy theory, we have then seen that (quark) projection operators can remove parity doubling from $M=1$ (meson) conspiracies, as well as from $M=\frac{1}{2}$ (baryon) conspiracies.

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